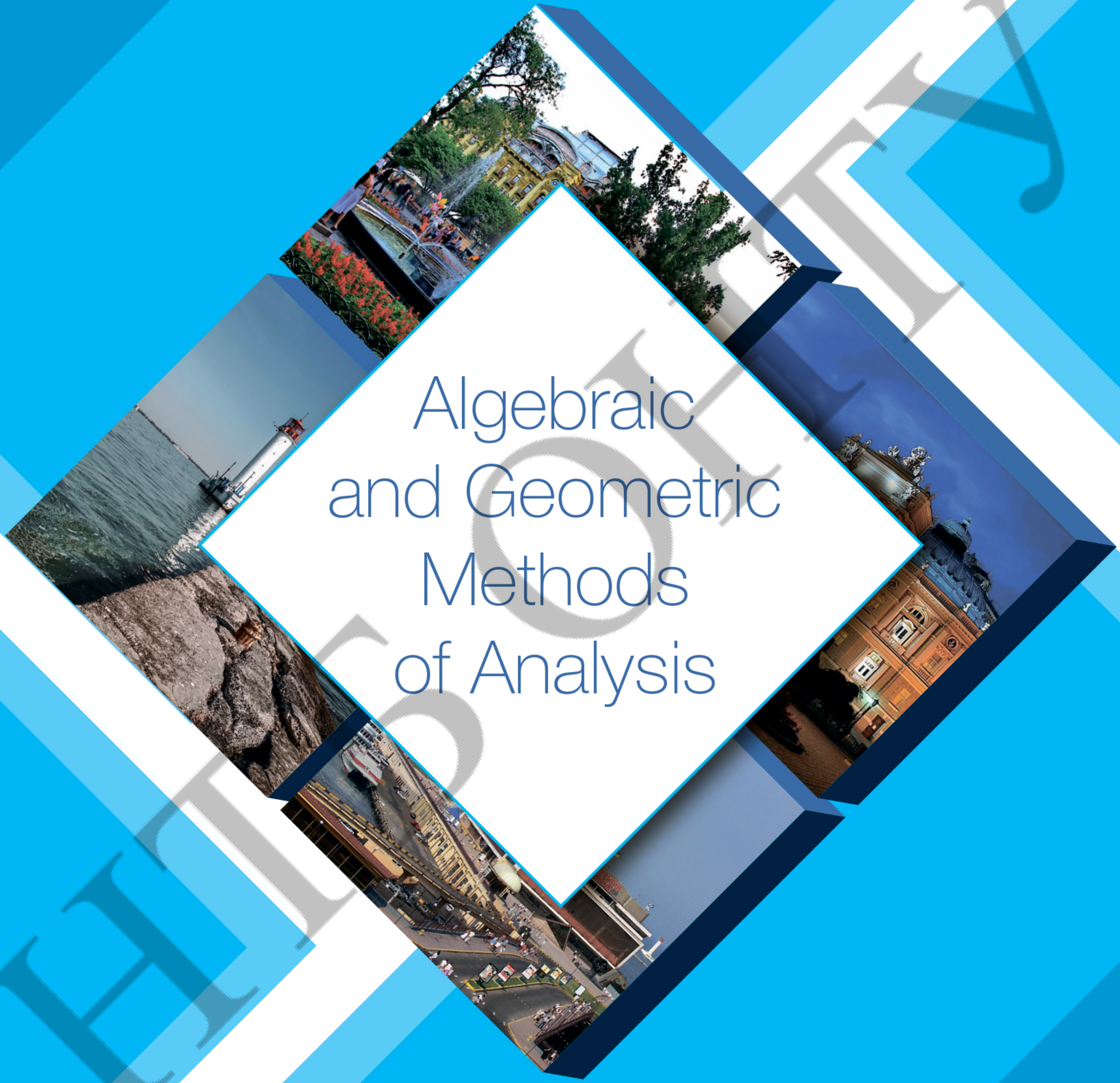
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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The solution built in the proof of Theorem 2 has the following form

$$u(|t|_p,x)=(\mathcal{F}^{-1}_{\xi\to x}\hat{u})(|t|_p,x)=\left((\mathcal{F}^{-1}_{\xi\to x}b)*u_0\right)(|t|_p,x),\ (t,x)\in\mathbb{Q}_p^+\times\mathbb{Q}_p^n, \tag{7}$$

We consider the problem (5)-(6) in the class of generalized functions, radial in t.

Denote by $\Phi'(\mathbb{Q}_p^+,\Phi'(\mathbb{Q}_p^n))$ the set of distributions over the test function space $\Phi(\mathbb{Q}_p^n)$, with values in $\Phi'(\mathbb{Q}_p^n)$.

Theorem 3. *Let $F\in\Phi'(\mathbb{Q}_p^+,\Phi'(\mathbb{Q}_p^n))$ be a generalised solution of the equation (5), that is*

$$\langle\langle F,D_t^\alpha\varphi_1\rangle,\varphi_2\rangle=\langle\langle F,\varphi_1\rangle,D_x^\beta\varphi_2\rangle,$$

for any $\varphi_1\in\Phi(\mathbb{Q}_p^+)$, $\varphi_2\in\mathbb{Q}_p$. If F is radial in t, then $F\in\mathcal{D}(\mathbb{Q}_p^+,\Phi'(\mathbb{Q}_p^n))$. If, in addition, $F(0,x)=0$, then $F(t,x)\equiv 0$.

It follows from Theorem 3 that the solutions of the Cauchy problems constructed in Theorem 2 are unique in the class of radial in t, bounded locally constant functions.

Theorem 4. *Suppose that the conditions of Theorem 2 hold. Then the solution of the problem (5)-(6), defined in (7), satisfies the following estimate in $L^1(\mathbb{Q}_p^n)$ in variable x*

$$||u(|t|_p,\cdot)||_{L_1(\mathbb{Q}_p^n)}\le p^{2n\gamma}||u_0||_{L_1(\mathbb{Q}_p^n)}, \tag{8}$$

where $\gamma\ge\dfrac{2}{K}$ is a positive constant.

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On the semigroup of injective monoid endomorphisms of a some extension of the bicyclic semigroup

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In this paper we shall follow the semigroup terminology of [5].

By ω we denote the set of all non-negative integers.

Let $\mathscr{P}(\omega)$ be the family of all subsets of ω. For any $F\in\mathscr{P}(\omega)$ and any integer n we put $n+F=\{n+k\colon k\in F\}$ if $F\neq\varnothing$ and $n+\varnothing=\varnothing$. A subfamily $\mathscr{F}\subseteq\mathscr{P}(\omega)$ is called *ω-closed* if $F_1\cap(-n+F_2)\in\mathscr{F}$ for all $n\in\omega$ and $F_1,F_2\in\mathscr{F}$. For any $a\in\omega$ we denote $[a)=\{x\in\omega\colon x\geqslant a\}$.

On the set $\boldsymbol{B}_\omega=\omega\times\omega$ we define the semigroup operation "$\cdot$" in the following way

$$(i_1,j_1)\cdot(i_2,j_2)=\begin{cases}(i_1-j_1+i_2,j_2), & \text{if } j_1\leqslant i_2;\\ (i_1,j_1-i_2+j_2), & \text{if } j_1\geqslant i_2.\end{cases}$$

It is well known that the bicyclic monoid is isomorphic to the semigroup $\boldsymbol{B}_\omega$.

The following construction is introduced in [1].

Let $\mathscr{F}$ be an ω-closed subfamily of $\mathscr{P}(\omega)$. On the set $\boldsymbol{B}_{\omega}\times\mathscr{F}$ we define the semigroup operation "$\cdot$" in the following way

$$(i_1,j_1,F_1)\cdot(i_2,j_2,F_2)=\left\{\begin{array}{ll}(i_1-j_1+i_2,j_2,(j_1-i_2+F_1)\cap F_2), & \text{if } j_1\leqslant i_2;\\ (i_1,j_1-i_2+j_2,F_1\cap(i_2-j_1+F_2)), & \text{if } j_1\geqslant i_2.\end{array}\right.$$

In [1] is proved that if the family $\mathscr{F}\subseteq\mathscr{P}(\omega)$ is ω-closed then $(\boldsymbol{B}_{\omega}\times\mathscr{F},\cdot)$ is a semigroup. Moreover, if an ω-closed family $\mathscr{F}\subseteq\mathscr{P}(\omega)$ contains the empty set $\varnothing$ then the set $\boldsymbol{I}=\{(i,j,\varnothing)\colon i,j\in\omega\}$ is an ideal of the semigroup $(\boldsymbol{B}_{\omega}\times\mathscr{F},\cdot)$. For any ω-closed family $\mathscr{F}\subseteq\mathscr{P}(\omega)$ the following semigroup

$$\boldsymbol{B}_{\omega}^{\mathscr{F}}=\left\{\begin{array}{ll}(\boldsymbol{B}_{\omega}\times\mathscr{F},\cdot)/\boldsymbol{I}, & \text{if } \varnothing\in\mathscr{F};\\ (\boldsymbol{B}_{\omega}\times\mathscr{F},\cdot), & \text{if } \varnothing\notin\mathscr{F}\end{array}\right.$$

is defined in [1].

In the paper [2] injective endomorphisms of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ with the two-elements family $\mathscr{F}$ of inductive nonempty subsets of ω are studies. Here the authors describe the elements of the semigroup $\boldsymbol{End}^1_*(\boldsymbol{B}_{\omega}^{\mathscr{F}})$ of all injective monoid endomorphisms of the monoid $\boldsymbol{B}_{\omega}^{\mathscr{F}}$, and show that Green's relations $\mathscr{R}$, $\mathscr{L}$, $\mathscr{H}$, $\mathscr{D}$, and $\mathscr{J}$ on $\boldsymbol{End}^1_*(\boldsymbol{B}_{\omega}^{\mathscr{F}})$ coincide with the relation of equality. In [3, 4] the semigroup $\boldsymbol{End}^1(\boldsymbol{B}_{\omega}^{\mathscr{F}})$ of all monoid endomorphisms of the monoid $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ is studied.

Example 1. Let $\mathscr{F}^3=\{[0),[1),[2)\}$. Fix an arbitrary positive integer k. We define the transformation $\alpha_{[k]}$ of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}^3}$ in the following way

$$(i,j,[p))\alpha_{[k]}=\left\{\begin{array}{ll}(ki,kj,[p)), & \text{if } p\in\{0,1\};\\ (k(i+1)-1,k(j+1)-1,[2)), & \text{if } p=2,\end{array}\right.$$

for all $i,j\in\omega$. It is obvious that $\alpha_{[k]}$ is an injective transformation of the monoid $\boldsymbol{B}_{\omega}^{\mathscr{F}^3}$.

Lemma 2. *For an arbitrary positive integer k the transformation $\alpha_{[k]}\colon\boldsymbol{B}_{\omega}^{\mathscr{F}^3}\to\boldsymbol{B}_{\omega}^{\mathscr{F}^3}$ is an injective monoid endomorphism of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}^3}$.*

Theorem 3. *Let $\mathscr{F}^3=\{[0),[1),[2)\}$ and ε be an injective monoid endomorphism of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}^3}$. Then $\varepsilon=\alpha_{[k]}$ for some positive integer k.*

By $(\mathbb{N},\cdot)$ we denote the multiplicative semigroup of positive integers.

Theorem 4. *Let $\mathscr{F}^3=\{[0),[1),[2)\}$. Then the monoid $\boldsymbol{End}^1_*(\boldsymbol{B}_{\omega}^{\mathscr{F}^3})$ of all injective endomorphisms of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}^3}$ is isomorphic to $(\mathbb{N},\cdot)$.*

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