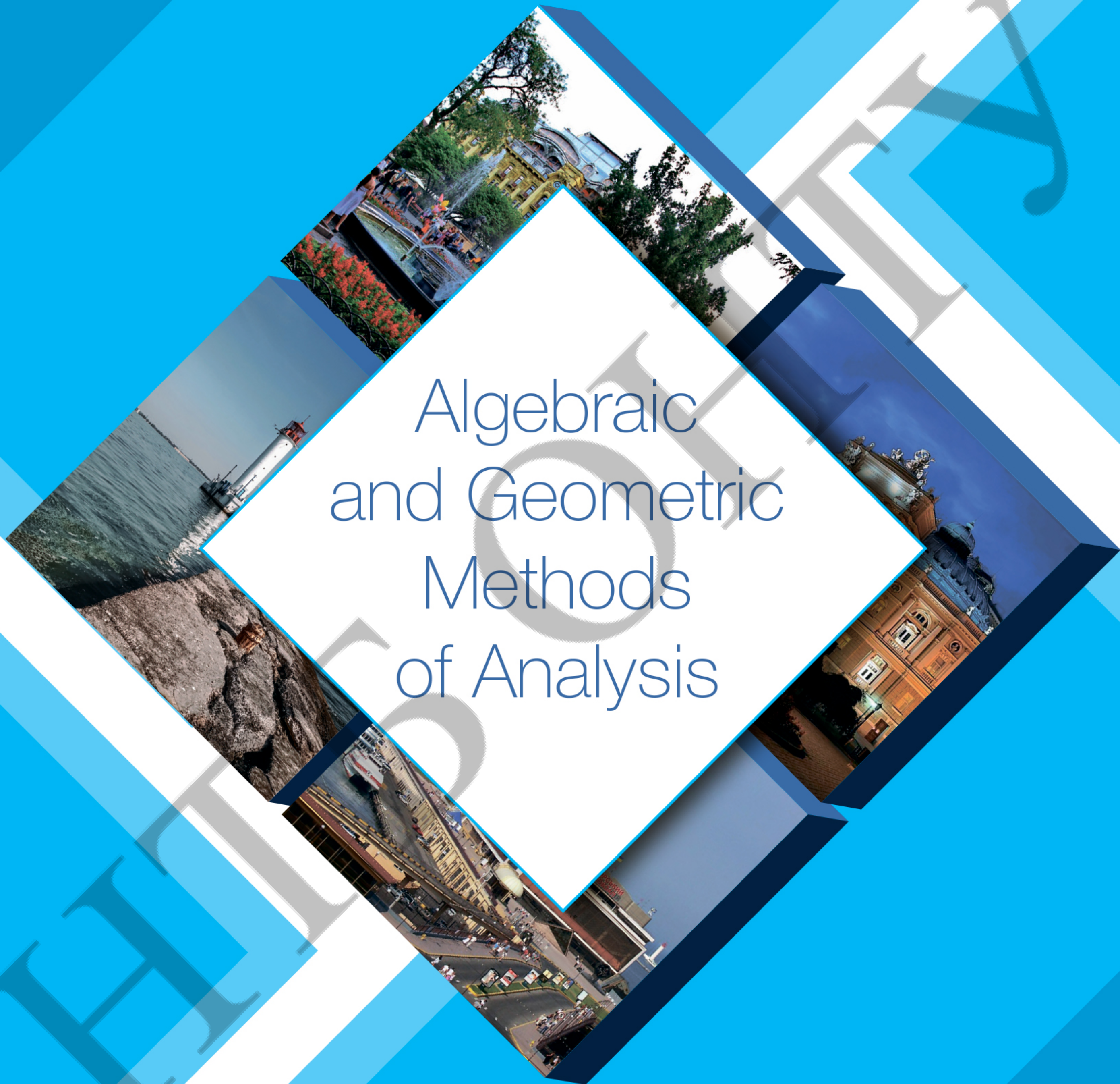
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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The volume entropy h_v is always bounded above by the topological entropy h_{top} of the geodesic flow on M. Moreover, if M has non-positive sectional curvature, then

$$h_v = h_{top}.$$

From the other hand we know, that

$$h_{top} \geq log|deg(f)|.$$

As the fixed point theorem was proved in 1912 by Brouwer [3], so any continuous mapping of a sphere onto itself has isolated point. Mapping of spaces ($R^{2d} \to R^{2d}$) for ($d = 1, \dots, n$) can be presented by $f : S^{2d} \to S^{2d}$ and determines the degree of mapping $degf = 2d$. Therefore, topological entropy of the system of n links of string length, $L = l_s n$, is the n sum,

$$h_{top} = n \cdot log(2d).$$

The Lefschetz number of one link

$$L(f^n) = 1 + (-1)^m degf^n$$

is equal to

$$L(f^n) = 1 + (2d)^n.$$

So, according to the Lefschetz formula we can calculate the index of isolated point on manifold M

$$L(f) = \sum ind_f x.$$

Considering the Hopf-Poincare theorem

$$\sum ind_f x = \chi(M)$$

we can receive the following formula

$$1 + ((2d)^n)^n = \chi(M).$$

Thus, using the theory of dynamical systems, we calculated the entropy, Lefschetz number and Euler characteristic of a black hole, represented as a multidimensional cubic space.

References

[1] Susskind Lectures on String Theory. July 10, 2022 version 2.1. URL: https: www.umsl.edu/∼ *majzoube/docs/string_theory_basics_v2.2.pdf*.

[2] Katok A., Hasselblatt B. Introduction to the modern theory of dynamical systems. Cambridge University Press 1995, 824 p.

[3] Brouwer L.E.J. Uber Abbildung von Mannigfaltigkeiten. *Mathematische Annalen*, Volume: 71, page 97-115, 1912.

Computing the Gromov–Hausdorf distance using gradient methods

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The Gromov–Hausdorff distance measures the difference in shape between metric spaces, which has been used in image processing and data analysis. Computing it can be formulated as a generalization of the NP-hard quadratic assignment problem without the bijectivity constraint. Continuous relaxations that are more tractable, such as the Gromov–Wasserstein distance, exist but fail to recover non-bijective solutions to the original problem. As a consequence, the approximation factor of the current relaxation algorithms is infinite in the general case.

We introduce a quadratic relaxation whose solutions provably deliver the Gromov–Hausdorff distance. The optimality guarantee is enabled in part by allowing non-bijections in the search space. We suggest a gradient method for approximating solutions to this relaxation, and show that it can efficiently compute the Gromov–Hausdorff distance between metric spaces of hundreds of points.

Magnetic trajectories on 2-step nilmanifolds

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From the mechanical perspective, the behaviour of a charged particle in presence of a force, known as Lorentz force, is described by an equation of the form:

$$\nabla_{\gamma'}\gamma' + F\gamma',$$

where γ is a differentiable curve on Riemannian manifold (M, g), ∇ is the corresponding Levi-Civita connection and F is a skew-symmetric $(1,1)$-tensor such that the corresponding 2-form $g(F\cdot,\cdot)$ is closed. Geodesics are the corresponding curves whenever $F \equiv 0$, that means that the particles do not experience any force. The magnetic trajectories are quite different from geodesics. For instance in the Euclidean plane, while geodesics are straight lines, magnetic trajectories are circles.

In this work, we concentrate the attention to magnetic trajectories associated to a left-invariant Lorentz force on a given 2-step nilpotent Lie group equipped with a left-invariant metric $(N, \langle\,,\,\rangle)$ and the associated compact quotients. The main purposes are:

(i) to describe the solutions of the magnetic equation;
(ii) to determine closedness conditions of magnetic trajectories on compact quotients.

To facilitate the description of magnetic trajectories through the identity element, we make use of the structure of the Lie algebra. Any 2-step nilpotent Lie algebra with a metric admits an orthogonal decomposition

$$\mathfrak{n} = \mathfrak{z} \oplus \mathfrak{v}, \qquad \mathfrak{v} = \mathfrak{z}^{\perp},$$

where $\mathfrak{z}$ denotes the center of $\mathfrak{n}$.

One proves that magnetic trajectories for left-invariant Lorentz forces preserving the decomposition above can be explicitly computed, see [1]. In other cases, the magnetic curves obey different features.

We shall show examples of magnetic trajectories. In particular on the Heisenberg Lie group of dimension three, one has examples related to elliptic integrals. As usual, once one finds the curves on the Lie group, one may induce them to compact quotients. In this situation, one search for closed magnetic trajectories.

On the other hand, we discuss obstructions to the existence of (left-invariant) Lorentz forces on 2-step nilpotent Lie algebras.

References

[1] G. Ovando, M. Subils. Magnetic Trajectories on 2-Step Nilmanifolds. *J. Geom. Anaysis*, 33, Art. 186, 2023.
[2] G. Ovando, M. Subils. Magnetic fields on non-singular 2-step nilpotent Lie groups, *J. Pure and Appl. Algebra*, 228 (6), Art. 107609, 2024.