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- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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# Some critical point results for Fréchet manifolds

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Linking techniques (see [1]) provide significant results in critical points theory. We present linking theorem and some of its corollaries, namely a mountain pass theorem and a three critical points theorem for Keller  $C^1$ -functional on  $C^1$ -Fréchet manifolds. We refer to [2] for the definitions.

**Theorem 1** (Linking Theorem, [2]). *Let  $M$  be a  $C^1$ -Fréchet manifold endowed with a complete Finsler metric  $\rho$  and let  $\varphi : M \rightarrow \mathbb{R}$  be a closed Keller  $C_c^1$ -functional. Suppose  $\{S_0, S, C\}$  is a linking set through  $\gamma \in C(S_0, \mathbb{T})$ ,  $C$  is closed and  $\rho(\gamma(S_0), C) > 0$ . Suppose the following conditions hold*

$$(1) \mathbf{s} := \sup_{\gamma(S_0)} \varphi \leq \inf_C \varphi :=: \mathbf{i},$$

(2)  $\varphi$  satisfies the Palais-Smale condition at

$$c := \inf_{h \in \mathcal{H}} \sup_{x \in S} \varphi(\gamma(x)), \quad (1)$$

$$\text{where } \mathcal{H} := \{h \in C(S, \mathbb{T}) : h|_{\partial S_0} = \gamma\}.$$

Then  $c$  is a critical value and  $c \geq \mathbf{i}$ . Furthermore, if  $c = \mathbf{i}$  then  $\text{Cr}(\varphi, c) \cap C \neq \emptyset$ .

The theorem yields the following corollaries:

**Theorem 2** (Mountain Pass Theorem, [2]). *Suppose that  $x_0, x_1 \in M$ ,  $x_0$  belongs to an open subset  $U \subset M$  and  $x_1 \notin \bar{U}$ . Let  $\varphi : M \rightarrow \mathbb{R}$  be a closed a Keller  $C_c^1$ -functional satisfying the following condition:*

$$(1) \max\{\varphi(x_0), \varphi(x_1)\} \leq \inf_{\partial U} \varphi(x) :=: \mathbf{i};$$

(2)  $\varphi$  satisfies the Palais-Smale condition at

$$c := \inf_{h \in \mathcal{C}} \sup_{t \in [0,1]} \varphi(h(t)), \quad (2)$$

$$\text{where } \mathcal{C} := \{h \in C([0,1], M) : h(0) = x_0, h(1) = x_1\}.$$

Then  $c$  is a critical value and  $c \geq \mathbf{i}$ . If  $c = \mathbf{i}$  then  $\text{Cr}(\varphi, c) \cap U \neq \emptyset$ .

**Theorem 3** (Three Critical Points Theorem, [2]). *Let  $M$  be a connected Fréchet manifold and  $\varphi : M \rightarrow \mathbb{R}$  a closed a Keller  $C_c^1$ -functional satisfying the Palais-Smale condition at all levels. If  $\varphi$  has two minima, then  $\varphi$  has one more critical point.*

We apply the mountain pass theorem and the Minimax principle to prove the following theorem which provides the sufficient conditions for a local diffeomorphism to be a global one.

**Theorem 4.** [2] *Let  $M, N$  be connected  $C^1$ -Fréchet manifolds endowed with complete Finsler metrics  $\delta, \rho$  respectively. Assume that  $\varphi : M \rightarrow N$  is a local diffeomorphism of class Keller  $C_c^1$ . Let  $\mathcal{I} : N \rightarrow [0, \infty]$  be a closed Keller  $C_c^1$ -functional such that  $\mathcal{I}(x) = 0$  if and only if  $x = 0$  and  $\mathcal{I}'(x) = 0$  if and only if  $x = 0$ . If for any  $q \in N$  the functional  $\phi_q$  defined by*

$$\phi_q(x) = \mathcal{I}(\varphi(x) - q)$$

*satisfies the Palais-Smale condition at all levels, then  $\varphi$  is a Keller  $C_c^1$ -global diffeomorphism.*

## REFERENCES

- [1] Martin Schetcher. *Linking Method in Critical Point Theory*. Berlin Birkhäuser Basel, 1999.
- [2] Kaveh Eftekharinasab. *Some critical point results for Fréchet manifolds*. <https://arxiv.org/abs/2205.01359>.

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