

International
Scientific Conference



Algebraic
and Geometric
Methods
of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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The following theorem is analogous to the Riemann's Theorem in \mathbb{R}^n , but for bounded functions defined on a space with a μ -disjoint fractal structure.

Theorem 8. *Let (X, \mathcal{S}, μ) be a measure space, $\Gamma = \{\Gamma_n : n \in \mathbb{N}\}$ be a μ -disjoint fractal structure on X , $f : X \rightarrow \mathbb{R}$ be a bounded function and $C \in \mathbb{R}$. The following statements are equivalent:*

- (1) $f \in R(X; \mu; \Gamma)$ and $\int_X^{(\mu, \Gamma)} f = C$.
- (2) Given $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $|C - S(f; \Gamma_n; \xi_n; \mu)| < \varepsilon$ for each $n \geq n_0$ and each selection for Γ_n, ξ_n .
- (3) Given $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that $|C - S(f; \Gamma_n; \xi; \mu)| < \varepsilon$ for each selection for Γ_n, ξ .
- (4) $S(f; \Gamma_m; \xi_m; \mu) \xrightarrow{m \rightarrow \infty} C$ for each sequence (ξ_m) such that ξ_m is a selection for Γ_m for each $m \in \mathbb{N}$.

The next result is crucial in order to justify that the Riemann integral of a bounded function with respect to a measure and a fractal structure does not depend on the fractal structure.

Proposition 9. *Let (X, \mathcal{S}, μ) be a measure space, $\Gamma = \{\Gamma_n : n \in \mathbb{N}\}$ and $\Gamma^* = \{\Gamma_n^* : n \in \mathbb{N}\}$ be two μ -disjoint fractal structures on X and $f : X \rightarrow \mathbb{R}$ be a bounded function. If $f \in R(X; \mu; \Gamma)$ and $f \in R(X; \mu; \Gamma^*)$, then $\int_X^{(\mu, \Gamma)} f = \int_X^{(\mu, \Gamma^*)} f$.*

Hence, it does make sense to introduce the following concept:

Definition 10. Let (X, \mathcal{S}, μ) be a measure space and $f : X \rightarrow \mathbb{R}$ be a bounded function. f is said to be μ -Riemann-integrable if there exists a μ -disjoint fractal structure Γ on X such that f is Riemann-integrable on X with respect to μ and Γ . Moreover, if so, the integral is defined as $\int_X^\mu f = \int_X^{(\mu, \Gamma)} f$.

Proposition 11. *Let (X, \mathcal{S}, μ) be a finite measure space and $f : X \rightarrow \mathbb{R}$ be a bounded measurable function. Then $f \in R(X; \mu)$ and $\int_X^\mu f = \int f d\mu$.*

Hence, if Γ is a μ -disjoint fractal structure on X such that f is Riemann-integrable with respect to μ and Γ , we can calculate $\int f d\mu$ as $\int_X^{(\mu, \Gamma)} f$. It also follows that if μ is a finite measure on the Borel σ -algebra of a topological space X and $f : X \rightarrow \mathbb{R}$ is a bounded continuous map, then f is μ -Riemann-integrable on X .

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Topological rigidity of quoric manifolds

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The basic problem in Geometric Topology is the topological classification of manifolds, spaces that are locally like the usual Euclidean spaces, like the surfaces. More precisely, we study manifolds that have the same algebraic properties (homotopy equivalences) and we would like to show that they are equivalent (homeomorphic). There are a lot of conjectures towards this direction with the strongest being the Isomorphism Conjecture of Farrell-Jones. Furthermore, there are the

corresponding conjectures when the manifolds are equipped with a group of symmetries (group actions). In this case, all the structures (homotopy equivalences, homeomorphisms) should preserve the group action (equivariant).

The original idea of the classification problems is Mostow's Rigidity Theorem in which it was proved that two hyperbolic manifolds, of dimension larger than 2, which are homotopy equivalent, they are isometric. This result is the basis of most of the conjectures of classification and rigidity. Usually, one of the two manifolds has nice properties (nonpositive curvature, hyperbolic fundamental group) and the other is simply homotopy equivalent to the first. The problem is to equip the second manifold with the properties of the first through the homotopy equivalence. After that, geometric methods, similar to the one in Mostow's Theorem, will give the result.

In the case of interest, we start with Euclidean spaces \mathbb{R}^n on which we can define a multiplication such that, if $x, y \neq 0$, then $xy \neq 0$. If we insist that the multiplication is associative, then $n = 1, 2, 4$ from Frobenius Theorem. In the first case we have the multiplication of the reals, in the second case we have the complex multiplication and in the third case, we have the multiplication on the quaternions which is not commutative. The corresponding spheres, in each case, are the elements of length one and for $n = 1$ is the group of two elements \mathbb{Z}_2 , for $n = 2$ is the unit circle S^1 , for $n = 4$ is the unit sphere in \mathbb{R}^4 , S^3 , and they are all groups with the induced multiplication. In each of these dimension we have the corresponding torus, $Z^n = \mathbb{Z}_2 \times \dots \times \mathbb{Z}_2$, $T^n = S^1 \times \dots \times S^1$ and $Q^n = S^3 \times \dots \times S^3$. In each case, starting with a polyhedron, we can construct a space on which the tori act and the quotient space in the original polyhedron. These manifolds are called standard models. The local action is given in fact as the corresponding multiplication.

In each case, we start with a manifold on which the tori act (locally linearly) and they are homotopy equivalent to the standard model, preserving the group action and we want to show that is homeomorphic to the standard model. In all case, the process is similar. Let N be the manifold that we study. First, we show that the action has the local properties of the standard model. Thus the quotient space is a polyhedron P . Next, we construct the standard model from P . The final result is consequence of two results: first that N is homeomorphic to the canonical model over P , which is homeomorphic to the original standard model.

For $n = 1$, we have a much richer structure than those of finite groups. In this case, we have actions of groups that are generated by reflections (Coxeter groups). The basic properties are given in [2]. The rigidity theorem is proved in [6]. In this case, we have to show that the elements that act as reflections in the standard model, act as reflections on N .

For $n = 2$ we have the toric manifolds, which are the non-singular toric varieties and their topological analogue, the quasitoric manifolds ([1], [3]). The result is given in [5]. To show that the action of T^n on N is locally standard, we study the representation s of T^n . To show that N is homeomorphic to the standard model of the action, we show that an element in the local Čech cohomology of the quotient map vanishes.

For the remaining case, we work along the lines of the Coxeter groups and quasitoric varieties. Let $Q^n = (S^3)^n$. We say that Q^n acts on a manifold M^{4n} locally regularly if, locally, the action is given by (quaternionic) multiplication or conjugation on each coordinate. Then the quotient is a manifold with corners. Conversely, starting with a manifold with corners and an appropriate function from its faces to the conjugacy classes of subgroups of Q^n , we can construct a locally regular (quoric) manifold. Our main result is the following.

Theorem 1. (*Rigidity of Quoric Manifolds*). *Let M^{4n} be a closed locally regular quoric manifold over a nice n -manifold with corners X and X is a homotopy polytope i.e. all the faces of X (and X itself) are contractible manifolds with corners. Let N^{4n} be a locally linear closed Q^n -manifold*

and $f : N^{4n} \longrightarrow M^{4n}$ a Q^n -equivariant homotopy equivalence. Then f is Q^n -homotopic to a Q^n -homeomorphism.

The proof of the main theorem follows the methods of [6] and [5].

- We show that the action on N^n is locally regular. For this result, we first prove that N^n has the same isotropy groups as M^n and f is an isovariant homotopy equivalence. Then we prove that the action on N^n is locally regular. That is quite different from the torus case. The reason is that this part depends on the representation theory of the underlying group. But Q^n is not abelian and thus its representation theory is more complicated than that of T^n . So, we need a more thorough analysis in this case.
- Let Y be the quotient manifold with corners of the action. We prove that N^n is Q^n -homeomorphic with the standard model constructed from Y . For this, there is an obstruction theory analogous to the torus case.
- The rest is standard. The map f induces a face preserving homotopy equivalence $\phi : Y \longrightarrow X$. Induction and standard surgery methods imply that ϕ is face homotopic to a face homeomorphism ψ . The map ψ induces a Q^n -homeomorphism $g : N^n \longrightarrow M^n$ that is homotopic to f .

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On the multiplicative order convergence on Banach lattice f -algebras

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Let E, F be Archimedean vector lattices. The Fremlin tensor product $E \overline{\otimes} F$ of E and F was introduced by Fremlin in [6]. $E \overline{\otimes} F$ contains the algebraic tensor product $E \otimes F$ as an ordered vector subspace satisfying density properties. The Fremlin projective tensor product $E \widehat{\otimes} F$ of Banach lattices E and F is a Banach lattice, [7]. It contains the Fremlin tensor product $E \overline{\otimes} F$ as a norm dense normed lattice. It is known that the Fremlin tensor product $A \overline{\otimes} B$ is an f -algebra if A and B are f -algebras, [4,5]. Also, we know that if A and B are Banach lattice f -algebras, then the Fremlin projective tensor product $A \widehat{\otimes} B$ of A and B is a Banach lattice f -algebra, [9].

A vector lattice E under an associative multiplication is called a lattice ordered algebra whenever the multiplication makes E an algebra with the usual properties and multiplication of positive elements in E is positive. A lattice ordered algebra A is called an f -algebra if $x \wedge y = 0$ for every $x, y \in A$ implies $(zx) \wedge y = (xz) \wedge y = 0$ for all $z \in A^+$, where A^+ denotes the positive part of A . A Banach algebra A is called a Banach lattice algebra if A is a Banach lattice and the multiplication

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