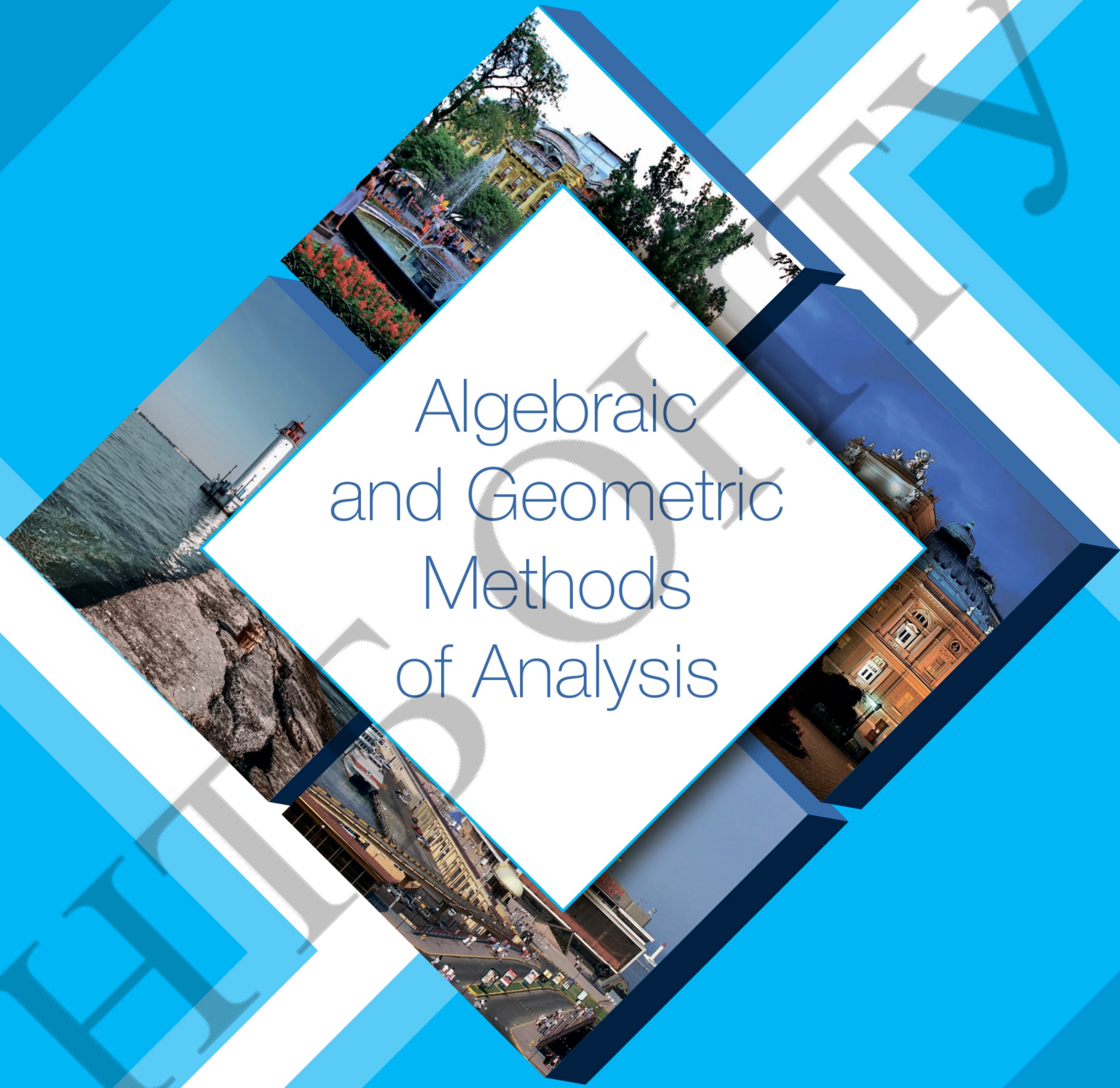
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Notes on the Quality of Non-compactness for Non-compact Sobolev Embeddings

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It is well known that when a Sobolev space on a bounded domain is embedded into the smallest possible Lebesgue or Lorentz space, the resulting embedding is non-compact. In this talk, we will closely examine non-compact Sobolev embeddings and describe the quality of their non-compactness from different points of view.

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Ordinary linear differential operators and connections. Application to curvilinear webs

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The framework is real analytic or holomorphic, the field $\mathbb{K}$ denoting $\mathbb{R}$ or $\mathbb{C}$ according to this framework.

We are given two vector bundles E and F, respectively of rank p and q, over a n-dimensional manifold V. We assume $q < p(n+1)$ (the rank of F is smaller than the rank of J^1E). A linear homogeneous differential operator of order one[1] is a linear morphism of vector bundles $D\colon J^1E \to F$. Associated to it is the partial order equation

$$(*) \quad \mathcal{D}s = 0, \text{ where we set, for any section } s \text{ of } E, \ \mathcal{D}s := D(j^1 s).$$

[1] The theory works also for differential operators of higher order.

The restriction $\sigma_1(D) : T^*(V) \otimes E \to F$ of D to the kernel $T^*(V) \otimes E$ of the projection $J^1E \to E$ is called the principal symbol of D.

More generally, for any integer h (≥ 1), we define by successive derivations the $(h-1)^{th}$ prolongation $D_h : J^hE \to J^{h-1}F$ of D : the solutions of $(*)$ are still the sections s of E such that $D_h(j^hs) = 0$. Recalling, the exact sequence

$$0 \to S^hT^*(V) \otimes E \to J^hE \to J^{h-1}E \to 0,$$

where $S^hT^*(V)$ denotes the h^{th} symetric power of the bundle $T^*(V)$ of 1-forms, we call *principal symbol* of D_h the restriction

$$\sigma_h(D) : S^hT^*(V) \otimes E \to J^{h-1}F$$

of D_h to the sub-bundle $S^hT^*(V) \otimes E$ of J^hE.

We denote by

$$c(n,h) := \frac{(n-1+h)!}{(n-1)!\ h!}$$

the dimension of the $\mathbb{K}$-vector space of homogeneous polynomials of degree h with n unknowns and coefficients in $\mathbb{K}$, which is also the rank of $S^hT^*(V)$, or the number of multi-indices $I = (i_1, ..., i_n)$ of partial derivatives of order $|I| = h$ with respect to n unknowns, (where $|I| = i_1 + i_2 + ... + i_n$).

Definition 1. The differential operator D is said to be ordinary if $q \leq pn$ and, for any h ($h \geq 1$), the principal symbol $\sigma_h(D)$ has maximal rank $(inf(q.c(n,h-1), p.c(n,h)))$

If D is ordinary, the kernel R_h of D_h is a vector bundle, which is the set of the formal solutions of $(*)$ at order h $\big($with $R_{h+1} = J^1R_h \cap J^{h+1}E$, the intersection being in $J^1(J^hE)\big)$.

Definition 2. The differential operator D is said to be calibrated if $\frac{p(n-1)}{q-p}$ is an integer, strictly positive.

This implies in particular $p < q$.

If $p < q$, we have only finitely many conditions to check for D to be ordinary. In fact, set :

$$h_0 := \Big[\frac{p(n-1)}{q-p}\Big], \quad \text{(the integral part of } \tfrac{p(n-1)}{q-p}\text{)}.$$

We then get:

Proposition 3. *For D to be ordinary when $p < q \leq p.n$, it is sufficient that the principal symbols $\sigma_h(D)$ have their rank maximal for $1 \leq h \leq h_0+1$ in general (resp. for $1 \leq h \leq h_0$ if D is moreover calibrated).*

We first prove:

Theorem 4. *If D is ordinary and $p < q \leq p.n$, the dimension of the vector space $\mathcal{S}_m$ of germs of solutions of the equation $\mathcal{D}s = 0$ at a point m of V is upper-bounded by the number*

$$\sum_{h=0}^{h_0} \binom{n-1+h}{h} . \frac{p(n-1)-(q-p)h}{n-1+h} \quad \Big(= p.c(n+1,h_0) - q.c(n+1,h_0-1)\Big).$$

If D is moreover, calibrated, we define a tautological connection on the vector bundle $\mathcal{E} := R_{h_0-1}$, such that

Theorem 5. *The space $\mathcal{S}$ of solutions of $(*)$ is isomorphic to the space of sections σ of $\mathcal{E}$ whose covariant derivative $\nabla\sigma$ vanishes. Hence, the dimension of $\mathcal{S}_m$ is maximal iff the curvature of this connection vanishes.*

We then apply these results by building, for any curvilinear d-web on V $(d > n)$, a linear differential operator which is *always ordinary and calibrated*, and for which *solutions of* $(*)$ *are the* $(n-1)$-*abelian relations* of the web. After theorem 1, we recover the upper-bound already given by Damiano ([3]), for the rank of the web (dimension of the space of $(n-1)$-abelian relations), by taking $p = d - n$ and $q = d - 1$ (hence $h_0 = d - n$).

As a corollary of Theorem 5, we can define for any curvilinear web a notion of "curvature", and prove :

Theorem 6. *The Damiano's upper-bound for the rank of a curvilinear web is reached iff its curvature vanishes.*

The main interest of this result is that it is no more necessary to exhibit the abelian relations for proving the maximality of the rank.

Such a definition for the curvature of a web, whose vanishing is equivalent to the maximality of the rank, goes back to Blaschke ([1]) in the case $n = 2, d = 3$. Various generalizations have been done since, mainly for planar webs ([6, 8, 9]), for webs of codimension one ([2, 4]) when they are "ordinary", and for d-curvilinear webs when $d = n + 1$ ([5]). The definition that we give below for d-curvilinear webs whatever be d is new ; it has been announced in a preprint ([7]).

As an example, we prove (using a computation with Maple) that *the curvature of the exceptional* 6-*web* $W_{0,6}$ *in dimension 3 vanishes*, hence recovering that it has has a maximal rank (as well as his 4 and 5-subwebs). In fact all $n+3$-webs $W_{0,n+3}$ (that we re-defined here in words of vector fields) have a maximal rank : this has been claimed by Damiano ([3]) for any n, and proved by him for n even. He made a mistake in the proof for n odd, which has been corrected by Pirio ([10]).

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