International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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On a flower-shape geometry

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Several important problems arising in many research fields, such as physics and differential geometry, lead to consider elliptic equations when a lack of compactness occurs. From the mathematical point of view, the main interest relies on the fact that often the tools of nonlinear functional analysis, based on compactness arguments, cannot be used, at least in a straightforward way, and some new techniques have to be developed.

Aim of the talk is to present some of these techniques, which strongly use symmetry, together with their applications to elliptic problems with a variational structure. In particular we deal with a group theoretical scheme, raised in the study of problems which are invariant with respect to the action of orthogonal subgroups, and we present a construction, called flower-shape geometry, and its applications to the study of nonlinear problems set in strip-like domains. These results appeared in a joint paper with Giuseppe Devillanova (Politecnico di Bari) and Giovanni Molica Bisci (Urbino).

On equicontinuity of families of mappings with one normalization condition by the prime ends

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Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, abbr. $\rho \in \operatorname{adm}\Gamma$, if $\int\limits_{\gamma} \rho(x)|dx| \geqslant 1$ for each (locally rectifiable) $\gamma \in \Gamma$. We define the quantity

$$M(\Gamma) = \inf_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^n(x)\, dm(x) \qquad (1)$$

and call $M(\Gamma)$ *a modulus* of Γ; here m stands for the n-dimensional Lebesque measure, see [1, 6.1].

Given sets E and F and a domain D in $\overline{\mathbb{R}^n} = \mathbb{R}^n \cup \{\infty\}$, we denote $\Gamma(E, F, D)$ the family of all paths $\gamma : [0, 1] \to \overline{\mathbb{R}^n}$ joining E and F in D, that is, $\gamma(0) \in E$, $\gamma(1) \in F$ and $\gamma(t) \in D$ for all $t \in [0, 1]$.

An *end* of a domain D is an equivalence class of chains of cross-cuts of D. We say that an end K is a *prime end* if K contains a chain of cross-cuts $\{\sigma_m\}$, such that

$$\lim_{m\to\infty} M(\Gamma(C,\sigma_m,D))=0$$

for some continuum C in D. Set $\mathbb{B}^n := \{x\in\mathbb{R}^n : |x|<1\}$. We say that the boundary of a domain D in $\mathbb{R}^n$ is *locally quasiconformal* if every point $x_0\in\partial D$ has a neighborhood U that admit a conformal mapping φ onto the unit ball $\mathbb{B}^n\subset\mathbb{R}^n$ such that $\varphi(\partial D\cap U)$ is the intersection of $\mathbb{B}^n$ and a coordinate hyperplane, see e.g. [2], cf. [3]. We say that a bounded domain D in $\mathbb{R}^n$ is *regular* if D may be mapped quasiconformally onto a bounded domain with a locally quasiconformal boundary. If $\overline{D}_P$ is the completion of a regular domain D by its prime ends and g_0 is a quasiconformal mapping of a domain D_0 with locally quasiconformal boundary onto D, then this mapping naturally determines the metric $\rho_0(p_1,p_2)=\left|\widetilde{g_0}^{-1}(p_1)-\widetilde{g_0}^{-1}(p_2)\right|$, where $\widetilde{g_0}$ is the extension of g_0 onto $\overline{D_0}$. Let $x_0\in\overline{D}$, $x_0\neq\infty$, $S(x_0,r)=\{x\in\mathbb{R}^n : |x-x_0|=r\}$, $A=A(x_0,r_1,r_2)=\{x\in\mathbb{R}^n : r_1<|x-x_0|<r_2\}$.

Let $f:D\to\mathbb{R}^n$, $n\geqslant 2$, and let $Q:\mathbb{R}^n\to[0,\infty]$ be a Lebesgue measurable function such that $Q(y)\equiv 0$ for $y\in\mathbb{R}^n\setminus f(D)$. Let $A=A(y_0,r_1,r_2)$ and let $\Gamma_f(y_0,r_1,r_2)$ denotes the family of all paths $\gamma:[a,b]\to D$ such that

$$f(\gamma)\in\Gamma(S(y_0,r_1),S(y_0,r_2),A(y_0,r_1,r_2))\,,$$

i.e., $f(\gamma(a))\in S(y_0,r_1)$, $f(\gamma(b))\in S(y_0,r_2)$, and $f(\gamma(t))\in A(y_0,r_1,r_2)$ for any $a<t<b$.

We say that, f *satisfies the inverse Poletsky inequality* at a point $y_0\in f(D)$, if the relation

$$M(\Gamma_f(y_0,r_1,r_2))\leqslant \int\limits_{f(D)\cap A(y_0,r_1,r_2)} Q(y)\cdot\eta^n(|y-y_0|)\,dm(y) \qquad (2)$$

holds for any Lebesgue measurable function $\eta:(r_1,r_2)\to[0,\infty]$ satisfying the relation

$$\int\limits_{r_1}^{r_2}\eta(r)\,dr\geqslant 1\,. \qquad (3)$$

We say that the boundary of D is *weakly flat* at a point $x_0\in\partial D$ if, for every number $P>0$ and every neighborhood U of the point x_0, there is a neighborhood $V\subset U$ such that $M(\Gamma(E,F,D))\geqslant P$ for all continua E and F in D intersecting ∂U and ∂V. We say that the boundary ∂D is weakly flat if the corresponding property holds at every point of the boundary.

Given domains $D,D'\subset\mathbb{R}^n$, $n\geqslant 2$, points $a\in D$, $b\in D'$ and a Lebesgue measurable function $Q:D'\to[0,\infty]$ denote $\mathfrak{S}_{a,b,Q}(D,D')$ a family of all open discrete and closed mappings f of D onto D', satisfying the relation (2) for any $y_0\in D'$, while $f(a)=b$. The following statement holds.

Theorem 1. *Assume that, D has a weakly flat boundary, any component of which does not degenerate into a point. If $Q\in L^1(D')$ and D' is regular, then any $f\in\mathfrak{S}_{a,b,Q}(D,D')$ has a continuous extension $\overline{f}:\overline{D}\to\overline{D'}_P$, $\overline{f}(\overline{D})=\overline{D'}_P$, and, in addition, a family $\mathfrak{S}_{a,b,Q}(\overline{D},\overline{D'})$ which consists of all extended mappings $\overline{f}:\overline{D}\to\overline{D'}_P$, is equicontinuous in $\overline{D}$.*

The result mentioned above is published in [4].

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Equiaffine immersions of codimension two with flat connection

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We consider the affine immersions by K. Nomizu, T. Sasaki [1], namely $f : (M^n, \nabla) \to \mathbb{R}^{n+2}$. For a transversal frame $\xi_1,\ \xi_2$ and tangent vector fields $X,\ Y$ we have the affine analogues of Gauss and Weingarten decompositions, namely

$$D_X f_*(Y) = f_*(\nabla_X Y) + h^\alpha(X,\ Y)\xi_\alpha,$$

$$D_X \xi_\alpha = -f_*(S_\alpha X) + \tau_\alpha^\beta(X)\xi_\beta,$$

where h^α are components of the affine fundamental form, S_α are shape operators, τ_α^β are forms of transversal connection (with respect to $\xi_1,\ \xi_2$).

The *Weingarten mapping* $S_x : Q_x \times T_x(M^n) \to T_x(M^n)$ is defined [2] as follows: $(\xi, X) \mapsto S_\xi X$ at every point $x \in M^n$ (where $T_x(M^n)$ and Q_x are tangent and transversal distributions.)

For an affine immersion $f : (M^n, \nabla) \to \mathbb{R}^{n+2}$ with a transversal frame $\{\xi_1,\ \xi_2\}$, an *induced volume element* θ on M^n is defined [1, 3, 4] as follows:

$$\theta(X_1, \dots, X_n) = \det(f_*(X_1), \dots, f_*(X_n), \xi_1, \xi_2).$$

The transversal distribution Q with frame $\{\xi_1,\ \xi_2\}$ is called *equiaffine* if $\nabla_X \theta = 0$ for all $X \in T_x(M^n), x \in M^n$. For two-codimension affine immersion this condition is equivalent [4] to

$$\tau_1^1(X) + \tau_2^2(X) \equiv 0.$$

With an equiaffine transversal distribution Q we have an *equiaffine structure* $(\nabla,\ \theta)$ on M^n.

We will consider an affine immersion $f : (M^n, \nabla) \to \mathbb{R}^{n+2}$ with flat connection ∇ and equiaffine transversal distribution. Two-codimensional affine surfaces with different additional properties have been studied by many authors. Flat affine surfaces in $\mathbb{R}^4$ with flat normal connection were studied in [3]. The description of a parallel affine immersions $(M^n, \nabla) \to \mathbb{R}^{n+2}$ with flat connection in dependence on the rank of the Weingarten mapping were given in [2].

Let us remind that in general case (codimension k) the kernel and the image of the Weingarten mapping is defined by $\ker S = \bigcap\limits_{\alpha=1}^{k} \ker S_\alpha,\ \text{im } S = \bigcup\limits_{\alpha=1}^{k} \text{im } S_\alpha$. We say that Weingarten mapping is p-dimensional if $\text{rank} S := \dim \text{im} S = p$. It was proved [6] that for the immersion