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# Algebraic and Geometric Methods of Analysis

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## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
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ІНТЕРНАЦІОНАЛЬНИЙ ЦЕНТР СПІВРОБІТНИЦТВА

## Estimate of maximum of the products of inner radii of mutually non-overlapping domains

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Let  $\mathbb{N}$ ,  $\mathbb{R}$  be the sets of natural and real numbers, respectively,  $\mathbb{C}$  be the complex plane,  $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$  be its one point compactification,  $\mathbb{U}$  be the open unit disk in  $\mathbb{C}$  and  $\mathbb{R}^+ = (0, \infty)$ . Let  $r(B, a)$  be an inner radius of the domain  $B \subset \overline{\mathbb{C}}$  relative to a point  $a \in B$  [1–4]. The inner radius of the domain  $B$  is connected with Green's generalized function  $g_B(z, a)$  of the domain  $B$  by the relations

$$g_B(z, a) = -\ln|z - a| + \ln r(B, a) + o(1), \quad z \rightarrow a,$$

$$g_B(z, \infty) = \ln|z| + \ln r(B, \infty) + o(1), \quad z \rightarrow \infty.$$

The system of points  $A_n := \{a_k \in \mathbb{C}, k = \overline{1, n}\}$ ,  $n \in \mathbb{N}$ ,  $n \geq 2$ , is called  $n$ -radial, if  $|a_k| \in \mathbb{R}^+$  for  $k = \overline{1, n}$  and

$$0 = \arg a_1 < \arg a_2 < \dots < \arg a_n < 2\pi.$$

Consider the following extremal problem.

**Problem.** For all values of the parameter  $\gamma \in \mathbb{R}^+$  to find estimate of the maximum of the functional

$$J_n(\gamma) = [r(B_0, 0) r(B_\infty, \infty)]^\gamma \prod_{k=1}^n r(B_k, a_k), \quad (1)$$

where  $n \in \mathbb{N}$ ,  $n \geq 2$ ,  $a_0 = 0$ ,  $A_n = \{a_k\}_{k=1}^n \in \overline{\mathbb{C}}/\{0, \infty\}$  be any fixed  $n$ -radial system of different points,  $B_0, B_\infty, \{B_k\}_{k=1}^n$  be a system of mutually non-overlapping domains,  $0 \in B_0 \subset \overline{\mathbb{C}}$ ,  $\infty \in B_\infty \subset \overline{\mathbb{C}}$ ,  $a_k \in B_k \subset \overline{\mathbb{C}}$ ,  $k = \overline{1, n}$ .

The following proposition is true.

**Theorem 1.** Let  $n \in \mathbb{N}$ ,  $n \geq 2$ ,  $\gamma \in \mathbb{R}^+$ . Then, for any fixed  $n$ -radial system of different points  $A_n = \{a_k\}_{k=1}^n \in \overline{\mathbb{C}}/\{0, \infty\}$  and any mutually non-overlapping domains  $B_0, B_\infty, B_k$ ,  $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$ ,  $\infty \in B_\infty \subset \overline{\mathbb{C}}$ ,  $a_k \in B_k \subset \overline{\mathbb{C}}$ ,  $k = \overline{1, n}$ , the following inequalities hold

$$J_n(\gamma) \leq \begin{cases} (n+1)^{-\gamma \frac{n+1}{n+2}} \left[ \prod_{k=1}^n r(B_k, a_k) \right]^{1 - \frac{2\gamma}{n+2}} \prod_{k=1}^n |a_k|^{\frac{2\gamma}{n+2}}, & \text{if } \gamma \in (0, \frac{n+2}{2}]; \\ (n+1)^{-\frac{n+1}{2}} \prod_{k=1}^n |a_k|, & \text{if } \gamma > \frac{n+2}{2}. \end{cases}$$

**Remark 2.** If  $\gamma = \frac{n+2}{2}$  and  $\prod_{k=1}^n |a_k| \leq 1$ , then from above posed Theorem 1, the following inequality holds

$$[r(B_0, 0) r(B_\infty, \infty)]^{\frac{n+2}{2}} \prod_{k=1}^n r(B_k, a_k) \leq (n+1)^{-\frac{n+1}{2}}.$$

In this case the structure of points and domains is not important.

For any  $n$ -radial system of points  $A_n = \{a_k\}_{k=1}^n$ ,  $|a_k| = 1$ , and any pairwise non-overlapping domains  $\{B_k\}_{k=1}^n$ ,  $a_k \in B_k \subset \overline{\mathbb{C}}$ ,  $k = \overline{1, n}$ , the inequality

$$\prod_{k=1}^n r(B_k, a_k) \leq 2^n \prod_{k=1}^n \alpha_k$$

is valid (see Corollary 5.1.3 [1]). In Theorem 6.11 [2] for any different points  $a_k$  on the circle  $|a_k| = 1$ ,  $k = \overline{1, n}$  ( $n \geq 2$ ), and any pairwise non-overlapping domains  $B_k \subset \overline{\mathbb{C}}$  such that  $a_k \in B_k$ ,  $k = \overline{1, n}$ , the inequality

$$\prod_{k=1}^n r(B_k, a_k) \leq \left(\frac{4}{n}\right)^n$$

is proved. Thus, from Theorem 1 we have next result.

**Corollary 3.** *Let  $n \in \mathbb{N}$ ,  $n \geq 2$ ,  $\gamma \in \mathbb{R}^+$ . Then, for any system of different points  $\{a_k\}_{k=1}^n$  of the unit circle  $|a_k| = 1$  and any mutually non-overlapping domains  $B_0, B_\infty, B_k$ ,  $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$ ,  $\infty \in B_\infty \subset \overline{\mathbb{C}}$ ,  $a_k \in B_k \subset \overline{\mathbb{C}}$ ,  $k = \overline{1, n}$ , the following inequalities hold*

$$J_n(\gamma) \leq \begin{cases} (n+1)^{-\gamma \frac{n+1}{n+2}} \left(\frac{4}{n}\right)^{n - \frac{2\gamma n}{n+2}}, & \text{if } \gamma \in (0, \frac{n+2}{2}]; \\ (n+1)^{-\frac{n+1}{2}}, & \text{if } \gamma > \frac{n+2}{2}. \end{cases}$$

If  $B_0 \subset \mathbb{U}$ , then from the proof of the Theorem 1, the following results are valid.

**Corollary 4.** *Let  $n \in \mathbb{N}$ ,  $n \geq 2$ ,  $\gamma \in \mathbb{R}^+$  and  $B_0 \subset \mathbb{U}$ . Then, for any system of different points  $\{a_k\}_{k=1}^n$  of the unit circle  $|z| = 1$  and any mutually non-overlapping domains  $B_k$ ,  $a_k \in B_k \subset \overline{\mathbb{C}}$ ,  $k = \overline{0, n}$ ,  $a_0 = 0$ , and the domains  $B_k$ ,  $k = \overline{1, n}$ , are mirror-symmetric relative to the unit circle  $|z| = 1$ , the inequality holds*

$$r^{2\gamma}(B_0, 0) \prod_{k=1}^n r(B_k, a_k) \leq (n+1)^{-\frac{n+1}{2}}.$$

**Corollary 5.** *Let  $n \in \mathbb{N}$ ,  $n \geq 2$ ,  $\gamma \in \mathbb{R}^+$ ,  $R > 0$  and  $B_0$  be an arbitrary domain belonging to the open circle  $|w| < R$ . Then, for any  $n$ -radial system of different points  $A_n = \{a_k\}_{k=1}^n$  such that  $|a_k| = R$ ,  $k = \overline{1, n}$ , and any mutually non-overlapping domains  $B_k$ ,  $a_k \in B_k \subset \overline{\mathbb{C}}$ ,  $k = \overline{0, n}$ ,  $a_0 = 0$ , and the domains  $B_k$ ,  $k = \overline{1, n}$ , are mirror-symmetric relative to the circle  $|w| = R$ , the inequality holds*

$$r^{2\gamma}(B_0, 0) \prod_{k=1}^n r(B_k, a_k) \leq (n+1)^{-\frac{n+1}{2}} \cdot R^{2\gamma}.$$

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