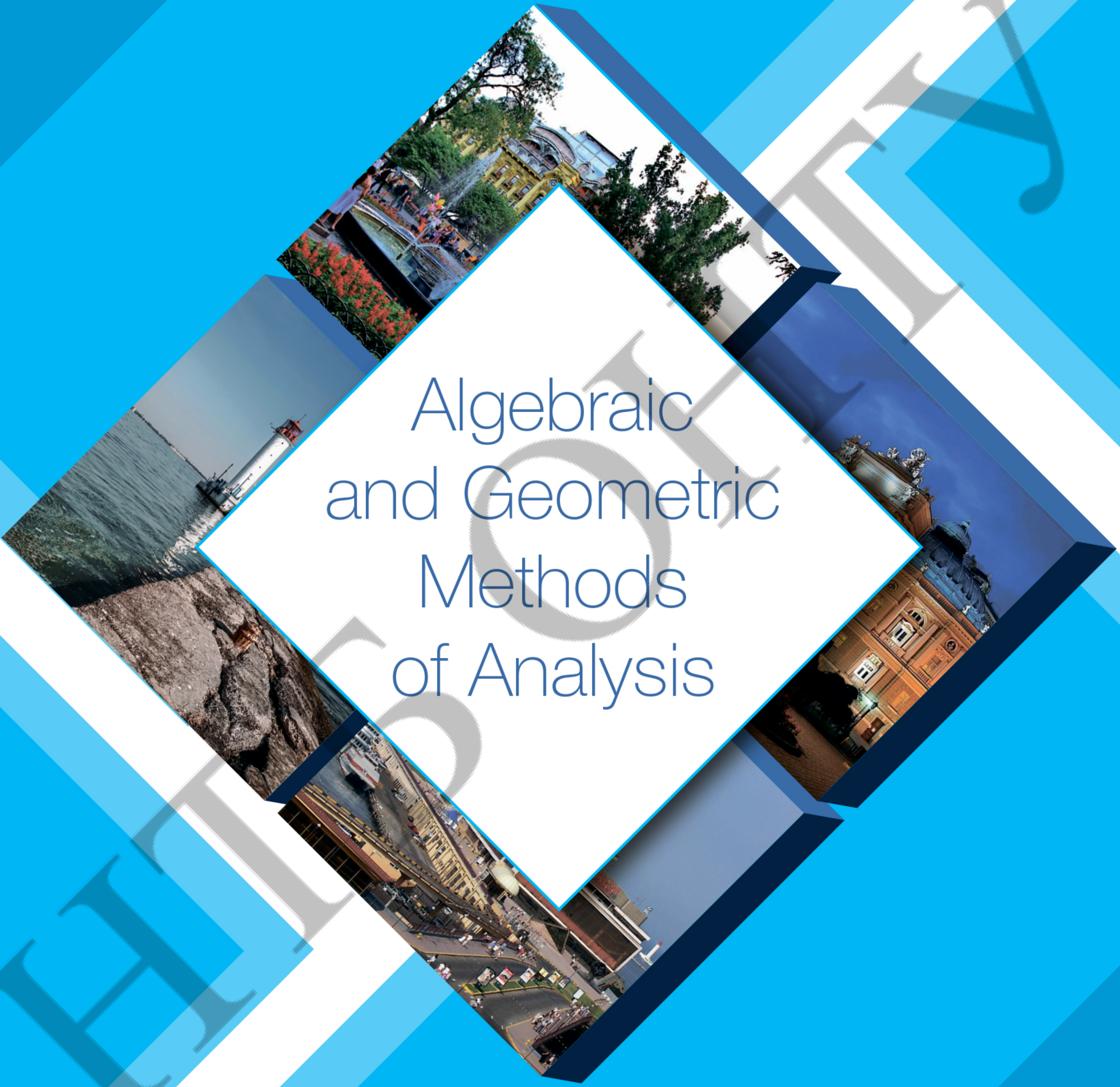
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Normal forms of functions with degenerate critical points on surfaces whose stabilizers are homotopically non-trivial

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Let M be a smooth compact surface, and P be either $\mathbb{R}$ or S^1. The group of diffeomorphisms $\mathcal{D}(M)$ acts on the space of smooth P-valued functions $C^\infty(M,P)$ by the rule:

$$C^\infty(M,P)\times\mathcal{D}(M)\to C^\infty(M,P)\qquad (f,h)\mapsto f\circ h.$$

For a smooth function $f\in C^\infty(M,P)$ we denote by

$$\mathcal{S}(f)=\{h\in\mathcal{D}(M)\,|\,f\circ h=f\},\qquad \mathcal{O}(f)=\{f\circ h\,|\,h\in\mathcal{D}(M)\}$$

the *stabilizer* and the *orbit* of f. Homotopy properties of $\mathcal{S}(f)$ and $\mathcal{O}(f)$ and their connected components are well studied for a large class of smooth functions with isolated singularities on surfaces, see [2]. We also denote by $\mathcal{S}_{\mathrm{id}}(f)$ a connected component of the identity map id in $\mathcal{S}(f)$.

We consider the following class $\mathcal{F}(M,P)$ of smooth functions: a function f belongs to $\mathcal{F}(M,P)$ if

(1) for each connected component V of the boundary ∂M a function $f|_V$ either takes a constant value or is a covering map,
(2) a set of critical points Σ_f of f is a disjoint union of smooth submanifolds of M and $\Sigma_f\subset\mathrm{Int}(M)$,
(3) for each connected component C of Σ_f and each critical point $p\in C$ there exist a local chart $(U,\phi:U\to\mathbb{R}^2)$ near p and a chart $(V,\psi:V\to\mathbb{R})$ near $f(p)\in P$ such that $f(U)\subset V$ and a local representation $f_p=\psi\circ f\circ\phi^{-1}:\phi(U)\to\psi(V)$ of f is
   (a) either a polynomial homogeneous polynomial f_p without multiple factors,
   (b) or is given by $f_C(x,y)=\pm y^{n_C}$ for some $n_C\in\mathbb{N}_{\geq 2}$ depending of C.

Connected components of Σ_f are isolated critical points and critical circles.

Let $\mathcal{F}^0(M,P)$ be a subset of $\mathcal{F}(M,P)$ of function which satisfy (1), (2), (3.b), but instead (3.a) the following condition holds:

(3.a') either a polynomial f_p given by $f_p(x,y)=\pm x^2\pm y^2$.

For a function $f\in\mathcal{F}(M,P)$ a stabilizer $\mathcal{S}_{\mathrm{id}}(f)$ is homotopy equivalent to S^1 if $f\in\mathcal{F}^0(M,P)$, and is contractible otherwise, [1, Theorem 1.2]. Our main result is an analytical characterization of functions from $\mathcal{F}^0(M,P)$, see Theorem 3. The following proposition contains basic facts about functions from $\mathcal{F}^0(M,P)$.

Proposition 1. *Let f be a function from $\mathcal{F}^0(M,P)$. Then*

(1) *M is one of the following surfaces: $S^1\times[0,1]$, D^2, S^2 or T^2.*
(2) *a function $f:M\to P$ is always null-homotopic if M is not a torus. A function on torus can be either null-homotopic or not null-homotopic.*
(3) *f has any finite number of critical circles if $M=S^1\times[0,1]$, D^2, S^1, or T^2 and f is not null-homotopic. If $M=T^2$ and f is null-homotopic, then f has at least 2 critical circles.*
(4) *If $M=S^1\times[0,1]$ or T^2, then f does not have isolated critical points. If $M=D^2$ or S^2, a function f has one and two non-degenerate extremes respectively.*

To state our main result we need the following definition.

Definition 2 (Primitive functions)**.** Let $f_0:M\to P$ be a smooth function

(1) $M = S^1 \times [0,1] = \{(z,s) \,|\, z \in \mathbb{C}, |z| = 1,\, 0 \le s \le 1\}$, and $f_0 : S^1 \times [0,1] \to \mathbb{R}$ is given by $f_0(\phi, s) = s$;
(2) $M = D^2 = \{(x,y) \in \mathbb{R}^2 \,|\, x^2 + y^2 \le 1\}$, and $f_0 : D^2 \to \mathbb{R}$ is given by $f_0(x,y) = \pm x^2 \pm y^2$;
(3) $M = S^2 = \{(x,y,z) \,|\, x^2 + y^2 + z^2 = 1\}$, and $f_0(x,y,z) : S^2 \to \mathbb{R}$ is given by $f_0(x,y,z) = z$;
(4) $M = T^2 = \left\{(x,y,z) \in \mathbb{R}^3 \,\middle|\, \left(\sqrt{x^2+y^2} - 2\right)^2 + z^2 = 1\right\}$, and $f_0 : T^2 \to \mathbb{R}$ is given by $f_0(x,y,z) = z$;
(5) $M = T^2 = \{(w,z) \in \mathbb{C}^2 \,|\, |z| = |w| = 1\}$, and $f_0 : T^2 \to S^1$ is given by $f_0(w,z) = z$.

Obviously that functions from (1)–(4) belongs to $\mathcal{F}^0(M,P)$. They are height functions for (1)–(4), and a function from (5) is an angular projection. These functions have a minimum possible number of critical submanifolds, and we will call them *primitive functions*.

Theorem 3. *Let f be a smooth function from $\mathcal{F}^0(M,P)$ and $f_0 \in \mathcal{F}^0(M,P)$ be a primitive function. A function f admits a decomposition*

$$f = \varkappa \circ f_0 \circ h^{-1} \tag{1}$$

for some diffeomorphism $h : M \to M$ and a smooth function $\varkappa : \mathrm{Im}(f_0) \to P$ satisfying the following conditions:

(A) *$\varkappa$ has the only finite number of critical points in which it is not flat, i.e., not all derivatives of $\varkappa$ at each critical point vanish,*
(B) *$\varkappa$ does not have extremes at $f_0(\Sigma^0_{f_0})$ and $f_0(\partial M)$.*

In particular, if $f \in \mathcal{F}^0(T^2, P)$ is null-homotopic, then f_0 is given by (4), and by (5) otherwise. A factorization (1) is not unique and depends on the choice of h.

REFERENCES

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[2] Sergiy Maksymenko. Deformations of functions on surfaces *Proceedings of Institute of Mathematics of NAS of Ukraine*, 17, no. 2 (2020) 150-199.

Nonlinear interpolation of α-Holderian mappings with applications to quasilinear PDEs

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The Marcinkiewicz interpolation theorems for linear operators acting on Lebesgue spaces turned out to be a powerful tool for studying regularity of solutions for linear PDEs in L^p-spaces. The K-method introduced by J. Peetre ([5, 6]) allowed to extend the study of regularity of solutions of linear equations on spaces different from L^p-spaces. The main difficulty to apply Peetre's definition is the identification of the interpolation spaces between two normed spaces embedded in a same topological space. In [2, 3, 4] we did such a study with applications to linear PDEs using new non-standard spaces as grand or small Lebesgue spaces and $G\Gamma$-gamma spaces.

In [7] L. Tartar gave interpolation results on nonlinear Hölderian mappings (which include Lipschitz mappings) and applied them to a variety of boundary value problems as bilinear applications, semi-linear PDEs but also on variational inequalities.

In this talk we present some results contained in the recent paper [1], were we extend Tartar's results