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Algebraic and Geometric Methods of Analysis

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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

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ІНТЕРНАЦІОНАЛЬНИЙ ЦЕНТР СПІВРОБОТИ

Trace Regularization Problem On a Banach Space

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Let \mathcal{H} be a separable Hilbert space and let $S_1[\mathcal{H}]$ be the trace class operators on \mathcal{H} (First Schatten Class, [3]).

Consider $\mathcal{H}_1 = L^2(\mathcal{H}; [0, \pi])$ and define an inner product on \mathcal{H}_1 by:

$$(f, g)_{\mathcal{H}_1} = \int_0^\pi (f(t), g(t))_{\mathcal{H}} dt$$

for all $f, g \in \mathcal{H}_1$.

- With this inner product, \mathcal{H}_1 is also a separable Hilbert space.

Here, we study the same problem in [2], with \mathcal{H} replaced by a arbitrary separable Banach space \mathcal{B} , under the following conditions:

- (1) $Q(t)$ has a weak second-order derivative in $[0, \pi]$ and for $t \in [0, \pi]$, $Q^{(i)}(t)$ ($i = 0, 1, 2$) is a self adjoint trace class operator on \mathcal{B} .
- (2) $\|Q\|_{\mathcal{H}_1} < 1$.
- (3) \mathcal{H}_1 has an o.n.b. $\{\varphi_n\}_{n=1}^\infty$ such that $\sum_{n=1}^\infty \|Q\varphi_n\|_{\mathcal{H}_1} < \infty$.
- (4) $\|Q^i(t)\|_{S_1[\mathcal{B}]}$ ($i = 0, 1, 2$) is a bounded measurable function on $[0, \pi]$.

It is clear from (4), that this is a nontrivial problem since, among other things, in the standard approach, there are a number of possible definitions of $S_1[\mathcal{B}]$ (see [3] and Pietsch [9]).

We assume that \mathcal{B} is a continuous dense embedding in a separable Hilbert space \mathcal{H} and for each $f, g \in \mathcal{B}$, $(f, g)_h = (f, g)_{\mathcal{H}}$ is the Hilbert functional on \mathcal{B} .

Theorem 1 (Polar Representation Theorem). *Let \mathcal{B} be a separable Banach space. If $A \in \mathbb{C}[\mathcal{B}]$, then there exists a partial isometry U and a self-adjoint operator T , with $D(T) = D(A)$ and $A = UT$. Furthermore, $T = [A^*A]^{1/2}$ in a well-defined sense.*

Def. If $A \in S_1[\mathcal{B}]$, we called it a trace class (or nuclear) operator on \mathcal{B} .

* Since $S_p[\mathcal{H}]$ is a two sided *ideal, it follows that the same is true for $S_p[\mathcal{B}]$.

* For $1 \leq p < \infty$, $A \in S_p[\mathcal{B}]$ and $B \in \mathcal{L}[\mathcal{B}]$ then $AB, BA \in S_p[\mathcal{B}]$ and

$$\|AB\|_{S_p[\mathcal{B}]} \leq \|B\|_{\mathcal{L}[\mathcal{B}]} \|A\|_{S_p[\mathcal{B}]}$$

$$\|BA\|_{S_p[\mathcal{B}]} \leq \|B\|_{\mathcal{L}[\mathcal{B}]} \|A\|_{S_p[\mathcal{B}]}$$

Lemma 2. *If $\lambda \notin \sigma(L_0)$ then $QR_0(\lambda) \in S_1[\mathcal{H}_1]$*

Lemma 3. *The operator valued function $R(\lambda) - R_0(\lambda)$ is analytic in $\rho(L)$, the resolvent set of L , with respect to the $S_1[\mathcal{H}_1]$ norm.*

Theorem 4. *The regularized trace formula for operator L on B with the conditions on operator function $Q(t)$ is given by*

$$\sum_{m=0}^{\infty} \left[\sum_{n=1}^{\infty} (\lambda_{mn} - \mu_m) - \frac{1}{\pi} \int_0^\pi \text{tr} Q(t) dt \right] = \frac{1}{4} [\text{tr}(Q(0)) + \text{tr} Q(\pi)]$$

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