International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

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Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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in Geometry by H. T. Croft, K. J. Falconer and R. K. Guy. We will present F. Nazarov's proof of Tóth's inequality and discuss its higher-dimensional analogues.

Gottlieb groups of some Moore spaces

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In this work, we present some computations of Gottlieb groups of Moore spaces $M(A,n)$ for some classes of finitely generated abelian groups A.

Given $m \geq 1$, recall that the m-th *Gottlieb group* $G_m(X)$ of a space X has been defined in [4, 5] as the subgroup of the homotopy group $\pi_m(X)$ consisting of all elements which can be represented by a map $f\colon \mathbb{S}^m \to X$ such that $f \vee \iota_X\colon \mathbb{S}^m \vee X \to X$ extends (up to homotopy) to a map $F\colon \mathbb{S}^m \times X \to X$. Notice that $\alpha \in G_m(\Sigma X)$ if and only if the generalized Whitehead product $[\alpha, \iota_{\Sigma X}] = 0$ (see [1, Proposition 5.1]).

First, we recall from [5, Theorems 5.2 and 5.4]:

Theorem 1. *Let A be a finitely generated abelian group and $n \geqslant 3$. Then,*

$$G_n(M(A,n)) = \begin{cases} 0, & \text{if } n \text{ is even,} \\ 0, & \text{if } n \text{ is odd and } \operatorname{rk}(A) \neq 1, \\ 2\mathbb{Z} \subseteq \mathbb{Z} = \pi_n(\mathbb{S}^n), & \text{if } n \neq 1,3,7 \text{ is odd and } A = \mathbb{Z}, \\ \mathbb{Z} = \pi_n(\mathbb{S}^n), & \text{if } n = 1,3,7 \text{ and } A = \mathbb{Z} \end{cases}$$

We point out that the result above has been stated also in [2] for $n \geqslant 3$. In addition, [2, Corollary 4.4] claims that if n is odd, then $G_n(M(\mathbb{Z} \oplus T, n))$ is infinite cyclic, where T is a finite abelian group.

As stated in [2, Remark 4.5], it would be interesting to compute other Gottlieb groups for some Moore spaces, such as $G_{n+1}(M(A,n))$. We will do this for a finitely generated abelian group A which its torsion subgroup has order 2 (mod 4). We notice that on [3, Chapter 3] there are some results on $G_{n+1}(M(A,n))$ only for A having torsion subgroup with odd order.

Our main result is:

Theorem 2. *Let A be a finite abelian group with order $|A| \equiv 2 \pmod 4$. Then $G_{n+1}(M(\mathbb{Z} \oplus A, n)) = 0$, for $n \geq 3$, and $G_{n+2}(M(\mathbb{Z} \oplus A, n)) = 0$, for $n \geq 4$.*

Furthermore, investigations of $G_{n+k}(M(\mathbb{Z} \oplus A, n))$ for $k = 3, 4, 5$ and A as above, is planned as well.

References

[1] Martin Arkowitz. The generalized Whitehead product. *Pacific J. Math.* 12 : 7–23, 1962.
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[3] Marek Golasiński and Juno Mukai. *Gottlieb and Whitehead center groups of spheres, projective and Moore spaces*, Springer, 2014.
[4] Daniel H Gottlieb. A certain subgroup of the fundamental group. *American Journal of Mathematics*, 87(4) : 840–856, 1965.
[5] Daniel H Gottlieb. Evaluation subgroups of homotopy groups. *American Journal of Mathematics*, 91(3) : 729–756, 1969.

Inner semi-continuity of medial axes and conflict sets

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A central notion in pattern recognition is that of the *medial axis* M_X of a closed, nonempty, proper subset $X \subset \mathbb{R}^n$. Namely, M_X consists of all those points $a \in \mathbb{R}^n$ for which there is more than one closest point (with respect to the Euclidean distance $d(a, X)$) in X:

$$M_X := \{a \in \mathbb{R}^n \mid \#m(a) > 1\} \text{ where } m(a) := \{x \in \mathbb{R}^n \mid ||a - x|| = d(a, X)\}.$$

The definition goes back to H. Blum (cf. [3]) who gave it for $X = \partial D$ where $D \subset \mathbb{R}^n$ is a bounded domain. Then, knowing the 'skeleton' $M_X \cap D$ and $d(\cdot, X)|_{M_X}$ ('compressed data') one can reconstruct the 'shape' D.

The medial axis has long been known for being highly unstable (cf. e.g. [4]): the smallest deformation of X may lead to an important change in M_X (think of X as a circle in the plane — M_X is its centre, while the same circle but now with the smallest $\mathscr{C}^\infty$ protuberance yields a medial axis that is a segment). However, this point of view has a flaw — it sees the modification as through a blackbox, there is an initial state and a final one with nothing in between.

Our aim is to provide the right setting for considering the deformation of X which is the *(Painlevé)-Kuratowski convergence of closed sets* and to show in this case the inner-semicontinuity of the medial axis. The most general result we have, and one that turns out ot be optimal already in $\mathbb{R}^n$, can be stated as follows:

Theorem 1. *Let $\mathcal{M}$ be a connected complete Riemannian manifold and Π a T_1 topological space of parameters with a distinguished non-isolated point 0 having a countable basis of*

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