



International
Scientific Conference



Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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How far apart can the projection of the centroid of a convex body and the centroid of its projection be?

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Let K be a convex body in \mathbb{R}^n , i.e., a compact convex set with non-empty interior. The centroid (the center of mass) of K is the point

$$c(K) = \frac{1}{|K|} \int_K x \, dx,$$

where $|K|$ denotes the volume of K and the integration is with respect to Lebesgue measure.

In this work we study the following question. Let H be a hyperplane in \mathbb{R}^n . Denote by $P_H c(K)$ the orthogonal projection of the centroid of K onto H and by $c(P_H K)$ the centroid of the projection of K onto H . For centrally symmetric bodies these two points coincide, but for non-symmetric bodies these points are generally different. Thus it is natural to ask how far apart these two points can be relative to some linear size of K . More precisely, we are interested in the smallest constant D_n such that for any convex body K in \mathbb{R}^n we have

$$|P_H c(K) - c(P_H K)| \leq D_n w_K(u),$$

where u is the unit vector parallel to the segment connecting $P_H c(K)$ and $c(P_H K)$, and $w_K(u)$ is the width of K in the direction of u , given by

$$w_K(u) = \max_{x \in K} \{\langle x, u \rangle\} - \min_{x \in K} \{\langle x, u \rangle\}.$$

Questions of this type began attracting attention several years ago in connection to Grünbaum-type inequalities for sections and projections; see [5], [2]. In particular, an analogue of the question above for sections of convex bodies is stated in [4, p. 127]. For other questions related to distances between various centroids the reader is referred to the book [1, p. 36] and the references contained therein.

Theorem 1 ([3]). Let D_n , $n \geq 3$, be the smallest number such that

$$|P_H c(K) - c(P_H K)| \leq D_n \cdot w_K(u), \quad (1)$$

for every convex body K in \mathbb{R}^n and every hyperplane $H \subset \mathbb{R}^n$, where

$$u = \frac{P_H c(K) - c(P_H K)}{|P_H c(K) - c(P_H K)|},$$

provided $P_H c(K) \neq c(P_H K)$. Then

- (i) $D_3 = 1 - \sqrt{\frac{2}{3}} \approx 0.1835$; the sequence $\{D_n\}_{n=3}^{\infty}$ is increasing; and $\lim_{n \rightarrow \infty} D_n \approx 0.2016$.
- (ii) Inequality (1) turns into equality if and only if K is a body obtained as follows. For a fixed hyperplane H and a vector u parallel to H , denote by θ a unit normal vector to H and take any $(n-2)$ -dimensional subspace U orthogonal to u and transversal to θ . Let L_0 be any convex body in U . Denote by tL_0 the dilation of L_0 with respect to its centroid by a factor of $t = t_{max}$, which will be defined later in the proof. Let λ, μ, ν be real numbers, $\mu \neq \nu$. Define $L_1 = tL_0 + \lambda u + \mu \theta$ and $L_2 = tL_0 + \lambda u + \nu \theta$. Then K is the convex hull of L_0, L_1 , and L_2 . Figure 1.1 shows an example of such a body in \mathbb{R}^3 when $H = \{x_3 = 0\}$, $u = e_1$, and U is the linear span of e_2 .

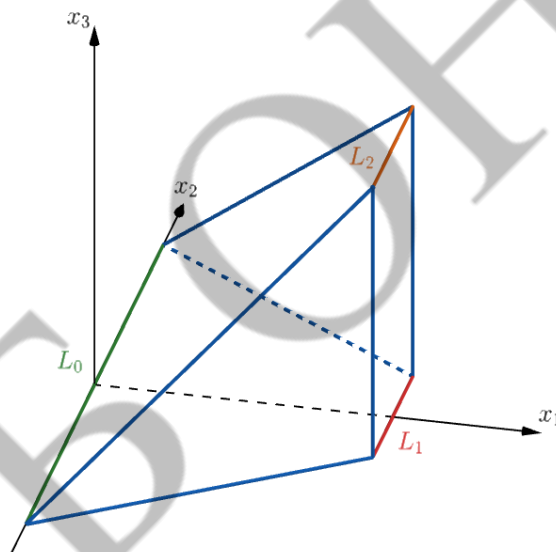


FIGURE 1.1

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Contractions of representations and realizations of Lie algebras

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Realizations (first-order differential operators) and representations (linear operators) of Lie algebras are widely applicable in modern group analysis of differential equations, in classification of gravity fields, in geometric control theory, in difference schemes for numerical solutions of differential equations, in theory of invariants, etc.

To study limit processes that connect different theories or their mathematical models it is useful to investigate contractions (limit connections) of their underlying symmetries. In practice, we first study possible limit processes between abstract Lie algebras, and then, we need to find a way how to introduce similar limits in the existing realizations or representations of Lie algebras. Unfortunately, the direct application of the known contraction to a realization or representation of a Lie algebra gives several zero operators, what makes it impossible for further application to real equations.

To overcome this obstacle, we propose to construct a parameterized series of realizations and representations based on the action of the contraction matrix on the tensor of structure constants. The realizations and representations obtained in this way coincide in the limit with the corresponding realizations and representations of contracted Lie algebras. We provide the algorithm for constructing parameterized series and present a number of illustrative examples.

For clarity, let's consider main definitions. Let $\mathcal{L}_n(V)$ be the variety of n -dimensional Lie algebras (set of Lie brackets) on a vector space V over the field \mathbb{R} , then each n -dimensional Lie algebra $\mathfrak{g} = (V, [\cdot, \cdot])$ corresponds to a multiplication rule $\mu \in \mathcal{L}_n$: $\forall x, y \in V \quad [x, y] = \mu(x, y)$.

General linear group $GL(V)$ acts on the variety of Lie brackets as follows:

$$\forall A \in GL(V), \forall \mu \in \mathcal{L}_n \quad (A\mu)(x, y) = A^{-1}(\mu(Ax, Ay)) \quad \forall x, y \in V.$$

Consider a continuous function $U(\varepsilon) = U: (0, 1] \rightarrow GL(V)$ and a parameterized family of Lie algebras $\mathfrak{g}_\varepsilon = (V, [\cdot, \cdot]_\varepsilon)$ with the Lie product defined for arbitrary elements of the vector space $[x, y]_\varepsilon = U_\varepsilon^{-1}[U_\varepsilon x, U_\varepsilon y]$. All such algebras are isomorphic to the initial algebra $\mathfrak{g} = (V, [\cdot, \cdot])$.

Definition 1. If $\forall x, y \in V$ there exists a limit

$$[x, y]_0 := \lim_{\varepsilon \rightarrow +0} [x, y]_\varepsilon = \lim_{\varepsilon \rightarrow +0} U_\varepsilon^{-1}[U_\varepsilon x, U_\varepsilon y]$$

then $[\cdot, \cdot]_0$ is a well-defined Lie bracket and Lie algebra $\mathfrak{g}_0 = (V, \mu_0)$ is called a *contraction* of the Lie algebra \mathfrak{g} .

Let $M \subset \mathbb{R}^m$ be an open domain. Let us denote the Lie algebra of smooth vector fields on M by $\text{Vect}(M)$.

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