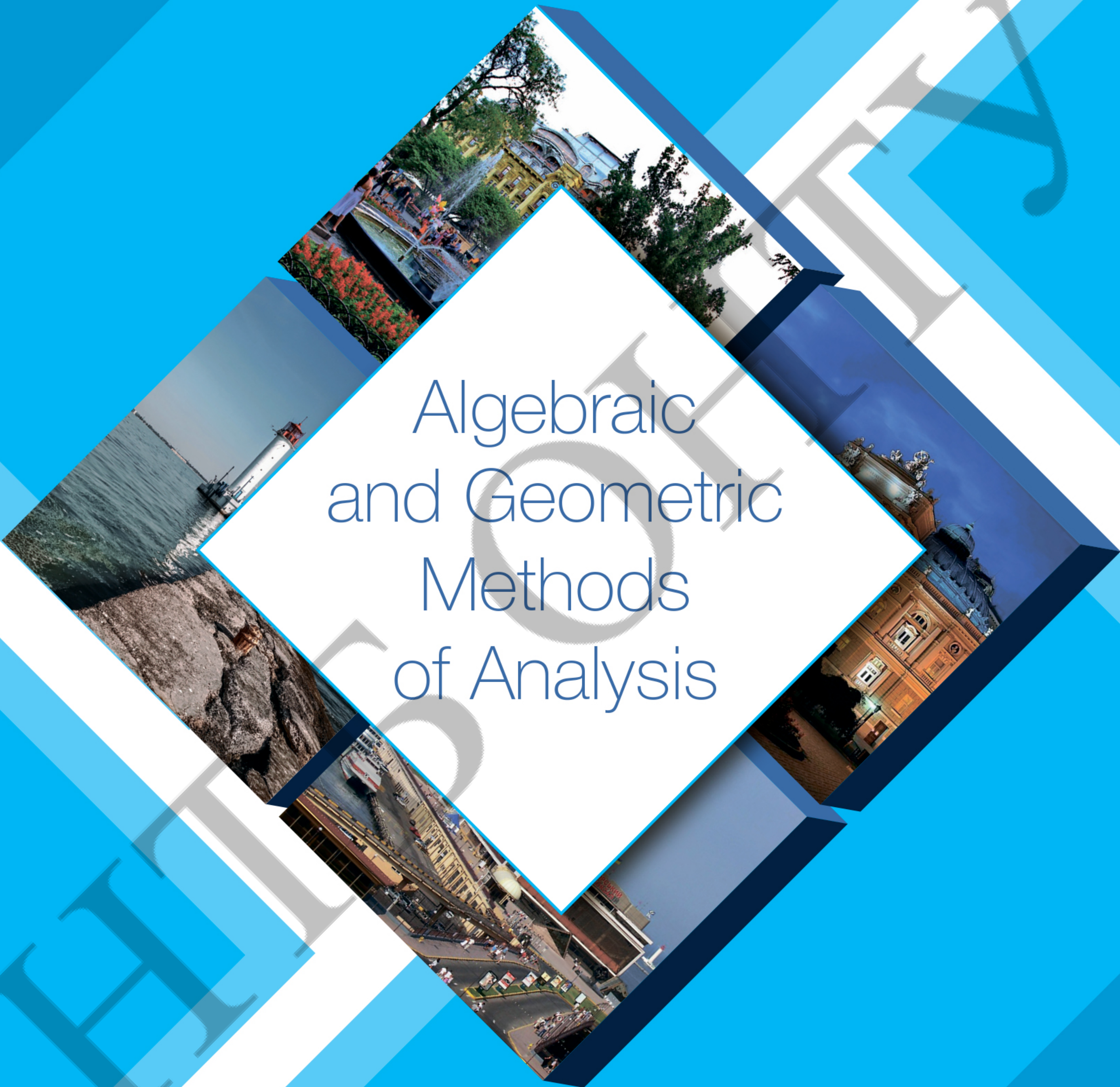
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Self-similar actions of the fundamental group of the Klein bottle

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A virtual endomorphism of a group G is a homomorphism of the form $\phi : H \to G$, where $H < G$ is a subgroup of finite index. A virtual endomorphism $\phi : H \to G$ is called simple if there are no nontrivial normal ϕ-invariant subgroups.

A recursive construction using a simple virtual endomorphism ϕ produces a so-called self-similar action of the group G on a d-regular rooted tree X^*. The X^* represents words over the alphabet X of size d. In general, a faithful action of a group G on rooted tree X^* is said to be *self-similar* if for every $g \in G$ and every $x \in X$ there exists unique pair $g|_x \in G$ and $y \in X$ such that $g(xw) = yg|_x(w)$. A self-similar action is called *self-replicating* if the associative simple virtual endomorphism ϕ is surjective. One can find more information regarding self-similar actions in [1].

Consider the fundamental group of the Klein bottle K. The group K is finitely generated by affine transformations $t(x, y) = (x, y + 1)$ and $s(x, y) = (x + 1/2, -y)$. We can show that for every virtual endomorphism $\phi : H \to K$ there exist subgroup of finite index $H_1 \sim \mathbb{Z}^2$ and associated matrix $B_\phi \in M_2(\mathbb{Q})$ of rational entities such that the restriction $\phi|_{H_1} : H_1 \to K$ is in fact a linear map $\phi|_{H_1}(x) = B_\phi x$.

Theorem 1. *Let $\phi : H \to K$ be a virtual endomorphism and $B_\phi \in M_2(\mathbb{Q})$ the associated matrix. Then ϕ is simple, and therefore produces a self-similar action, if and only if B_ϕ is not of the forms:*

$$\begin{pmatrix} \alpha & \frac{n}{m}\beta \\ \frac{m}{n}\gamma & \delta \end{pmatrix}, \frac{1}{2}\begin{pmatrix} \alpha+\beta+\gamma+\delta & \frac{n}{m}(\alpha+\beta-\gamma-\delta) \\ \frac{m}{n}(\alpha-\beta+\gamma-\delta) & \alpha-\beta-\gamma+\delta \end{pmatrix}, \begin{pmatrix} k & b_1 \\ 0 & b_2 \end{pmatrix} \text{ or } \begin{pmatrix} b_1 & 0 \\ b_2 & k \end{pmatrix} \tag{1}$$

for $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$; $n, m, k \in \mathbb{Z}$; $b_i \in \mathbb{Q}$.

Theorem 2. *1) The group K admits a transitive self-similar action on a d-regular rooted tree if and only if $d \geq 2$ is not an odd prime.*

2) The group K admits a self-replicating action on a d-regular rooted tree if and only if d is not a prime or a power of 2.

A self-similar action (G, X^*) is called *finite-state* if for every $g \in G$ the set of its sections $\{g|_v : v \in X^*\}$ is finite. In other words, the action of every element can be emulated by a finite-state transducer.

A self-similar action (G, X^*) is called *contracting* if there exists finite set $\mathcal{N} \subset G$ such that for every $g \in G$ there exists $n \in \mathbb{N}$ such that $g|_v \in \mathcal{N}$ for all $v \in X^*$ of length $\geq n$.

Theorem 3. *Let (K, X^*) be a transitive self-similar action and B_ϕ the matrix of the associated virtual endomorphism ϕ.*

1) The (K, X^*) is contracting if and only if the eigenvalues of B_ϕ are less than 1 in modulus.
2) If (K, X^*) is self-replicating, then (K, X^*) is finite-state if and only if it is contracting.
3) The group K admits a transitive finite-state (contracting) action of degree d if and only if $d \geq 2$ is not an odd prime.

References

[1] Nekrashevych, V.: *Self-similar groups.* Providence, RI: American Mathematical Society, 2005.