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Order continuity properties of lattice ordered algebras

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First, we give some fundamental notions.

Definition 1. A linear ordering of a real linear space X is an ordering satisfying these conditions:

(1) $x \leq y$ implies $x + z \leq y + z$, for all x, y, z in X,
(2) $x \leq y, \lambda \geq 0$ implies $\lambda x \leq \lambda y$.

Definition 2. An ordered linear space is a linear space with a linear ordering. Let A be an algebra with the unit e and A^+ be positive cone of A. For elements x, y of A $x \leq y$ means $x - y \in A^+$. A is an ordered linear space with this ordering.

Definition 3. If $xy \geq 0$ whenever $x \geq 0, y \geq 0$, then A is called an ordered algebra. If A is a Banach algebra with a closed cone A^+, then A is called an ordered Banach algebra.

Definition 4. If A is a real vector lattice and is associative but not necessarily commutative or unital algebra such that the multiplication and the partial ordering in A are compatible, i.e. $x, y \in A^+ \Rightarrow xy \in A^+$, then A is called a lattice-ordered algebra($l-$algebra).

Definition 5. An $l-$algebra is called

(1) a $d-$algebra whenever the multiplications by positive elements are lattice (Riesz) homomorphisms of A, that is, $(x \vee y)z = xz \vee yz$ and $z(x \vee y) = zx \vee zy$ for all $x, y \in A, z \in A^+$.
(2) an almost $f-$algebra if $x \wedge y = 0$ implies $xy = 0$.
(3) an $f-$algebra if $x \wedge y = 0$ implies $xz \wedge y = zx \wedge y = 0$ for all $z \in A^+$.

In this work, we mainly deal with lattice ordered algebras such as $f-$algebras, $d-$algebras and almost $f-$algebras and their properties.

References

[1] Y.A. Abramovich, C.D. Aliprantis An Invitation to Operator Theory. *Graduate Studies in Mathematics*, vol.50. American Mathematical Society, Providence, 2002.
[2] C.D. Aliprantis, Owen Burkinshaw. *Positive Operators.*, Springer, Dordrecht, 2006.
[3] H. G. Dales. *Banach Algebras and Automatic Continuity.*, *London Mathematical Society Monographs, New Series*, vol.24, Oxford University Press, Oxford, 2004.
[4] M. Messerschmidt. *Normality of spaces of operators and quasi-lattices.*,*Positivity*, 19(4), 695–724,2015.
[5] S. Mouton, H. Raubenheimer. *More spectral theory in ordered Banach algebras.*, *Positivity*, 1(4), 305–317, 1997.
[6] H. Raubenheimer, S. Rode. *Cones in Banach algebras.*, Indag. Math. N.S. 7(4), 489–502, 1996.
[7] Ch. E. Rickart. *General Theory of Banach Algebras.*, Van Nostrand, Princeton, 1974.
[8] A. C. Zaanen. *Riesz Spaces II.*, North-Holland, Amsterdam, 1983.
[9] E. A. Alekhno. *The order continuity in ordered algebras.*, Positivity 21(2), 539–574, 2017.
[10] C. B. Huijsmans. *Lattice-Ordered Algebras and f-Algebras: A Survey. In: Positive Operators, RieszSpaces, and Economics.*, *Studies in Economic Theory*, vol.2. Springer, Berlin, Heidelberg, 1991.

Зміст