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Formation of algorithmic culture of students in the classroom of higher mathematics

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The paper deals with the study and application of algorithms in the lessons of higher mathematics. The course of higher mathematics has sufficiently broad possibilities for the formation, study and application of algorithms, since its content naturally lays down the algorithmic line. The task of the formation of universal computer literacy should be solved when teaching all academic subjects of higher educational institutions. A significant role is given to the course of higher mathematics. When studying this course, students develop stable mathematical skills more successfully if special educational instructions and plans for solving important problems are introduced. They serve as propedeutics of the formation of the algorithmic culture in the future. On the other hand, a firm knowledge of the plans for solving the basic problems of a course in higher mathematics is the initial foundation of students' mathematical preparation.

Applying plans for solving problems in the process of teaching higher mathematics, students should be guided by the fact that they should not just remember one plan or another, but the main thing is to understand which theoretical sentences its application is based on, and each step of the training activity perform consciously, not automatically. Students are familiarized with plans by solving problems at a lecture, their further refinement is carried out in practical classes for various forms of work (frontal, group, individual).

Algorithmic culture of the future teacher of mathematics is an integral part of his general culture. The general culture of the future teacher of mathematics can be characterized as an expression of the maturity of the entire system of professionally significant personal qualities, productively implemented in the process of individual activity. General culture is the result of the qualitative development of knowledge, skills, abilities, interests, beliefs, norms of professional activity and behavior, abilities and social feelings of a future teacher of mathematics.

From the point of view of learning mathematical activity, algorithmic culture is part of mathematical culture. Algorithmic training contributes to the formation and development among students, and through them, students of specific ideas and skills related to understanding the essence of the algorithm and its properties, the essence of the programming language as a means of recording the algorithm, the algorithmic nature of mathematics methods and their applications associated with owning techniques and means of recording problem solving in an algorithmic language.

An algorithmic culture is understood as a set of specific "algorithmic" ideas, knowledge and skills that should be part of the general culture of a future teacher of mathematics at the present stage of society's development and, therefore, determine a purposeful component of a general cultural pedagogical education and student competence.

In conclusion, we note that the line of forming the algorithmic culture of students suggests the prospect of its further convergence at the level of interdisciplinary connections both with the course of mathematics and with other natural-mathematical and humanitarian academic disciplines.

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Behavior of the trajectories of a single cubic operator

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In the paper for one cubic Volterra operator on a two-dimensional simplex found all the fixed points and fully understood the behavior of the trajectories generated by this operator.

One of the main tasks in the study of a dynamic system is to study the evolution of the state of the system. Usually, the "descendants" of the state of the system are determined by some law. Numerous problems of biology are solved using the theory of measure and the theory of dynamical systems. These dynamical systems are determined by iterations of nonlinear operators. We give the definition of such operators:

Let $E = \{1, 2, ..., n\}$.

Consider the set

$$S^{n-1} = \left\{ x = (x_1,\, x_2, ..., x_n) \in R^n : x_i \geq 0,\ \sum_{i=1}^{n} x_i = 1 \right\}.$$

The set S^{n-1} is called the $n-1$ dimensional simplex. Each the element $x \in S^{n-1}$ is a probability measure on E and its can be interpreted as a state of the biological (physical, sociological, etc.) system consisting of n elements.

One of the main tasks for this system is to study the evolution of the system state. Usually, the descendants of the state of the system are determined by certain laws. For solving problems arising in mathematical genetics is used quadratic operators whose theory is currently well developed (see for example [1-3]). In [4] for one all fixed points were found on a Volterra cubic operator on a two-dimensional simplex. A description is given of the limit set of trajectories for some subclasses of such operators.

In this paper, we study dynamical systems defined by cubic operators. Fully studied trajectory of a single cubic operator on S^2, which arises naturally in the study of certain problems population biology.

In the simplest problem of population genetics is considered biological system E, consisting of n species $1, 2, ..., n$. We consider that the species of parents i, j, k uniquely determine the probability of each species l for an immediate descendant. Denote this probability by $P_{ijk,l}$. Then $P_{ijk,l} \geq 0, \sum_{l=1}^{n} P_{ijk,l} = 1$ and the values of $P_{ijk,l}$ do not change with any permutation i, j, k if the varieties are not related to gender. Population status is described by the set $x = (x_1, x_2, ..., x_n)$ probabilities of varieties. Therefore, $x \in S^{n-1}$.

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On nonexistence of Kenmotsu structure on Kirichenko–Uskorev-hypersurfaces of Kählerian manifolds

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1. The almost contact metric (*acm*-) structure is one of the most important differential-geometrical structures on manifolds. As it is known [2], an almost contact metric structure on a odd-dimensional manifold N is a system $\{\Phi,\, \xi,\, \eta,\, g\}$ of tensor fields on this manifold, where Φ is a tensor of type $(1,1)$, ξ is a vector, η is a covector and $g = \langle\cdot,\, \cdot\rangle$ is a Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1;\ \Phi(\xi) = 0;\ \eta\circ\Phi = 0;\ \Phi^2 = -id + \xi\otimes\eta;$$
$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta(X)\eta(Y),\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of smooth vector fields on N. As one of the most meaningful and interesting *acm*-structure we mark ut the Kenmotsu structure that is defined by the following condition [2]:

$$\nabla_X(\Phi)Y = \langle \Phi X, Y\rangle\, \xi - \eta(Y)\Phi X, \quad X, Y \in \aleph(N),$$

In [3], V. F. Kirichenko and I. V. Uskorev have introduced a new class of almost contact metric structure. Namely, they have defined the almost contact metric structure with the close contact form as the structures of cosymplectic type. V. F. Kirichenko and I. V. Uskorev have also proved that their structure is invariant under canonical conformal transformations [3].

Evidently, a trivial example of Kirichenko–Uskorev structure is the cosymplectic structure, and as a non-trivial example we can consider the Kenmotsu structure.

2. Now let us consider the *acm*-structure induced on a hypersurface N of a Kählerian manifold M^{2n}, $n \geq 3$. The Cartan structural equations of such *acm*-structure are the following [4]:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\sigma^\alpha_\beta \omega^\beta \wedge \omega + i\sigma^{\alpha\beta}\omega_\beta \wedge \omega,$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\sigma^\beta_\alpha \omega_\beta \wedge \omega - i\sigma_{\alpha\beta}\omega^\beta \wedge \omega,$$
$$d\omega = -i\sigma^\alpha_\beta\, \omega^\beta \wedge \omega_\alpha + i\sigma_{n\beta}\omega \wedge \omega^\beta - i\sigma^\beta_n \omega \wedge \omega_\beta.$$

Here σ is the second fundamental form of the immersion of N into M^{2n}; $\omega_\alpha = \omega^{\hat{a}}$; $\alpha,\ \beta = 1\,,\ldots,\, n-1$; $\hat{a} = a + n$.

Taking into account the results on the matrix of the second fundamental form [5], we obtain the first Theorem.

Theorem 1. *The Cartan structural equations of Kirichenko–Uskorev acm-structure induced on a hypersurface of a Kählerian manifold M^{2n}, $n \geq 3$ are the following:*

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\sigma^{\alpha\beta}\omega_\beta \wedge \omega;$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\sigma_{\alpha\beta}\omega^\beta \wedge \omega;$$
$$d\omega = 0.$$

Comparing these equations with well-known Cartan structural equation of a Kenmotsu structure [2]

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + \omega \wedge \omega^\alpha;$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + \omega \wedge \omega_\alpha;$$
$$d\omega = 0,$$

we obtain our second result.

Theorem 2. *Krichenko–Uskorev almost contact metric structure induced on a hypersurface of a Kählerian manifold M^{2n}, $n \geq 3$, cannot be a Kenmotsu structure.*

Note that the presented Theorems develop some results on hypersurfaces of Kählerian manifolds [5], [6].

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НТБ ОНАХТ

On almost contact metric hypersurfaces in W_4-manifolds

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1. The famous work by Alfred Gray and Luis M. Hervella [1] contains a classification of the almost Hermitian structures on first order differential-geometrical invariants. In accordance to this classification, all the almost Hermitian structures are divided into 16 classes. Analytical criteria for each concrete structure to belong to one or another class have been obtained [1].

The class of W_4-manifolds is one of so-called small Gray–Hervella classes of almost Hermitian manifolds. Some specialists identify this class with the class of locally conformal Kählerian (lcK-) manifolds that is not absolutely correct. In fact, the W_4-class contains all locally conformal Kählerian manifolds, but coincides with the class of lcK-manifolds only for dimension at least six [2]. W_4-manifolds were studied in detail from diverse points of view by such outstanding mathematicians as Alfred Gray (USA), Vadim Feodorovich Kirichenko (Russian Federation) and Izu Vaisman (Israel).

We remark also that the present communication is a continuation of researches of the author in the area of W_4-manifolds (see, for example, [3], [4], [5] and others).

2. As it is known, an almost Hermitian manifold is a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n} [1]. The fundamental form of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY \rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

An almost Hermitian structure belongs to the W_4-class, if

$$\nabla_X\ (F)\ (Y, Z)\ =\ -\frac{1}{2\,(n-1)}\ \{\langle X, Y \rangle\ \delta\, F\,(Z) - \langle X, Z \rangle\ \delta\, F\,(Y) -$$
$$- \langle X, JY \rangle\ \delta\, F\,(JZ) + \langle X, JZ \rangle\ \delta\, F\,(JY)\}\,, \quad X, Y, Z \in \aleph(M^{2n}),$$

where δ is the codifferentiation operator and ∇ is the Riemannian connection of the metric $g = \langle \cdot, \cdot \rangle$ [1].

3. The main results are the following:

1) The Cartan structural equations of the general type almost contact metric structure on an oriented hypersurface in a W_4-manifold are obtained;

2) The Cartan structural equations of almost contact metric structures on an oriented hypersurface with type number 0, 1 or 2 in a W_4-manifold are selected;

3) A characterization in terms of the type number (Takagi–Kurihara characterization [6]) of some important classes of almost contact metric structures on hypersurfaces in W_4-manifolds is obtained;

4) A criterion of the minimality of such hypersurfaces in the terms of their type number is established;

5) It is proved that 2- and 3-hypersurfaces in W_4-manifolds do not admit almost contact metric structures belonging to any well-studied classes of almost contact metric structures (cosymplectic, nearly cosymplectic, Kenmotsu, Sasaki etc).

Using the above mentioned fact that the class of W_4-manifolds contains all lcK-manifolds, we conclude that the obtained result are also related to almost contact metric structures on oriented hypersurfaces in lcK-manifolds.

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НТБ ОНАХТ

Quantum calculus and singularities of quasi-discriminant sets

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Let $g : \mathbb{R} \to \mathbb{R} : x \mapsto g(x)$ be a given smooth one-to-one map of the real axis, which is the domain of polynomial $f(x)$ with arbitrary coefficients. We want to find conditions on the coefficients of the polynomial under which it has at least a pair of roots t_i, t_j satisfying the relation $g(t_i) = t_j$ and investigate the structure of the algebraic variety in the space of coefficients possessing such property.

Here we consider a generalization of the classical discriminant of the polynomial. This generalization naturally includes the classical discriminant and its analogs emerging when the q-differential and difference operators are used. The aim of this research is to propose an efficient algorithm for calculating the parametric representation of all components of the g-discriminant set $\mathcal{D}_g(f)$ of the monic polynomial $f_n(x)$ of degree n.

Define the q-**bracket** $[a]_q$, q-**Pochhammer symbol** $(a;q)_n$, q-**factorial** $[n]_q!$, q-**binomial coefficients (Gaussian) coefficients** $\begin{bmatrix} n \\ k \end{bmatrix}_q$, g-**binomial** $\{x;t\}_{n;g}$ as follows: $[a]_q = \dfrac{q^a-1}{q-1}$, $(a;q)_n = \prod\limits_{k=0}^{n-1}\left(1-aq^k\right)$, $(a;q)_0 = 1$, $[n]_q! = \prod\limits_{k=1}^{n}[k]_q = \dfrac{(q;q)_n}{(1-q)^n}$, $q \neq 1$, $\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q!\,[k]_q!} = \prod\limits_{i=1}^{k}\frac{q^{n-i+1}-1}{q^i-1}$, $\{x;t\}_{n;g} \equiv \prod\limits_{i=0}^{n-1}(x-g^i(t))$, $\{x;t\}_{0;q} = 1$. Here g^k is the k-th iteration of the diffeomorphism g, $k \in \mathbb{Z}$. As $q \to 1$, all these objects become classical.

Let $f_n(x)$ be is a monic polynomial of degree n with complex coefficients defined by $f_n(x) \stackrel{\text{def}}{=} x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$. Let $\mathbb{P}$ be the space of polynomials over $\mathbb{R}$ and let $g : \mathbb{R} \to \mathbb{R} : x \mapsto qx + \omega$, $q, \omega \in \mathbb{R}$, $q \neq \{-1, 0\}$, be a linear diffeomorphism on $\mathbb{R}$ that induces a linear **Hahn operator** $\mathcal{A}_g$ on $\mathbb{P}$, satisfying the following two conditions: (1) the degree reduction: $\deg(\mathcal{A}_g f_n)(x) = n-1$; in particular, $\mathcal{A}_g x = 1$; (2) Leibnitz rule analogue: $(\mathcal{A}_g x f_n)(x) = f_n(x) + g(x)(\mathcal{A}_g f_n)(x)$.

The Hahn operator $\mathcal{A}_g$ called below g-**derivative** has the form

$$(\mathcal{A}_g f)(x) \stackrel{\text{def}}{=} \begin{cases} \dfrac{f(qx+\omega)-f(x)}{(q-1)x+\omega}, & x \neq \omega_0, \\ f'(\omega_0), & x = \omega_0, \end{cases}$$

where $\omega_0 = \omega/(1-q)$ is the fixed point of g. Parameters q and ω are satisfied the following conditions $q, \omega \in \mathbb{R}$, $q \neq \{-1, 0\}$ and $(q, \omega) \neq (1, 0)$. The g-derivative $\mathcal{A}_g$ can be considered as a generalization of the q-differential Jackson operator $\mathcal{A}_q$ at $\omega = 0$, $q \neq 1$, as the difference operator Δ_ω at $q = 1$ and as the classical derivative d/dx in the limit $q \to 1$ and $\omega = 0$.

The q-calculus has became a part of the more general construct called quantum calculus [1, 2]. It has numerous applications in various fields of modern mathematics and theoretical physics. The pair of roots t_i, t_j, $i, j = 1, \ldots, n$, $i \neq j$ of the polynomial $f_n(x)$ is called g-**coupled** if $g(t_i) = t_j$.

Problem 1. In the coefficient space $\Pi \equiv \mathbb{C}^n$ of the polynomial $f_n(x)$, investigate the g-**discriminant set** denoted $\mathcal{D}_g(f_n)$ on which this polynomial has at least one pair of g-coupled roots.

The sequence $\mathrm{Seq}_g^{(k)}(t_1)$ of g-coupled roots of length k is defined as the finite sequence $\{t_i\}$, $i = 1, \ldots, k$ in which each term, beginning with the second one, is a g-coupled root of the preceding term: $g(t_i) = t_{i+1}$. The initial root t_1 is called the generating root of the sequence $\mathrm{Seq}_g^{(k)}(t_1)$.

For each fixed set of parameters q, ω, the g-discriminant set $\mathcal{D}_g(f_n)$ consists of a finite set of varieties $\mathcal{V}_k$ on each of which $f_n(x)$ has k sequences $\mathrm{Seq}_g^{(l_i)}(t_i)$ of g-coupled roots of length $_i$ with different generating roots t_i, $i = 1, \ldots, k$. To obtain an expression for the generalized (sub)discriminant of the polynomial $f_n(x)$ in terms of its coefficients, any method available in the classical elimination theory can be used. If we replace the derivative $f_n'(x)$ by the polynomial $\mathcal{A}_g f_n(x)$, then any matrix method for calculating the resultant of a pair of polynomials gives an expression of the generalized k-th subdiscriminant $D_g^{(k)}(f_n)$ [3].

Theorem 2. *The polynomial $f_n(x)$ has exactly $n-d$ different sequences of g-coupled roots, iff the first nonzero element in the sequence of i-th generalized subdiscriminants $D_g^{(i)}(f_n)$ is the subdiscriminat $D_g^{(d)}(f_n)$ with the index d.*

Consider the partition $\lambda = [1^{n_1}2^{n_2}3^{n_3}\ldots]$ of a natural number n. Every partition λ of n determines the structure of the g-coupled roots of the polynomial $f_n(x)$, and this structure is associated with the algebraic variety $\mathcal{V}_l^i$, $i = 1, \ldots, p_l(n)$ of dimension l corresponding to the number of different generating roots t_i in the coefficient space Π. The partition $\left[n^1\right]$ corresponding to the case when there is a unique sequence of roots of length n specified by the generating root t_1. Then, the polynomial $f_n(x)$ is a g-binomial $\{x; t_1\}_{n;g}$ and its coefficients a_i can be represented in terms of the elementary symmetric polynomials $\sigma_i(x_1, x_2, \ldots, x_n)$ calculated on the roots $g^j(t_1)$, $j = 0, \ldots, n-1$, $a_i = (-1)^i \sigma_i\left(t_1, g(t_1), \ldots, g^{n-1}(t_1)\right)$, $i = 1, \ldots, n$.

Theorem 3 ([4]). *Let there be a variety $\mathcal{V}_l$, $\dim \mathcal{V}_l = l$ on which the polynomial $f_n(x)$ has different sequences of g-coupled roots and the sequence of roots $\mathrm{Seq}_g^{(m)}(t_1)$ has length $m > 1$. The roots of the other sequences are not g-coupled with all roots of the sequence $\mathrm{Seq}_g^{(m)}(t_1)$. Let $\mathbf{r}_l(t_1, \ldots, t_l)$ be a parameterization of the variety $\mathcal{V}_l$. Then for $0 < k < n$, the formula*

$$\mathbf{r}_l(t_1, \ldots, t_l, t_{l+1}) = \mathbf{r}_l(t_1, \ldots, t_l) + \sum_{i=1}^{k} \begin{bmatrix} k \\ i \end{bmatrix}_q \frac{[m-i]_q!}{[m]_q!} \left(\mathcal{A}_g^i \mathbf{r}_l\right)(t_1)\{t_{l+1}; t_1\}_{i;g} \tag{1}$$

specifies a polynomial parameterization of the part of $\mathcal{V}_{l+1}$ on which there are two sequences of roots $\mathrm{Seq}_g^{(m-k)}(g^k(t_1))$ and $\mathrm{Seq}_g^{(k)}(g(t_{l+1}))$, and the other sequences of roots are the same as on the original variety $\mathcal{V}_l$.

The structure of singular points of each variety $\mathcal{V}_{l+1}$ can be described in terms of varieties $\mathcal{V}_l$, connected with it by (1).

The same results on the structure and parametrization of the g-discriminant set $\mathcal{D}_g(f_n)$ can be obtained for other variants of g-derivative, e.g. for the case of Hahn symmetric derivative [2]

$$\widetilde{\mathcal{A}}_{q,\omega} f(t) = \frac{f(qt+\omega) - f(q^{-1}(t-\omega))}{(q-q^{-1})t + (1+q^{-1})\omega}$$

as well.

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Derivative Thomae formula for singular half-periods

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A complete generalization of Thomae theorems [1] in hyperelliptic case is obtained, that is values at zero of the lowest non-vanishing derivatives of theta functions with singular characteristics of arbitrary multiplicity are expressed in terms of branch points $\{e_i\}_{i=1}^{2g+2}$ and period matrix ω.

Theorem 1. *Let $\mathcal{I}_\mathfrak{m} \cup \mathcal{J}_\mathfrak{m}$ with $\mathcal{I}_\mathfrak{m} = \{i_1, \ldots, i_{g+1-2\mathfrak{m}}\}$ and $\mathcal{J}_1 = \{j_1, \ldots, j_{g+1+2\mathfrak{m}}\}$ be a partition of the set of indices of all $2g+2$ branch points of hyperelliptic curve, and $[\mathcal{I}_\mathfrak{m}]$ denotes a singular characteristic of multiplicity $\mathfrak{m}$ corresponding to $\mathcal{A}(\mathcal{I}_\mathfrak{m}) - K$. Let $\Delta(\mathcal{I}_\mathfrak{m})$ and $\Delta(\mathcal{J}_\mathfrak{m})$ be Vandermonde determinants built from $\{e_i \mid i \in \mathcal{I}_\mathfrak{m}\}$ and $\{e_j \mid j \in \mathcal{J}_\mathfrak{m}\}$. Then with a set $\mathcal{K} \subset \mathcal{J}_\mathfrak{m}$ of cardinality $\mathfrak{k} = 2\mathfrak{m} - 1$ or $2\mathfrak{m}$ the following relation holds*

$$\frac{\partial}{\partial v_{n_1}} \cdots \frac{\partial}{\partial v_{n_\mathfrak{m}}} \theta[\mathcal{I}_\mathfrak{m}](v)\big|_{v=0} = \epsilon \left(\frac{\det \omega}{\pi^g}\right)^{1/2} \Delta(\mathcal{I}_\mathfrak{m})^{1/4} \Delta(\mathcal{J}_\mathfrak{m})^{1/4} \times$$
$$\times \sum_{\substack{p_1,\ldots,p_\mathfrak{m} \in \mathcal{K} \\ \text{all different}}} \prod_{i=1}^{\mathfrak{m}} \frac{\sum_{j=1}^{g} (-1)^{j-1} s_{j-1}(\mathcal{I}_\mathfrak{m} \cup \mathcal{K}^{(p_i)}) \omega_{j n_i}}{\prod_{k \in \mathcal{K} \setminus \{p_1,\ldots,p_\mathfrak{m}\}} (e_{p_i} - e_k)}. \quad (1)$$

where $s_j(\mathcal{I})$ denotes an elementary symmetric polynomial of degree j in branch points with indices from $\mathcal{I}$, and $\mathcal{K}^{(p_i)} = \mathcal{K} \backslash \{p_i\}$, and ϵ satisfies $\epsilon^8 = 1$.

Theta function with characteristic $[\varepsilon]$ is defined by the formula

$$\theta[\varepsilon](v;\tau) = \exp\left(\mathrm{i}\pi(\varepsilon'^t/2)\tau(\varepsilon'/2) + 2\mathrm{i}\pi(v + \varepsilon/2)^t \varepsilon'/2\right) \theta(v + \varepsilon/2 + \tau\varepsilon'/2; \tau). \quad (2)$$

All half-integer characteristics are represented by partitions of $2g+2$ indices of the form $\mathcal{I}_m \cup \mathcal{J}_m$ with $\mathcal{I}_m = \{i_1, \ldots, i_{g+1-2m}\}$ and $\mathcal{J}_m = \{j_1, \ldots, j_{g+1+2m}\}$, where m runs from 0 to $[(g+1)/2]$, and $[\cdot]$ means the integer part. Number m is called *multiplicity*. Infinity with index $2g+2$ is usually omitted in the sets. The characteristic $[\mathcal{I}_m]$ corresponds to partition $\mathcal{I}_m \cup \mathcal{J}_m$ in the following way

$$\sum_{i \in \mathcal{I}_m} \mathcal{A}(e_i) - K = \varepsilon(\mathcal{I}_m)/2 + \tau\varepsilon'(\mathcal{I}_m)/2,$$

where K denotes the vector of Riemann constants. According to Riemann theorem $\theta(v + \mathcal{A}(\mathcal{I}_m) - K)$ vanishes to order m at $v = 0$, Characteristics of multiplicity 0 and 1 are called non-singular even and odd, respectively. All other characteristics are called *singular*.

Some further results are derived from Theorem 1.

Corollary 2. *Let $\mathcal{I}_2 \cup \mathcal{J}_2$ with $\mathcal{I}_2 = \{i_1, \ldots, i_{g-\mathfrak{k}}\}$ and $\mathcal{J}_2 = \{j_1, \ldots, j_{g+1+\mathfrak{k}}\}$, where $\mathfrak{k} = 3$ or 4, be a partition of the set of $2g+1$ indices of finite branch points, such that singular characteristic $[\mathcal{I}_2]$, corresponding to $\mathcal{A}(\mathcal{I}_2) - K$, has multiplicity 2. Let $\Delta(\mathcal{I}_2)$ and $\Delta(\mathcal{J}_2)$ be Vandermonde determinants built from $\{e_i \mid i \in \mathcal{I}_2\}$ and $\{e_j \mid j \in \mathcal{J}_2\}$. Then*

$$\frac{\partial}{\partial v_{n_1}} \frac{\partial}{\partial v_{n_2}} \theta[\mathcal{I}_2](v)\big|_{v=0} = \epsilon \left(\frac{\det \omega}{\pi^g}\right)^{1/2} \Delta(\mathcal{I}_2)^{1/4} \Delta(\mathcal{J}_2)^{1/4} \sum_{i,j=1}^{g} (\hat{S}[\{\mathcal{I}_2\}])_{i,j} \omega_{i n_1} \omega_{j n_2} \quad (3)$$

with $g \times g$ matrix

$$(\hat{S}[\mathcal{I}_2])_{i,j} = (-1)^{i+j}\Big(2s_{i-\mathfrak{k}+1}(\mathcal{I}_2)s_{j-\mathfrak{k}+1}(\mathcal{I}_2) - s_{i-\mathfrak{k}+2}(\mathcal{I}_2)s_{j-\mathfrak{k}}(\mathcal{I}_2) - s_{i-\mathfrak{k}}(\mathcal{I}_2)s_{j-\mathfrak{k}+2}(\mathcal{I}_2)\Big), \qquad (4)$$

where ϵ satisfies $\epsilon^8 = 1$, and elementary symmetric functions $s_l(\mathcal{I}_2)$ are replaced by zero when $l < 0$.

Theorem 3. *For hyperelliptic curves of genera $g \geqslant 3$, when characteristics of multiplicity 2 exist, rank of every matrix of second derivative theta constants equals three, that is*

$$rank\big(\partial_v^2\theta[\mathcal{I}_2]\big) = 3.$$

Therefore, $\det\big(\partial_v^2\theta[\mathcal{I}_2]\big) = 0$ *in genera $g > 3$.*

Conjecture 4. *With a characteristic $[\mathcal{I}_\mathfrak{m}]$ of multiplicity $\mathfrak{m}$ corresponding to a partition $\mathcal{I}_\mathfrak{m} \cup \mathcal{J}_\mathfrak{m}$ with $\mathcal{I}_\mathfrak{m} = \{i_1, \ldots, i_{g-\mathfrak{k}}\}$ and $\mathcal{J}_\mathfrak{m} = \{j_1, \ldots, j_{g+1+\mathfrak{k}}\}$, where $\mathfrak{k} = 2\mathfrak{m} - 1$ or $2\mathfrak{m}$, of indices of $2g+1$ finite branch points the following holds*

$$\partial_u^\mathfrak{m}\theta[\mathcal{I}_\mathfrak{m}](\omega^{-1}u)\big|_{u=0} = \epsilon\left(\frac{\det\omega}{\pi^g}\right)^{1/2}\Delta(\mathcal{I}_\mathfrak{m})^{1/4}\Delta(\mathcal{J}_\mathfrak{m})^{1/4}\hat{S}[\mathcal{I}_\mathfrak{m}], \qquad (5)$$

where u are non-normalized variables, and order $\mathfrak{m}$ tensor $\hat{S}[\mathcal{I}_\mathfrak{m}]$ belongs to the $\mathfrak{m}$-th tensor power $\mathcal{S}_{2\mathfrak{m}-1}^{\otimes\mathfrak{m}}$ of the vector space $\mathcal{S}_{2\mathfrak{m}-1}$ spanned by $2\mathfrak{m}-1$ vectors $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{2\mathfrak{m}-2}$ such that $\mathbf{s}_d = \big(s_{j-\mathfrak{k}+d}(\mathcal{I}_\mathfrak{m})\big)_{j=1}^g$. The basis spanning $\hat{S}(\mathcal{I}_\mathfrak{m})$ could be found from partitions of $\mathfrak{m}(\mathfrak{m}-1)$ of length $\mathfrak{m}$ formed from numbers $\{0, 1, \ldots, 2\mathfrak{m}-2\}$.

As a byproduct a generalization of Bolza formulas [2] are deduced.

Proposition 5. *Let $\mathcal{I}_\mathfrak{m}$ be a set of $g-\mathfrak{k}$ indices, and $\mathfrak{m} = [(\mathfrak{k}+1)/2]$. Elementary symmetric polynomials in branch points $\{e_i \mid i \in \mathcal{I}_\mathfrak{m}\}$ of genus g hyperelliptic curve with period matrix ω are defined by*

$$s_j(\mathcal{I}_\mathfrak{m}) = (-1)^j \frac{\partial^\mathfrak{m}_{u_{2\mathfrak{k}-4(\mathfrak{m}-1)-1},\ldots,u_{2\mathfrak{k}-5},u_{2\mathfrak{k}+2j-1}}\theta[\mathcal{I}_\mathfrak{m}](\omega^{-1}u)}{\partial^\mathfrak{m}_{u_{2\mathfrak{k}-4(\mathfrak{m}-1)-1},\ldots,u_{2\mathfrak{k}-5},u_{2\mathfrak{k}-1}}\theta[\mathcal{I}_\mathfrak{m}](\omega^{-1}u)}\Big|_{u=0}.$$

In particular,

$$e_\iota = -\frac{\partial^{[g/2]}_{u_{2(g mod 2)+1},\ldots,u_{2g-7},u_{2g-1}}\theta[\{\iota\}](\omega^{-1}u)}{\partial^{[g/2]}_{u_{2(g mod 2)+1},\ldots,u_{2g-7},u_{2g-3}}\theta[\{\iota\}](\omega^{-1}u)}\Big|_{u=0}.$$

Here $u = \omega v$ are non-normalized coordinates of Jacobian of the curve.

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Kuratowski limits of subsets of real line and their applications to pretangent spaces

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Let (X,d) be an unbounded metric space and $\tilde{r} = (r_n)_{n\in\mathbb{N}}$ be a scaling sequence of positive real numbers tending to infinity. We define the pretangent and tangent spaces $\Omega^X_{\infty,\tilde{r}}$ to (X,d) at infinity as metric spaces whose points are equivalence classes of sequences $(x_n)_{n\in\mathbb{N}} \subset X$ which tend to infinity with the speed of $\tilde{r}$. The detailed description of constructions of these spaces and their basic properties see, e. g., in [2].

Let (Y,δ) be a metric space. For any sequence $(A_n)_{n\in\mathbb{N}}$ of nonempty sets $A_n \subseteq Y$, the *Kuratowski limit inferior* of $(A_n)_{n\in\mathbb{N}}$ is the subset $\mathop{Li}\limits_{n\to\infty} A_n$ of Y defined by the rule:

$$\left(y \in \mathop{Li}\limits_{n\to\infty} A_n\right) \Leftrightarrow \left(\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N}\ \forall n \geq n_0 \colon B(y,\varepsilon) \cap A_n \neq \varnothing\right),$$

where $B(y,\varepsilon)$ is the open ball of radius $\varepsilon > 0$ centered at the point $y \in Y$,

$$B(y,\varepsilon) = \{x \in Y : \delta(x,y) < \varepsilon\}.$$

Similarly, the *Kuratowski limit superior* of $(A_n)_{n\in\mathbb{N}}$ can be defined as the subset $\mathop{Ls}\limits_{n\to\infty} A_n$ of Y for which

$$\left(y \in \mathop{Ls}\limits_{n\to\infty} A_n\right) \Leftrightarrow \left(\forall \varepsilon > 0\ \forall n \in \mathbb{N}\ \exists n_0 \geq n : B(y,\varepsilon) \cap A_{n_0} \neq \varnothing\right).$$

The Kuratowski limit inferior and limit superior are basic concepts of set-valued analysis in metric spaces and have numerous applications (see, for example, [1]).

We denote $tA := \{tx : x \in A\}$ for any nonempty set $A \subseteq \mathbb{R}$ and $t \in \mathbb{R}$, and, $\nu_0 := \tilde{X}^0_{\infty,\tilde{r}} \in \Omega^X_{\infty,\tilde{r}}$ for any pretangent space $\Omega^X_{\infty,\tilde{r}}$ of an unbounded metric space (X,d). Moreover, for every scaling sequence $\tilde{r}$, we denote by $\boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}}$ the set of all pretangent at infinity spaces to (X,d) with respect to $\tilde{r}$. Write

$$Sp\left(\Omega^X_{\infty,\tilde{r}}\right) := \{\rho(\nu_0,\nu) \colon \nu \in \Omega^X_{\infty,\tilde{r}}\} \text{ and } Sp(X) := \{d(p,x) \colon x \in X\}.$$

Proposition 1. *Let (X,d) be an unbounded metric space, $p \in X$, $\tilde{r} = (r_n)_{n\in\mathbb{N}}$ be a scaling sequence and let $\tilde{\mathbf{R}}$ be the set of all infinite subsequences of $\tilde{r}$. Then the equalities*

$$\bigcup_{\Omega^X_{\infty,\tilde{r}} \in \boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}}} Sp\left(\Omega^X_{\infty,\tilde{r}}\right) = \mathop{Li}\limits_{n\to\infty} \left(\frac{1}{r_n} Sp(X)\right),$$

$$\bigcup_{\Omega^X_{\infty,\tilde{r}'} \in \boxtimes^X_{\infty,\tilde{r}'},\ \tilde{r}' \in \tilde{\mathbf{R}}} Sp(\Omega^X_{\infty,\tilde{r}'}) = \mathop{Ls}\limits_{n\to\infty} \left(\frac{1}{r_n} Sp(X)\right)$$

hold.

Corollary 2. *Let (X,d) be an unbounded metric space, $\tilde r$ be a scaling sequence and let ${}^1\Omega^X_{\infty,\tilde r}$ be tangent and separable. Then we have*

$$\mathop{Li}_{n\to\infty}\left(\frac{1}{r_n}Sp(X)\right)=\mathop{Ls}_{n\to\infty}\left(\frac{1}{r_n}Sp(X)\right)=Sp\left({}^1\Omega^X_{\infty,\tilde r}\right).$$

Corollary 3. *Let (X,d) be an unbounded metric space, $\tilde r$ be a scaling sequence. Then the sets*

$$\bigcup_{\Omega^X_{\infty,\tilde r}\in\boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}}} Sp(\Omega^X_{\infty,\tilde r}) \quad \textit{and} \quad \bigcup_{\Omega^X_{\infty,\tilde r'}\in\boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}'},\ \tilde r'\in\tilde{\mathbf{R}}} Sp(\Omega^X_{\infty,\tilde r'})$$

are closed subsets of $[0,\infty)$.

Recall that a metric space (Y,δ) is said to be *strongly rigid* if for all $x,y,z,w\in Y$ the conditions $\delta(x,y)=\delta(w,z)$ and $x\neq y$ imply that $\{x,y\}=\{z,w\}$. Let us consider a strongly rigid metric space (Y,δ) such that:

(i_1) $\delta(x,y)<2$ for all points $x,\,y\in Y$; (i_2) $\sup\{\delta(x,y):x,y\in Y\}=2$;

(i_3) The cardinality of the open ball $B(y^*,r)=\{y\in Y:\delta(y,y^*)<r\}$ is finite for every $r\in(0,2)$ and every $y^*\in Y$.

Corollary 4. *Let (X,d) be an unbounded metric space, $\tilde r$ be a scaling sequence, $\Omega^X_{\infty,\tilde r}$ be tangent and let (Y,δ) be a strongly rigid metric space satisfying conditions (i_1)-(i_3). If $Y_1\subseteq Y$ and $f:\Omega^X_{\infty,\tilde r}\to Y_1$ is an isometry, then $\Omega^X_{\infty,\tilde r}$ is finite.*

Example 5. Let (Y,δ) be a metric space with $Y=\mathbb{N}$ and the metric δ defined such that:

$$\delta(1,2)=1+\frac{1}{2};$$
$$\delta(1,3)=1+\frac{2}{3},\quad \delta(2,3)=1+\frac{3}{4};$$
$$\delta(1,4)=1+\frac{4}{5},\quad \delta(2,4)=1+\frac{5}{6},\quad \delta(3,4)=1+\frac{6}{7};$$
$$\delta(1,5)=1+\frac{7}{8},\quad \delta(2,5)=1+\frac{8}{9},\quad \delta(3,5)=1+\frac{9}{10},\quad \delta(4,5)=1+\frac{10}{11};$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .$$

Then (Y,δ) is a countable, complete and strongly rigid metric space satisfying conditions (i_1)-(i_3). By Corollary 4 no tangent space $\Omega^X_{\infty,\tilde r}$ is isometric to (Y,δ).

Corollary 6. *Let (X,d) be an unbounded metric space and let $\tilde r$ be a scaling sequence. Then the following statements are equivalent:*

(i) *There is a single-point pretangent space $\Omega^X_{\infty,\tilde r}$;*

(ii) *All $\Omega^X_{\infty,\tilde r}$ are single-point;*

(iii) *The equality*

$$\mathop{Li}_{n\to\infty}\left(\frac{1}{r_n}Sp(X)\right)=\{0\}$$

holds.

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Algebraic and geometric questions about a FTL physics

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The recent proposal of a negative mass fluid to explain both the dark matter and energy [7] has renovated the interest for cosmological solutions based upon non-ordinary masses. Challenging the Λ-CDM paradigm, some fringe models are grounded on hypothetical interactions with antimatter [5] whereas others suppose the influence of faster than light (FTL) imaginary mass ([4], [6], [8]). More than a decade ago ([1, 2, 3]) we supplied an organic description of all the possible states (positive, negative and imaginary mass) subsequent to modified Lorentz's equations giving physical significance to the energetic condition $absE < m_0c^2$. Namely, we assumed that a fermion could pass from negative energy (identified as antimatter) to positive levels (i.e., the ordinary matter) through the interval between $-m_0c^2$ and $+m_0c^2$ where it would behave like a luxon ($v = c$)or a tachyon ($v > c$) keeping its half-integer spin. We wish to illustrate the algebraic and geometric questions behind a so formulated FTL physics, included a falsification test currently being assembled at CERN's Antiproton Decelerator.

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НТБ ОНАХТ

Algorithms for solving an algebraic equation

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For finding global approximate solutions to an algebraic equation in n unknowns, the Hadamard open polygon for the case $n = 1$ and Hadamard polyhedron for the case $n = 2$ are used. The solutions thus found are transformed to the coordinate space by a translation (for $n = 1$) and by a change of coordinates that uses the curve uniformization (for $n = 2$). Next, algorithms for the local solution of the algebraic equation in the vicinity of its singular (critical) point for obtaining asymptotic expansions of one-dimensional and two-dimensional branches are presented for $n = 2$ and $n = 3$. Using the Newton polygon (for $n = 2$), the Newton polyhedron (for $n = 3$), and power transformations, this problem is reduced to situations similar to those occurring in the implicit function theorem. In particular, the local analysis of solutions to the equation in three unknowns leads to the uniformization problem of a plane curve and its transformation to the coordinate axis. Then, an asymptotic expansion of a part of the surface under examination can be obtained in the vicinity of this axis. Examples of such calculations are presented.

Для нахождения глобальных приближённых решений алгебраического уравнения с n неизвестными при $n = 1$ предлагается ломаная Адамара, а при $n = 2$ — многогранник Адамара. Найденные решения переводятся в координатное подпространство: для $n = 1$ — сдвигом, а для $n = 2$ — заменой координат, использующей униформизацию кривой. Затем излагаются алгоритмы локального решения алгебраического уравнения вблизи особой (критической) точки для $n = 2$ и $n = 3$ для получения асимптотических разложений одномерных и двумерных ветвей. С помощью многоугольника Ньютона (при $n = 2$), многогранника Ньютона (при $n = 3$) и степенных преобразований эта задача сводится к ситуациям, аналогичным теореме о неявной функции. В частности, при локальном анализе решений одного уравнения от трёх неизвестных приходим к задаче об униформизации плоской алгебраической кривой и преобразовании её в координатную ось. После этого вблизи этой оси можно получить асимптотическое разложение куска изучаемой поверхности. Приведены примеры таких вычислений.

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Around the homologous sphere of Poincare and its applications

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Topology of a 3-dim manifolds defined by the system of equations

$$|z_1|^2 + |z_2|^2 + |z_3|^2 - 1 = 0, \quad z_1^l + z_2^m + z_3^n = 0, \tag{1}$$

where $z_k = x_k + iy_k$ are the complex coordinates, depends from the values of the parameters $l,\ m,\ n$. In the case $l = 2,\ \ m = 3,\ \ n = 5$ the manifold defined by the conditions (10) is a famous homologous sphere of Poincare, which has a set of homologies same with standard $3D$-sphere $|z_1|^2 + |z_2|^2 = 1$, but differs from it by self fundamental group. It has an important applications in various branch of modern algebraic topology (J.Milnor,1968).

In this report will be told how to represent the homologous sphere defined by intersection of the fife-dimensional sphere with singular manifold ($l = 2,\ \ m = 3,\ \ n = 5$)

$$|z_1|^2 + |z_2|^2 + |z_3|^2 = 1, \quad z_1^2 + z_2^3 + z_3^5 = 0. \tag{2}$$

in the form of an explicit expression for one function between of the four variables $H(x, y, u, v) = 0$.

Theorem 1. *In the Eulerian coordinates*

$$z_1 = \cos(\theta)\,\mathrm{e}^{-2/3\,i\sqrt{3}\phi},\ \ z_2 = -\sin(\theta)\sin(1/2\,\beta)\,\mathrm{e}^{-1/2\,i\left(\alpha-\delta+4/3\,\sqrt{3}\phi\right)},$$

$$z_3 = \sin(\theta)\cos(1/2\,\beta)\,\mathrm{e}^{1/2\,i\left(\alpha+\delta-4/3\,\sqrt{3}\phi\right)}, \tag{3}$$

the equation of the unit five-dimensional sphere is identically satisfied and the equation of the orbifold $z_1^2 + z_2^3 + z_3^5 = 0$ *takes the form*

$$\begin{gathered}
(\cos(\theta))^2\, e^{-4/3\,i\sqrt{3}\phi} - \sin(\theta)\sin(1/2\,\beta) e^{-1/2\,i\sqrt{3}\left(\sqrt{3}\alpha-\sqrt{3}\delta+4\,\phi\right)} + \\
+\sin(\theta)\sin(1/2\,\beta) e^{-1/2\,i\sqrt{3}\left(\sqrt{3}\alpha-\sqrt{3}\delta+4\,\phi\right)}\,(\cos(1/2\,\beta))^2 + \\
+\sin(\theta)\sin(1/2\,\beta) e^{-1/2\,i\sqrt{3}\left(\sqrt{3}\alpha-\sqrt{3}\delta+4\,\phi\right)}\,(\cos(\theta))^2 - \\
-\sin(\theta)\sin(1/2\,\beta) e^{-1/2\,i\sqrt{3}\left(\sqrt{3}\alpha-\sqrt{3}\delta+4\,\phi\right)}\,(\cos(\theta))^2\,(\cos(1/2\,\beta))^2 + \\
+\sin(\theta)\,(\cos(1/2\,\beta))^5\, e^{5/6\,i\sqrt{3}\left(\sqrt{3}\alpha+\sqrt{3}\delta-4\,\phi\right)} - \\
-2\,\sin(\theta)\,(\cos(1/2\,\beta))^5\, e^{5/6\,i\sqrt{3}\left(\sqrt{3}\alpha+\sqrt{3}\delta-4\,\phi\right)}\,(\cos(\theta))^2 + \\
+\sin(\theta)\,(\cos(1/2\,\beta))^5\, e^{5/6\,i\sqrt{3}\left(\sqrt{3}\alpha+\sqrt{3}\delta-4\,\phi\right)}\,(\cos(\theta))^4 = 0.
\end{gathered} \tag{4}$$

Using then the variable χ, defined by the condition $\mathrm{e}^{5/2\,i\alpha+5/2\,i\delta-10/3\,i\sqrt{3}\phi} - \mathrm{e}^{5\,\chi} = 0$, we express the variable ϕ as $\phi = -1/4\,i\,(i\alpha + i\delta - 2\,\chi)\sqrt{3}$ and after separation of the real and imaginary parts of complex equation ((4)), are obtained two equations into the five variables $\alpha,\ \delta,\ \chi$ and $\theta,\ \beta$.

As result of elimination of the variable χ from both equations is derived equation of homologous sphere of Poincare in the form of one function of the four variables. The equation is the summa of the functions $\sin()$ *and* $\cos()$ *with linear arguments. It contains more than* 200 *items.*

НТБ ОНАУ

By analogy can be considered the case of tetrahedral space which corresponds to the intersection of the fife-dimensional sphere with singular manifold ($l = 2,\quad m = 3,\quad n = 4$)

$$|z_1|^2 + |z_2|^2 + |z_3|^2 = 1, \quad z_1^2 + z_2^3 + z_3^4 = 0. \tag{5}$$

and the octahedral space defined by the condition

$$|z_1|^2 + |z_2|^2 + |z_3|^2 = 1, \quad z_1^2 + z_2^3 + z_2 z_3^3 = 0. \tag{6}$$

Proposition 2. *The relation between a four variables $F(x, y, a, b) = 0$ which defines some $3D$-variety can be considered as General Integral of the par of the second order ODE's $f(x, y, y', y'') = 0$ and $g(a, b, b;, b'') = 0$.*

$$\swarrow F(x, y, a, b) = 0 \searrow$$

$$y'' = f(x, y, y') \quad \longleftrightarrow \quad b'' = h(a, b, b')$$

The Liouville-Tresse invariants of both equations with respect to non degenerate transformations of the variables $x = X(u, v), y = Y(u, v)$ or $a = A(p, q), b = B(p, q)$ can be used for the studies of topological properties of the manifold $F(x, y, a, b) = 0$.

Theorem 3. *Spatial homogeneous the first order system of the equations*

$$\begin{aligned}
\frac{d}{ds}x(s) &= 4\,a_0\,z^2 + (4\,a_2\,y + (3\,a_1 - b_2)\,x)\,z + 4\,a_{22}\,y^2 + \\
&\quad + (3\,a_{12} - 2\,b_{22})\,xy + (2\,a_{11} - b_{12})\,x^2, \\
\frac{d}{ds}y(s) &= 4\,b_0\,z^2 + ((3\,b_2 - a_1)\,y + 4\,b_1\,x)\,z + (2\,b_{22} - a_{12})\,y^2 + \\
&\quad + (-2\,a_{11} + 3\,b_{12})\,xy + 4\,b_{11}\,x^2, \\
\frac{d}{ds}z(s) &= (-b_2 - a_1)\,z^2 + ((-2\,b_{22} - a_{12})\,y - b_{12}\,x - 2\,a_{11}\,x)\,z,
\end{aligned} \tag{7}$$

is projective extension of planar polynomial differential systems

$$\frac{dx}{ds} = a_0 + a_1x + a_2y + a_{11}x^2 + a_{12}xy + a_{22}y^2, \quad \frac{dy}{ds} = b_0 + b_1x + b_2y + b_{11}x^2 + b_{12}xy + b_{22}y^2 \tag{8}$$

with the parameters a_i, a_{ij} and b_i, b_{ij}.

After eliminating of the variables ($y(x)$ or $z(x)$ it is reduced to the second-order differential equations $F(x, y, y', y'') = 0$.

As a result of the exclusion of the function y'' from the system

$$F(x, y, y', y'') = 0, \quad \frac{\partial F}{\partial y''} = 0,$$

a first order differential equation is arised $C(x, y, y') = 0$. Through each point M of integral curve $C = Q(x, y)$ passes the integral curve of the equation $F(x, y, y', y') = 0$, for which the point M is the return point of the second type and this allow to study the limit cycles of the system (6).

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On the generalization of the Darboux theorems

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We refer to [1] for the definitions concerning the category of MC^k-Frechet manifolds.
We prove that vector fields have local flows.

Theorem 1. *Let F be a Fréchet space, X an MC^k-vector field on $U \subset F$, $k \geqq 1$. There exists a real number $\alpha > 0$ such that for each $x \in U$ there exists a unique integral curve $\ell_x(t)$ satisfying $\ell_x(0) = x$ for all $t \in I = (-\alpha, \alpha)$. Furthermore, the mapping $\mathbb{F} : I \times U \to F$ given by $\mathbb{F}_t(x) = \mathbb{F}(t, x) = \ell_x(t)$ is of class MC^k.*

Therefore we are able to apply Moser's approach, that is constructing an appropriate isotopoy generated by a time dependent vector field that provides the chart transforming of symplectic forms to constant ones to prove the Darboux theorem in the category of MC^k-manifolds.

Definition 2. Let M be a bounded Fréchet manifold. We say that M is weakly symplectic if there exists a closed smooth 2-form ω such that it is weakly non-degenerate i.e. for all $x \in M$ and $v_x \in T_x M$

$$\omega_x(v_x, w_x) = 0 \tag{1}$$

for all $w_x \in T_x M$ implies $v_x = 0$.

Let F'_b be the strong dual of F and define the map $\omega_x^{\#} : F \to F'_b$ by

$$\langle w, \omega_x^{\#}(v) \rangle = \omega_x(w, v),$$

where $\langle \cdot, \cdot \rangle$ is a duality pairing. Condition 1 implies that $\omega^{\#}$ is injective.

Let $x \in U$ be fixed and define $H_x = \{\omega_x(y, .) \mid y \in F\}$, this is a subset of F'_b and its topology is induced from it. We assume that all Fréchet spaces are reflexive.

Lemma 3. *$\omega_x^{\#} : F \to H_x$ is an isomorphiam.*

Theorem 4. *Let (M, ω) be a weakly symplectic smooth bounded Fréchet manifold modeled on F. Let $\omega^t = \omega_0 + t(\omega - \omega_0)$ for $t \in [0, 1]$. Suppose that following hold*

(1) There exits an open star-shaped neighborhood $\mathcal{U}$ of zero such that for all $x \in \mathcal{U}$ the map $\omega_x^{t\#} : F \to H_x$ is isomorphism for each t,
(2) for $x \in \mathcal{U}$ the map $(\omega_x^{t\#})^{-1} : H_x \to E$ is smooth for each t.

Then there exists a coordinate chart $(\mathcal{V}, \varphi)$ around zero such that $\varphi^* \omega = \omega_0$.

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Discrete sets, discrete measures, quasicrystals Fourier, pure crystals

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Let μ be a measure in $\mathbb{R}^d$ which is a tempered distribution, let $\hat{\mu}$ be its Fourier transform, in a general case it is a tempered distribution. If μ and $\hat{\mu}$ are measures with closed discrete supports, then μ is called Fourier quasicrystal. For example let $\mu = \sum_{k\in\mathbb{Z}^d} \delta(x-n)$, where δ is Dirac's measure. By Poisson's formula, we get $\hat{\mu} = \mu$. If the support of μ is a finite union of translates of a single full-rang lattice, then μ is called a pure crystal. If the support of μ is a finite union of translates of several full-rang lattices, then μ is called a comb.

In our talk we show some well-known and new results when Fourier quasicrystal is a pure crystal or a comb. Some of these results we expand to the class of tempered distributions.

НТБ ОНАХТ

Algebraic-geometric aspects of function field analogues to abelian varieties

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This communication is a continuation of [6, 7, 8, 9].

Let p be a prime number, $q = p^n$, $\mathbb{F}_q$ be the field with q elements and characteristic p, $\mathbb{F}$ be a finite field extension of a finite field $\mathbb{F}_q$.
We extend the case of algebraic number fields [6, 7] to the case of function fields in characteristic $p > 0$ and construct function field analogues to abelian varieties of elliptic and hyperelliptic curves appeared in [8, 9]. In the last case we investigate function field analogues to abelian varieties which are Jacobian varieties of hyperelliptic curves in characteristic $p > 0$. Recall that for hyperelliptic curves the function field analogues to abelian varieties are function field analogues to Jacobian varieties of the curves. For Jacobians it is possible to define corresponding $p-$divisible groups. We plan to present results on function field analogues to p-divisible groups of the Jacobian varieties.

Moduli and estimates for hyperelliptic curves of genus $g \geq 2$ over $\mathbb{F}_p$.

Let

$$C : y^2 = f(x)$$

be an algebraic curve and let $Disk(C)$ be the discriminant of $f(x)$. Consider hyperelliptic curve of genus $g \geq 2$ over prime finite field $\mathbf{F}_p$

$$C_g : y^2 = f(x), \; D(f) \neq 0.$$

For projective closure of C_g the quasiprojective variety

$$S_{g,p} = \{\mathbf{P}^{2g+2}(\mathbf{F}_p) \setminus (Disk(C_g) = 0)\}$$

parametrizes all hyperelliptic curves of genus g over $\mathbf{F}_p$. By well known Weil bound (affine case)

$$|\#C_g(\mathbf{F}_p) - p| \leq 2g\sqrt{p}.$$

where $\#C$ is the number of points on the curve C over ground field. As we can see from Weil (and some more strong) bounds, for $p \geq 17$ any hyperelliptic curve of genus $g = 2$ has points in $\mathbb{F}_p$ for these prime p. Also for $g = 3$ every hyperelliptic (h) curve of genus 3 has points in $\mathbb{F}_p$ for $p \geq 37$. For $p = 2, 3, 5, 7, 11$ there are examples of h-curves of genus 2 that have not points in $\mathbb{F}_p$. By author's computations any h-curve of genus 2 over $\mathbb{F}_{13}$ has points in the field. Similarly, for $p = 2, 3, 5, 7, 11, 13, 17$ there are examples of h-curves of genus 3 that have not points in $\mathbb{F}_p$.

Theorem 1. [8]. *Let $p \equiv 3 \bmod 4$. Under $p \geq 11$ there is such $a \in \mathbb{F}_p$ that the equation*

$$y^2 = x^{\frac{p-1}{2}} + a$$

has no solutions in $\mathbb{F}_p$.

Global $\mathfrak{G}$-shtukas and local $\mathbb{P}$-shtukas [1, 2, 3, 4].

Definition 2. (Hartl, Rad [1, 2]) Let C be a smooth projective geometrically irreducible curve over $\mathbb{F}_q$. A global $\mathfrak{G}$-shtuka $\overline{\mathcal{G}}$ over an $\mathbb{F}_q$-scheme S is a tuple $(\mathcal{G}, s_1, \ldots, s_n, \tau)$ consisting of a $\mathfrak{G}$-torsor $\mathcal{G}$ over $C_S := C \times_{\mathbb{F}_q} S$, an n-tuple of (characteristic) sections $(s_1, \ldots, s_n) \in C^n(S)$ and a Frobenius connection τ defined outside the graphs of the sections s_i.

For Jacobian varieties it is possible to define corresponding p-divisible groups and their function field analogues.

Definition 3. (Hartl, Rad [1]) Let $\mathbb{P}$ be a flat affine group scheme of finite type over $Spec\, \mathbb{F}[[z]]$ and $\mathfrak{G}$ is a flat affine group scheme of finite type over a smooth projective geometrically irreducible curve over $\mathbb{F}_q$

Recall that local $\mathbb{P}$-shtukas are the functional field analogs of p-divisible groups with additional structure and moduli stacks of global $\mathfrak{G}$-shtukas are the functional field analogs for Shimura varieties. In some cases $\mathbb{P}$ is a paraholic Bruhat-Tits group scheme by Pappas, Rapoport [5] and $\mathfrak{G}$ is a parahoric Bruhat-Tits group scheme over a smooth projective curve over finite field $\mathbb{F}_q$ with q elements of characteristic p. Investigations by U. Hartl [3], by Hartl, Arasteh Rad [1, 2], by U. Hartl, E. Viehmann [4] continue works of V. G. Drinfeld , L. Lafforgue, G. Faltings.

If will sufficient time we plan to give a short review of history of these research.

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НТБ ОНАХТ

Extensions of almost orthosymmetric lattice bimorphisms

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Let E ,F and G be vector lattices. We say that a linear operator $T: E \to F$ is a lattice homomorphism if $T(x \vee y) = Tx \vee Ty$ for every $x, y \in E$. A bilinear map $\Phi : E \times F \to G$ is said to be positive if $|\Phi(x, y)| \leq \Phi(|x|, |y|)$ for all $x \in E$ and $y \in F$. The bilinear map $\Phi : E \times F \to G$ is said to be lattice bilinear map (or lattice bimorphism) whenever it is separately lattice homomorphisms for each variable or equivalently, $|\Phi(x, y)| = \Phi(|x|, |y|)$ for all $x \in E$ and $y \in F$. Let E and F be Archimedean vector lattices. A bilinear map $T : E \times E \to F$ is called an orthosymmetric if $x \wedge y = 0$ implies $T(x, y) = 0$ for all $x, y \in E$. A vector lattice E is called Dedekind complete if every non-empty subset of E which is bounded from above has a supremum. A Dedekind complete vector lattice M is said to be a Dedekind completion of the vector lattice E whenever E is Riesz isomorphic to a majorizing order dense Riesz subspace of M. Denote by E^δ the Dedekind completion of E. Every Archimedean vector lattice has a unique Dedekind completion. A vector lattice E is said to be universally complete if E is Dedekind complete and every pairwise disjoint positive vectors in E has a supremum in E. Every Archimedean vector lattice E has a universal completion E^u. It means that there exists a unique (up to a lattice isomorphism) universally complete vector lattice E^u such that E is Riesz isomorphic to an order dense Riesz subspace of E^u.

Definition 1. Let E and F be Archimedean vector lattices. A bilinear map $T : E \times E \to F$ is called an almost orthosymmetric if $x \wedge y = 0$ implies $T(x, y) \wedge T(y, x) = 0$ for all $x, y \in E$, [13].

Every orthosymmetric bilinear map is an almost orthosymmetric, but the converse is not always true.

Let E be an Archimedean vector lattice and F be a Dedekind complete vector lattice.

In this talk, we show that if $T : E \times E \to F$ is an almost orthosymmetric lattice bimorphism, then extension of T , $T^\sim : E^\delta \times E^\delta \to F$, is an almost orthosymmetric lattice bimorphism.

Theorem 2. *Let E be an Archimedean vector lattice and let E^δ be its Dedekind completion and let F be a Dedekind complete vector lattice. If $\Phi : E \times E \to F$ is an almost orthosymmetric lattice bimorphism, then Φ can be extended to an almost orthosymmetric lattice bimorphism $\Psi : E^\delta \times E^\delta \to F$.*

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The properties of the surface of Minkowski space, which determine the type of its Grassmann image

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The properties of the surface of the Euclidean space, which determine the values or boundaries of the values of the curvature of the Grassmann manifolds along planes tangential to the Grassmann image of the surface have been investigated [1], [2], [3]. The results of solving the similar problems for the surfaces of the Minkowski space depend on the type of their Grassmann image. In this paper, the properties of the surfaces of different types that determine the type of their Grassmann image are investigated.

In Minkowski space ${}^1R_4$ (with metric $ds^2 = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$), the submanifolds of the space-like and the time-like planes of the Grassman manifold $PG(2,4)$ are pseudo-Riemannian four-dimensional manifolds of the pseudo-Euclidean space ${}^3R_6$. The tangent space for each of these sub-manifolds is the space with a signature metric $(--++)$ [4].

The Grassmann image of the space-like (time-like) two-dimensional surface of the space ${}^1R_4$ is a two-dimensional submanifold of the manifold of time-like (space-like) planes [4]. The induced metric of the Grassmann image may be sign-definite, sign-indefinite or degenerated, and, hence, the Grassmann image can be a two-dimensional space-like, time-like or isotropic surface.

Proposition 1. *If the time-like surface $V^2 \subset {}^1R_4$ has a flat normal connection, then its Grassmann image is a time-like surface.*

Proposition 2. *If the time-like (space-like) surface $V^2 \subset {}^1R_4$ is minimal (maximum), then its Grassmann image is a space-like surface.*

Proposition 3. *If the surface $V^2 \subset {}^1R_4$ is a hypersurface of a some three-dimensional subspace, then the type of its Grassmann image coincides with the type of the surface.*

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The semigroup of star partial homeomorphisms of a finite deminsional Euclidean space

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We follow the terminology of [1, 2].

We shall introduce the notion of a star partial homeomorphism of a finite dimensional Euclidean space $\mathbb{R}^n$. By $\mathbf{St}_0(\mathbb{R}^n)$ we denote the set of all stars at the origin $\mathbf{0}$ of $\mathbb{R}^n$.

We describe the structure of the semigroup $\mathbf{PStH}_{\mathbb{R}^n}$ of star partial homeomorphisms of the space $\mathbb{R}^n$.

Proposition 1. *$\mathbf{PStH}_{\mathbb{R}^n}$ is an inverse submonoid of the symmetric inverse monoid $\mathscr{I}_{\mathfrak{c}}$.*

Proposition 2. *(i) An element α of $\mathbf{PStH}_{\mathbb{R}^n}$ is an idempotent if and only if $\alpha\colon S \to S$ is the identity map for some star $S \in \boldsymbol{St}_0(\mathbb{R}^n)$.*

(ii) The band $E(\mathbf{PStH}_{\mathbb{R}^n})$ is isomorphic to the semilattice $(\boldsymbol{St}_0(\mathbb{R}^n), \cap)$.

(iii) $\varepsilon \leqslant \iota$ in $E(\mathbf{PStH}_{\mathbb{R}^n})$ if and only if $\operatorname{dom}\varepsilon \subseteq \operatorname{dom}\iota$.

(iv) $\alpha \leqslant \beta$ in $\mathbf{PStH}_{\mathbb{R}^n}$ if and only if $\beta|_{\operatorname{dom}\alpha} = \alpha$.

Proposition 3. *Let be $\alpha, \beta \in \mathbf{PStH}_{\mathbb{R}^n}$. Then the following statements hold:*

(i) $\alpha\mathscr{R}\beta$ in $\mathbf{PStH}_{\mathbb{R}^n}$ if and only if $\operatorname{ran}\alpha = \operatorname{ran}\beta$;

(ii) $\alpha\mathscr{L}\beta$ in $\mathbf{PStH}_{\mathbb{R}^n}$ if and only if $\operatorname{dom}\alpha = \operatorname{dom}\beta$;

(iii) $\alpha\mathscr{H}\beta$ in $\mathbf{PStH}_{\mathbb{R}^n}$ if and only if $\operatorname{ran}\alpha = \operatorname{ran}\beta$ and $\operatorname{dom}\alpha = \operatorname{dom}\beta$.

Proposition 4. *$\mathbf{PStH}_{\mathbb{R}^n}$ is a bisimple inverse semigroup.*

Corollary 5. *Every two maximal subgroup in $\mathbf{PStH}_{\mathbb{R}^n}$ are isomorphic. Moreover every maximal subgroup in $\mathbf{PStH}_{\mathbb{R}^n}$ id isomorphic to the group of all star homeomorphisms of the unit ball $\mathbf{B}_1$ in $\mathbb{R}^n$.*

Theorem 6. *Every non-unit congruence on $\mathbf{PStH}_{\mathbb{R}^n}$ is a group congruence.*

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Extensions of semigroups by symmetric inverse semigroups of a bounded finite rank

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We study the semigroup extension $\mathscr{I}_\lambda^n(S)$ of a semigroup S by symmetric inverse semigroups of a bounded finite rank.

Definition 1. An subset D of a semigroup S is said to be ω-unstable if D is infinite and $aB \cup Ba \nsubseteq D$ for any $a \in D$ and any infinite subset $B \subseteq D$.

Definition 2. An subset D of a semigroup S is said to be *strongly ω-unstable* if D is infinite and $aB \cup Bb \nsubseteq D$ for any $a, b \in D$ and any infinite subset $B \subseteq D$.

It is obvious that a subset D of a semigroup S is strongly ω-unstable then D is ω-unstable.

Definition 3. An *ideal series* (see, for example, [1]) for a semigroup S is a chain of ideals

$$I_0 \subseteq I_1 \subseteq I_2 \subseteq \cdots \subseteq I_n = S.$$

We call the ideal series *(strongly) tight* if I_0 is a finite set and $D_k = I_k \setminus I_{k-1}$ is an (strongly) ω-unstable subset for each $k = 1, \ldots, n$.

A finite direct product of semigroups with tight ideal series is a semigroup with a tight ideal series and a homomorphic image a semigroup with a tight ideal series with finite preimages is a semigroup with a tight ideal series too [2].

Proposition 4. *Let S be a semigroup which admits a strongly tight ideal series. Then the direct power $(S)^n$ admits a strongly tight ideal series too.*

Theorem 5. *Let λ be an infinite cardinal and n be a positive integer. If S is a finite semigroup then*

$$I_0 = \{0\} \subseteq I_1 = \mathscr{I}_\lambda^1(S) \subseteq I_2 = \mathscr{I}_\lambda^2(S) \subseteq \cdots \subseteq I_n = \mathscr{I}_\lambda^n(S)$$

is the strongly tight ideal series for the semigroup $\mathscr{I}_\lambda^n(S)$.

Theorem 6. *Let S be a semigroup which admits a strongly tight ideal series. Then for every non-zero cardinal λ and any positive integer $n \leqslant \lambda$ the semigroup $\mathscr{I}_\lambda^n(S)$ admits a strongly tight ideal series too.*

Definition 7 ([2])**.** An algebraic semigroup S is called *algebraically complete* in the class of semitopological semigroups $\mathfrak{S}$, if S with any Hausdorff topology τ such that $(S,\tau)\in\mathfrak{S}$ is H-closed in $\mathfrak{S}$.

Theorem 8. *Let S be an inverse semigroup which admits a strongly tight ideal series. Then for every non-zero cardinal λ and any positive integer $n \leqslant \lambda$ the semigroup $\mathscr{I}_\lambda^n(S)$ is algebraically complete in the class of Hausdorff semitopological inverse semigroups with continuous inversion.*

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НТБ ОНАХТ

Non-Oriented Heegaard Diagrams

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Let $(F; u_1, u_2; v_1, v_2)$ be a genus 2 Heegaard diagram of a closed 3-manifold M. Here F is a closed surface which decomposes M into two handlebodies H_1, H_2 of genus 2, u_1, u_2 are meridians of H_1, and v_1, v_2 are meridians of H_2.

Assume that all crossing points of the meridians are transversal and that the diagram is normalized (the latter means that among the regions into which the meridians split F here are no biangles). The total number of those crossing points is called the *Heegaard complexity* of the diagram.

Let us cut F along u_1, u_2. We obtain a sphere with four holes $D_1^{\pm}$, $D_2^{\pm}$ which are conveniently interpreted as distinguished disks on the sphere. The meridians v_1, v_2 will then be cut into arcs which join the holes. We agree to depict k parallel arcs as one arc marked by the number $k \geq 0$. We also take into account the orientations of the glued boundaries $\partial D_i^{\pm}$ by assigning $(+1)$ for orientable gluing and (-1) for non-orientable one. Therefore we get $(+1, +1)$, $(+1, -1)$, $(-1, +1)$ and $(-1, -1)$ for pairs of holes.

It is well known that the set of all genus 2 Heegaard diagrams can be decomposed into three types. Each such diagram can be determined by a 7-tuple (a, b, c, d, e, f, g), where a, b, c, d are the arcs joining the holes, e, f determine the gluing maps $\varphi_i\colon \partial D_i^- \to \partial D_i^+$, $i = 1, 2$, and g is defined to be 0, 1, 2 and 3 for $(+1, +1)$, $(+1, -1)$, $(-1, +1)$ and $(-1, -1)$ respectively.

In order to give exact descriptions of φ_1, we introduce topological symmetries $s_i\colon \partial D_i^- \to \partial D_i^+$ and topological rotations $r_i\colon \partial D_i^- \to \partial D_i^+$ by the following results:

(1) $s_i\colon \partial D_i^- \cap (v_1 \cup v_2) \to \partial D_i^+ \cap (v_1 \cup v_2)$ such that the endpoint of each b-arc (respectively, c-arc) is taken to the other endpoint of the same arc.

(2) r_i shifts each point of $D_i^+ \cap (v_1 \cup v_2)$ to the next point of $D_i^+ \cap (v_1 \cup v_2)$.

Now we define φ_1, φ_2 as follows: $\varphi_1 = r_1^e s_1$, and $\varphi_2 = r_2^f s_2$.

A Heegaard diagram $(F; u_1, u_2; v_1, v_2)$ of genus 2 has type I, II or III if it can be described as follows:

Type I

- Each a-arc joins D_1^+ and D_2^+ as well as D_1^- and D_2^-.
- Each b-arc joins D_1^+ and D_1^-.
- Each c-arc joins D_2^+ and D_2^-.
- Each d-arc joins D_1^+ and D_2^- as well as D_1^- and D_2^+.

Type II

- Each a-arc, b-arc and d-arc can be described in the same way as type I while each c-arc joins D_1^+ and D_1^-.

Type III

- Each a-arc, b-arc and c-arc can be described in the same way as type II while each d-arc is a loop with ends at D_1^+ and D_1^- embracing D_2^+ and D_2^- respectively.

Theorem 1. *If the Heegaard complexity of a non-orientable 3-manifold M is no more than 5, then M is homeomorphic to $L_{p,q} \# S^1 \widetilde{\times} S^2$.*

There exists a manifold of complexity 6 that is not homeomorphic to $L_{p,q}\#S^1\widetilde{\times}S^2$*. Its diagram can be represented by the 7-tuple* $(1,1,1,1,0,0,3)$.

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НТБ ОНАХТ

Some many-dimensional extremal geometric problems

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The talk deals with many-dimensional analogs of some classical two-dimensional and three-dimensional geometric problems on an extremum. The asymptotic behavior of parameters of extremal geometric objects with increasing dimension of the space is studied.

It is shown that in some extremal problems (such as in the problem of a cylinder of fixed volume with a minimum total surface area and the problem of the shape of a right circular cone with a maximum volume of an inscribed ball in it) these parameters do not depend on the dimension of the space.

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НТБ ОНАХТ

On a regularized solution of the Cauchy problem for matrix factorizations of the Helmholtz equation in $m-$dimensional bounded domain

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In the paper it is considered the problem of regularization of the Cauchy problem for matrix factorisations of the Helmholtz equation in $m-$dimensional bounded domain of the type of a curvilinear triangle. Using the results of works [1]-[2], is constructed explicitly Carleman matrix and, based on the regularized solution of the Cauchy problem.

Let $\mathbb{R}^m$ be a $m-$dimensional real Euclidean space,

$$x = (x_1, \, \dots \, , x_m) \in \mathbb{R}^m, \;\; y = (y_1, \, \dots \, , y_m) \in \mathbb{R}^m.$$

Let $G_\rho \subset \mathbb{R}^m$ be a bounded simply connected domain whose boundary consists of the surface of a cone

$$\left|y'\right| = \tau y_m, \;\; \tau = tg\frac{\pi}{2\rho}, \;\; y_m > 0, \;\; \rho > 1,$$

and a smooth piece of the surface S lying inside the cone, i.e. $\partial G_\rho = S \bigcup T, \;\; T = \partial G_\rho \backslash S$.

We consider in the domain G_ρ a system of differential equations

$$D\left(\frac{\partial}{\partial x}\right) U(x) = 0, \tag{1}$$

where $D\left(\frac{\partial}{\partial x}\right)$ is the matrix of differential operators is of the first order.

We denote by $A(G_\rho)$ the class of vector functions in the domain G, of continuous on $\overline{G}_\rho = G_\rho \bigcup \partial G_\rho$ and satisfying system (1).

Problem 1. Let $U(y) \in A(G_\rho)$ and

$$U(y)|_S = f(y), \;\; y \in S. \tag{2}$$

Here, $f(y)$ is a given continuous vector function on S.

It is required to restore the vector function $U(y)$ in the domain G_ρ, based on its values $f(y)$ on S.

Theorem 2. *Let $U(y) \in A(G_\rho)$ on the entire boundary of ∂G_ρ satisfy the boundary condition*

$$|U(y)| \leq 1, \;\; y \in T.$$

Then we have the estimate

$$\left|U(x) - U_{\sigma(\delta)}(x)\right| \leq C_\rho(\lambda, x)\sigma\delta^{\left(\frac{\gamma}{R}\right)^\rho}, \;\; \sigma > 1, \;\; x \in G_\rho.$$

Corollary 3. *The limiting equality*

$$\lim_{\delta \to 0} U_{\sigma(\delta)}(x) = U(x),$$

holds uniformly on each compact set in the domain G_ρ.

Thus, the functional $U_{\sigma(\delta)}(x)$ determines the regularization of the solution of problem (1)-(2).

НТБ ОНАХТ

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НТБ ОНАХТ

Neighborhood maps on combinatorial trees and their Markov graphs

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A combinatorial tree is a finite connected acyclic undirected graph. For a self-map $\sigma : V(X) \to V(X)$ on the vertex set $V(X)$ of a combinatorial tree X, its Markov graph $\Gamma = \Gamma(X, \sigma)$ is defined as a directed graph with the vertex set $V(\Gamma) = E(X)$ and the arc set $A(\Gamma) = \{(u_1v_1, u_2v_2) : u_2, v_2 \in [\sigma(u_1), \sigma(v_1)]_X\}$ (here $[a, b]_X$ denotes the metric interval between a, b in X).

A map $\sigma : V(X) \to V(X)$ on a tree X is a neighborhood map provided $\sigma(u) \in N_G[u]$ for all vertices $u \in V(X)$ (i.e. if $\sigma(u) = u$ or $u\sigma(u) \in E(X)$ for all $u \in V(X)$).

Denote by $d_G(u)$ the vertex degree of u in a graph G and by $L(G)$ the set of all leaf vertices (i.e. vertices u with $d_G(u) = 1$) in G. Also, let $\mathrm{fix}\sigma$ denotes the set of σ-fixed points.

Theorem 1. *For any neighborhood map σ on a tree X with $|V(X)| \geq 2$ the number of weak components in the corresponding Markov graph $\Gamma(X, \sigma)$ equals $\sum_{u\in \mathrm{fix}\sigma} d_X(u) - |\mathrm{fix}\sigma| + 1$.*

Corollary 2. *A neighborhood map σ on a tree X with $|V(X)| \geq 2$ has a weakly connected Markov graph if and only if $\mathrm{fix}\sigma \subset \mathrm{L(X)}$.*

For a map σ on a tree X an edge $uv \in E(X)$ is called σ-positive (σ-negative) provided $d_X(\sigma(u), u) \leq d_X(\sigma(u), v)$ and $d_X(\sigma(v), v) \leq d_X(\sigma(v), u)$ ($d_X(\sigma(u), v) \leq d_X(\sigma(u), u)$ and $d_X(\sigma(v), u) \leq d_X(\sigma(v), v)$). Let $p(X, \sigma)$ and $n(X, \sigma)$ denote the number of σ-positive and σ-negative edges in X, respectively.

Theorem 3. *For any neighborhood map σ on a tree X the number of arcs in the corresponding Markov graph $\Gamma(X, \sigma)$ equals $|E(X)| + 2p(X, \sigma) - \sum_{u\in \mathrm{fix}\sigma} d_X(u)$.*

For a number $\alpha \in \mathbb{R} - \{0, 1\}$ the first general Zagreb index [4] of G is defined as the sum $Z_1^\alpha(G) = \sum_{u\in V(G)} d_G^\alpha(u)$. Similarly, for every number $\alpha \in \mathbb{R} - \{0\}$ the general Randic index [1] of a graph G is the sum $R^\alpha(G) = \sum_{uv\in E(G)} (d_G(u)d_G(v))^\alpha$.

Theorem 4. *For every n-vertex tree X the average number of arcs in Markov graphs for neighborhood maps on X equals*

$$(3 - \frac{2}{n}) \cdot Z_1^{-1}(K_1 + X) + 2R^{-1}(K_1 + X) + \frac{2}{n^2} - \frac{3}{n} - 3.$$

Given a graph G, its Narumi-Katayama index [5] is the product $\mathrm{NK(G)} = \prod_{\mathrm{u\in V(G)}} \mathrm{d_G(u)}$ of degrees over all vertices in G.

Proposition 5. *For every n-vertex tree X the number of its neighborhood maps equals* $\frac{1}{n} \cdot \mathrm{NK(K_1+X)}$.

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First Betti numbers of orbits of Morse functions on surfaces

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Let $\mathcal{G}$ be a minimal class of groups satisfying the following conditions: 1) $1 \in \mathcal{G}$; 2) if $A, B \in \mathcal{G}$, then $A \times B \in \mathcal{G}$; 3) if $A \in \mathcal{G}$ and $n \geq 1$, then the wreath product $A \wr_n \mathbb{Z} \in \mathcal{G}$.

In other words a group G belongs to the class $\mathcal{G}$ iff G is obtained from trivial group by a finite number of operations $\times, \wr_n \mathbb{Z}$. It is easy to see that every group $G \in \mathcal{G}$ can be written as a word in the alphabet $\mathcal{A} = \{1, \mathbb{Z}, (\,,\,), \times, \wr_2, \wr_3, \wr_4, \dots\}$. We will call such word a *presentation* of the group G in the alphabet $\mathcal{A}$. Evidently, the presentation of a group is not uniquely determined.

Denote by $Z(G)$ and $[G, G]$ the center and the commutator subgroup of G respectively.

Theorem 1. *Let $G \in \mathcal{G}$, ω be an arbitrary presentation of G in the alphabet $\mathcal{A}$, and $\beta_1(\omega)$ be the number of symbols $\mathbb{Z}$ in the presentation ω. Then there are the following isomorphisms:*

$$Z(G) \cong G/[G, G] \cong \mathbb{Z}^{\beta_1(\omega)}.$$

In particular, the number $\beta_1(\omega)$ depends only on the group G.

The groups from the class $\mathcal{G}$ appear as fundamental groups of orbits of Morse functions on surfaces. Let M be a compact surface and $\mathcal{D}$ be the group of C^∞-diffeomorphisms of M. There is a natural right action of the group $\mathcal{D}$ on the space of smooth functions $C^\infty(M, \mathbb{R})$ defined by the rule: $(f, h) \longmapsto f \circ h$, where $h \in \mathcal{D}$, $f \in C^\infty(M, \mathbb{R})$. Let $\mathcal{O}(f) = \{f \circ h \mid h \in \mathcal{D}\}$ be the *orbit* of f under the above action. Endow the spaces $\mathcal{D}$, $C^\infty(M, \mathbb{R})$ with Whitney C^∞-topologies. Let $\mathcal{O}_f(f)$ denote the path component of f in $\mathcal{O}(f)$.

A map $f \in C^\infty(M, \mathbb{R})$ will be called *Morse* if all its critical points are non-degenerate. Homotopy types of stabilizers and orbits of Morse functions were calculated in a series of papers by Sergiy Maksymenko [3], [2], Bohdan Feshchenko [4], and Elena Kudryavtseva [1]. As a consequence of Theorem 1 we get the following.

Corollary 2. *Let M be a connected compact oriented surface distinct from S^2 and T^2, f be a Morse function on M, $G = \pi_1 \mathcal{O}_f(f) \in \mathcal{G}$, ω be an arbitrary presentation of G in the alphabet $\mathcal{A}$, and $\beta_1(\omega)$ be the number of symbols $\mathbb{Z}$ in the presentation ω. Then the first integral homology group $H_1(\mathcal{O}(f), \mathbb{Z})$ of $\mathcal{O}(f)$ is a free abelian group of rank $\beta_1(\omega)$:*

$$H_1(\mathcal{O}(f), \mathbb{Z}) \simeq \mathbb{Z}^{\beta_1(\omega)}.$$

In particular, $\beta_1(\omega)$ is the first Betti number of $\mathcal{O}(f)$.

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Diffeomorphisms preserving Morse-Bott foliations

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Let M be a smooth compact manifold and $\mathcal{F}$ be a codimension one foliation on M having singular leaves of Morse-Bott type. This means that there the set Σ of singular leaves of $\mathcal{F}$ is a disjoint union of compact submanifolds Let also $\mathcal{D}(\mathcal{F})$ be the group of diffeomorphisms of M leaving each leaf invariant, and $\mathcal{D}(\mathcal{F}, \Sigma)$ be the subgroup of $\mathcal{D}(\mathcal{F})$ consisting of diffeomorphisms fixed on Σ.

Theorem 1. [1] *The "restriction to Σ map"*

$$\rho\colon \mathcal{D}(\mathcal{F}) \to \mathcal{D}(\Sigma), \qquad \rho(h) = h|_{\Sigma_f},$$

is a locally trivial fibration with fibre $\mathcal{D}(\mathcal{F}, \Sigma)$.

This result can be regarded as a "foliated" variant of the well know results by Cerf and Palais on local triviality of restrictions. In particular, the map ρ has a path-lifting property, and so it contains a "foliated" variant of isotopy extension theorem:

Corollary 2. [1] *Let $H : \Sigma \times [0,1] \to \Sigma$ be a C^∞ isotopy with $H_0 = \mathrm{id}_\Sigma$. Then it extends to an isotopy $H : M \times \to M$ such that $H_t \in \mathcal{D}(\mathcal{F}, \Sigma)$ for all $t \in [0,1]$.*

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НТБ ОНАХТ

Singular monotonic functions defined by a convergent positive series and a double stochastic matrix

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Let

1) $1=\frac{2}{3}+\sum\limits_{n=1}^{\infty}\frac{1}{(-2)^n}=\frac{2}{3}-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\ldots$ be normalized alternating binary series that defines a binary negapositional image of the number of the segment $[0;1]$:

$$x=\frac{2}{3}+\frac{\alpha_1(x)}{(-2)^1}+\frac{\alpha_2(x)}{(-2)^2}+\frac{\alpha_3(x)}{(-2)^3}+\ldots+\equiv\overline{\Delta}^2_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots};$$

2) $\|p_{ik}\|=\begin{pmatrix}p_{00} & p_{01}\\ p_{10} & p_{11}\end{pmatrix}$ be a positive double stochastic matrix i.e. $p_{ij}>0,\ p_{i0}+p_{i1}=1,\ p_{0j}+p_{1j}=1,\ i=0,1,\ j=0,1;$

3) $\overline{p}=(p_0;p_1)$ be a vector $p_0=\frac{p_{10}}{p_{01}+p_{10}}=\frac{1}{2}$ and $p_1=\frac{p_{01}}{p_{01}+p_{10}}=\frac{1}{2}$;

It is known that a binary negapositional number representation is a recoding of a classical binary representation:

$$x=\frac{a_1}{2}+\frac{a_2}{2^2}+\ldots+\frac{a_n}{2^n}+\ldots\equiv\Delta^2_{a_1(x)a_2(x)\ldots a_n(x)\ldots},\qquad a_n\in\{0;1\}$$

Considered in the talk is function F, defined by equality

$$F(x)=F(\overline{\Delta}^2_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots})=\beta_{\alpha_1(x)}+\frac{1}{2}\sum_{k=1}^{\infty}(\beta^{(k)}_{\alpha_k(x)\alpha_{k+1}(x)}\prod_{i=1}^{k-1}p_{\alpha_i(x)\alpha_{i+1}(x)}),\quad \textit{where} \tag{1}$$

$$\beta_{\alpha_1(x)}=\begin{cases}0, & \text{if}\quad \alpha_1(x)=1,\\ \frac{1}{2}, & \text{if}\quad \alpha_1(x)=0,\end{cases}$$

$$\beta^{(2n-1)}_{\alpha_{2n-1}(x)\alpha_{2n}(x)}=\beta^{(1)}_{\alpha_{2n-1}(x)\alpha_{2n}(x)}=\begin{cases}0, & \text{if}\quad \alpha_{2n}(x)=0,\\ p_{00}, & \text{if}\quad \alpha_{2n-1}(x)\neq\alpha_{2n}(x)=1,\\ p_{10}, & \text{if}\quad \alpha_{2n-1}(x)=\alpha_{2n}(x)=1,\end{cases}$$

$$\beta^{(2n)}_{\alpha_{2n}(x)\alpha_{2n+1}(x)}=\beta^{(0)}_{\alpha_{2n}(x)\alpha_{2n+1}(x)}=\begin{cases}0, & \text{if}\quad \alpha_{2n+1}(x)=1,\\ p_{01}, & \text{if}\quad \alpha_{2n}(x)=\alpha_{2n+1}(x)=0,\\ p_{00}, & \text{if}\quad \alpha_{2n}(x)\neq\alpha_{2n+1}(x)=0,\end{cases}$$

and $\alpha_k(x)$ is k negapositional digit of representation of the number x.

Definition 1. Let $(c_1,c_2,\ldots,c_m)$ be a orderly set of positive integers. *The Cylinder of m rank with basis $c_1c_2\ldots c_m$* is called a set $\overline{\Delta}^2_{c_1c_2\ldots c_m}$ of numbers of $x\in(0;1]$ that is first m negapositional digits of which are $c_1,c_2,\ldots,c_m$ respectively, i.e.

$$\overline{\Delta}^2_{c_1c_2\ldots c_m\ldots}=\left\{x:\ x=\overline{\Delta}^2_{c_1c_2\ldots c_m a_{m+1}a_{m+2}\ldots},\quad a_{m+i}\in\mathbb{N},\ i=1,2,3,\ldots\right\}.$$

Lemma 2. *For a function F defined by the equality (1) the mapping of the cylinder $\overline{\Delta}^2_{c_1c_2\ldots c_m}$ is a segment $[a;b]$, where*

$$a=\beta_{c_1}+\frac{1}{2}\sum_{k=1}^{m-1}\left(\beta^{(k)}_{c_kc_{k+1}}\prod_{j=1}^{k-1}q_{c_jc_{j+1}}\right),\qquad b=a+\frac{1}{2}\prod_{j=1}^{m-1}q_{c_jc_{j+1}},$$

Theorem 3. *Images of different cylinders of the same rank with the mapping F do not overlap and in the union give the whole segment* $[0, 1]$.

Theorem 4. *The function $F(x)$ denoted by the equality (1) is:*

1) correctly identified,

2) continuous,

3) strictly increasing,

4) linear for $p_{00} = 0.5$ and singular for $p_{00} \neq 0.5$ (has a derivative equal to zero almost everywhere in the sense of the Lebesgue measure).

The report proposes the results of studies of the above-mentioned functions.

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НТБ ОНАХТ

A Flat $(CHR)_3$-curvature tensor in a Trans-Sasakian Manifold

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Recently, we defined a $(CHR)_3$-curvature tensor in almost contact Riemannian manifolds([3]) using M. Prvanovic's paper ([4]).

On, 2009, A. A. Shaikh and Y. Matsuyama considered a trans-Sasakian manifold which is a generalization of a Kenmotsu and Sasakian manifold and got some interesting results([5]).

In this paper, we consider this tensor field in a trans-Sasakian manifold. Moreover, we define the notion of the $(CHR)_3$-flatness in an almost contact Riemannian manifold. Then, we consider this notion in a trans-Sasakian manifold and determine the curvature tensor, the Ricci tensor and the scaler curvature. Finally, we get a condition which the Ricci tensor becomes a generalized quasi- or quasi-Einstein ([1], [2]).

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НТБ ОНАХТ

On some fractal-based estimations of subsidence volume for various types of soils

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In [1, 2], the particle size distribution $N_s(L > d_s)$ was defined as the number of particles being of any size L larger than d_s, where d_s runs over the real numbers. In the same way we can introduce the particle size distribution by volume $V_s(L > d_s)$ (and by mass $M_s(L > d_s)$) as the volume (mass) of particles being of any size L larger than d_s, where d_s runs over the real numbers. Certainly, $N_s(L > d_s)$, $V_s(L > d_s)$ and $M_s(L > d_s)$ are real functions.

The fractal dimension DV_s of the particle size distribution by volume is defined then as the following:

$$DV_s = \lim_{d_s \to 0} -\frac{\ln(V_s(L > d_s))}{\ln(d_s)}$$

It implies that $-DV_s \ln(d_s) \approx \ln(V_s(L > d_s))$ hence $\ln({d_s}^{-DV_s}) \approx \ln(V_s(L > d_s))$ and finally

$$V_s(L > d_s) \approx \gamma {d_s}^{-DV_s},$$

where γ is a constant coefficient and the sign $\approx$ means "approximately".

On the basis of the fractal characteristics of the pore and particle structure, there were obtained theoretical models describing diffusion, deformation of the compaction and the shift of the medium [3], [4]. Under some additional conditions of fractal nature of the loess soil and developing methods introduced in [5, 6] we obtained certain predictive estimations of the coefficient of porosity after the disintegration of micro-aggregates. In this note we obtain some estimations of soil subsidence volume, based on the introduced above fractal dimension.

The particles forming the ground may have only a finite set of sizes. We denote these sizes $d_1, d_2, ..., d_{n-1}, d_n$ ranging in decreasing order from the largest. We assume that $\alpha = \alpha_j = d_j/d_{j-1}$, where $2 \leqslant j \leqslant n$, does not depend on j. This assumption corresponds to the idea of the self-similarity of fractal structures. In addition, all known mathematical fractals are constructed on this principle. As the structures formed by particles of a fixed size are self-similar, we also assume that all these structures have the same coefficient of porosity k_p as well as the same porosity $K_p = k_p/(1 + k_p)$. We discovered that under such conditions two different situations may occurred. Let k' be the coefficient of porosity and K' be the porosity of the soil after after the disintegration of micro-aggregates.

Theorem 1. *In the above denotations we have :*

1. *if* $K_p \geq \alpha^{DV_s}$ *then* $k' = \frac{(1+k_p)(\alpha^{DV_s}-1)}{(\alpha^{DV_s})^n-1} - 1$ *and* $K' = 1 - \frac{(\alpha^{DV_s})^n-1}{(1+k_p)(\alpha^{DV_s}-1)}$ *;*
2. *if* $K_p < \alpha^{DV_s}$ *then* $k' = \frac{k_p(1-\alpha^{-DV_s})}{1-(\alpha^{-DV_s})^n}$ *(5.18) and* $K' = \frac{k_p(1-\alpha^{-DV_s})}{k_p(1-\alpha^{-DV_s})+1-(\alpha^{-DV_s})^n}$.

Since we estimate volumetric characteristics of soils subsidence, the introduced above fractal dimension of the particle size distribution by volume is more convenient and allows us essentially clarify and simplify our calculations.

The results of our experiments and calculations show that on the basis of a new theoretical models and the "Microstructure" technique, having the values of the fractal dimension of the particle size distribution by volume, it is possible to forecast the volume deformations after the disintegration of the micro-aggregates. Depending on the type of soils and the specific experimental conditions, this may be the amount of subsidence deformation, swelling or suffusion. The details of our experiments and techniques are described in [6].

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НТБ ОНАХТ

The Shanin number and the predshanin number of N_τ^φ-kernel of a topological spaces

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A cardinal $\tau \geq \aleph_0$ is said to be a caliber of the space X if for any family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open in X sets such that $|A| = \tau$, there exists $B \subset A$, for which $|B| = \tau$, and $\bigcap\{U_\alpha : \alpha \in B\} \neq \emptyset$. Set $k(X) = \{\tau : \tau$ is a caliber of the space $X\}$.

The cardinal number $min\{\tau : \tau^+$ is caliber of $X\}$ is called the Shanin number of X and denoted by $sh(X)$, where τ^+ is the least cardinal number from all cardinals strictly greater that τ.

A cardinal $\tau \geq \aleph_0$ is said to be a precaliber of the space X if for any family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open in X sets such that $|A| = \tau$, there exists $B \subset A$, for which $|B| = \tau$, and $\{U_\alpha : \alpha \in B\}$ is centered. Set $pk(X) = \{\tau : \tau$ is a precaliber of the space $X\}$.

The cardinal number $min\{\tau : \tau^+$ is precaliber of $X\}$ is called the predshanin number of X and denoted by $psh(X)$, where τ^+ is the least cardinal number from all cardinals strictly greater that τ.

A system $\xi = \{F_\alpha : \ \alpha \in A\}$ of closed subsets of a space X is called *linked* if any two elements from ξ intersect [1].

A.V. Ivanov defined the space NX of complete linked systems (CLS) of a space X in a following way:

Definition 1. A linked system M of closed subsets of a compact X is called *a complete linked system* (a CLS) if for any closed set of X, the condition

"Any neighborhood OF of the set F consists of a set $\Phi \in M$"

implies $F \in M$[2].

A set NX of all complete linked systems of a compact X is called *the space NX of CLS of X*. This space is equipped with the topology, the open basis of which is formed by sets in the form of

$E = O(U_1, U_2, \ldots, U_n)\langle V_1, V_2, \ldots, V_s\rangle = \{M \in NX$: for any $i = 1, 2, \ldots, n$ there exists $F_i \in M$ such that $F_i \subset U_i$, and for any $j = 1, 2, \ldots, s$, $F \cap V_j \neq \varnothing$ for any $F \in M\}$, where $U_1, U_2, \ldots, U_n$, $V_1, V_2, \ldots, V_s$ are nonempty open in X sets [2].

Definition 2. Let X be a compact space, φ be a cardinal function and τ be an arbitrary cardinal number. We call *an N_τ^φ - kernel* of a topological space X the space

$$N_\tau^\varphi X = \{M \in NX : \exists F \in M : \varphi(F) \leq \tau\}.$$

Theorem 3. *Let X be an infinity compact space and $\varphi = d, \tau = \aleph_0$. Then:*
1) $sh(N_\tau^\varphi X) \leq sh(X)$;
2) $psh(N_\tau^\varphi X) \leq psh(X)$.

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The effectiveness of the use of computer programs in the teaching of mathematics in academic lyceums

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The article substantiates the role of mathematics in the general education system and reveals the expediency and possibility of using a computer program in mathematics lessons in academic lyceums.

The use of computer in the learning process, in essence, is a formative experiment aimed at studying and developing new personality traits. Important for the modern period of computerization of education is the realization of the fact that the use of computer technology will make the learning process more effective if they are used as a tool of knowledge, and not the transfer of knowledge. The computer is able to realize the many benefits of technical training. Modern computer programs allow you to create texts, various types of graphics, animation with sound, video. With their help, you can simulate the objects under study and conduct experiments to study their properties, simulate processes and phenomena, etc.

Interconnected training in computer science, mathematics and physics provides an opportunity to acquaint students with the use of applied mathematical packages as a tool in solving typical problems. Modeling is one of the difficult sections in the math course. The content-structural component of the model and mathematical modeling is an important component of the discipline, which is constantly being improved, as a result of which the study of the methodology for its study has not yet been completed. At the moment there is a large number of methodical training in computer modeling, which are actively used in the lessons of mathematics.

A model is a simplified resemblance of a real object or process. A key concept in modeling is considered a goal. The purpose of the simulation is the purpose of the future model. The target determines the properties of the original object to be reproduced in the model. You can model both material objects and processes. An information model is a description of a simulation object. On the basis of the presentation of the model are divided into tabular, graphical, object-informational and mathematical.

One of the available modeling tools is the Microsoft Excel office application, since almost all computer labs have MS Office. Microsoft Excel is a spreadsheet program that allows you to analyze large amounts of data. This program uses more than 600 mathematical, financial, statistical and other specialized functions, with which you can link various tables to each other, select arbitrary data presentation formats, create hierarchical structures. Mathcad is an application for engineering and mathematical computing, an industry standard for performing, distributing and storing calculations. Mathcad is a universal system, i.e. can be used in any field of science and technology - wherever mathematical methods are used. Blender is a free program for 3D modeling. The trick in this program is that during the creation of a 3-dimensional scene, the utility window can be divided into parts, each of which will be an independent window with a certain type of 3D scene, a timeline ruler, object settings.

Thus, the construction of simple graphical models, such as solving simple mathematical problems, is appropriate in the basic course of computer science. Independent development of graphical models requires programming knowledge, and this applies to material of increased difficulty, which is studied in a specialized computer science course or as part of an elective course [1].

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Gromov-Witten invariants and identification of the energy levels of solitonic states

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Calabi-Yau manifold can be represented in terms of toric data [1]. Such presentation allows us to go to the dual polyhedron, through which are defined the gauge groups. As an example, we can consider Calabi-Yau manifold $X_{24}(1,1,2,8,12)^{3,243}_{-480}$ determined by Gromov-Witten invariants [2], presented in table 1. Calabi-Yau manifold is defined by n - Gromov-Witten invariants, the analogue of the principal

Table 1

Gromov-Witten invariants for $X_{24}(1,1,2,8,12)^{3,243}_{-480}$

a, b, c	n	a, b, c	n	a, b, c	n	a, b, c	n
(0,0,1)	-1	(0,1,1)	-1	(0,1,2)	-2	(0,1,3)	-3
(0,1,4)	-4	(0,1,5)	-5	(0,2,3)	-3	(0,2,4)	-16
(1,0,0)	240	(1,0,1)	240	(1,1,1)	240	(1,1,2)	720
(1,1,3)	1200	(1,1,4)	1680	(1,2,3)	1200	(2,0,0)	240
(2,0,2)	240	(2,2,2)	240	(3,0,0)	240	(3,0,3)	240
(4,0,0)	240	(5,0,0)	240	(6,0,0)	240	(0,1,0)	0

quantum number in physics and by (a, b, c) - the internal quantum numbers.

An embedding of Gromov-Witten invariants is presented by the formula

$$n_{a,b,c} = \sum_d n_{a,b,c,d} \ . \qquad (1)$$

Such an embedding suggests the possibility of a phase transition between different manifolds characterized by different n. In [3], for the case of an elliptic fibration representing Calabi-Yau manifolds, the matter content of the charged fields of the effective theory is associated with divisors of the base of the fibration. The representations of groups of these fields is determined by the of intersection number of these divisors, in other words, gauge groups are associated with curves of singularities of the manifold and intersecting singularity curves define charged fields classified by gauge groups. Table 2 from [3] represents such a set of fields and the corresponding groups. From table 2 it can be seen that matter content presented by gauge groups can be embeded according to formula (0.1) through transitions between gauge groups

$$E_7 \to E_6 \to SU(6) \to SU(5) \to SU(4) \to SU(3) \to SU(2) \ ,$$

characterized by different sets of the matter content of the charged fields.

The presence of Gromov-Witten invariants, n signals about the presence of a central charge, Z or mass, M of a solitonic object, [4]

$$e^{\int_\eta S} = e^{2\pi i \tau \eta}, \ \eta = n\varphi, \ M = Z = \int_D \Omega = n_i \varphi_i$$

Table 2

Matter content of Calabi-Yau models, characterized by gauge groups (for n=2,4,6,8,10,12)

Group	Matter content
SU(2)	*(6n+16)**2***
SU(3)	*(6n+18)**3***
SU(4)	*(n+2)**6**+(4n+16)**4***
SU(5)	*(3n+16)**5**+(2+n)**10***
SO(10)	*(n+4)**16**+(n+6)**10***
E_6	*(n+6)**27***
E_7	$\left(\frac{n}{2}+4\right)$ ***56***

Here the holomorphic 3-form Ω is defined on Calabi-Yau X, (D is a cycle in X). Gromov-Witten invariants, n_i are connected with the cohomology classes, φ_i of rational curves in X into central charge, Z.

Solitonic objects of instanton type are characterized by the condition on the action, S

$$S \geq \left(\frac{8\pi^2}{g^2}\right)|Q| = E$$

where

$$Q = -\frac{1}{16\pi^2}\int d^4xTr[\tilde{F_{\mu\nu}}F_{\mu\nu}]$$

Pontryagin's homotopy index, defined by the Yang-Mills field strength, $F_{\mu\nu}$, [5]. Consequently, the central charge has an interpretation of a multiple topological quantum number or the number of the Bogomolny-Prasad-Sommerfeld, corresponding to the multiplicities of degeneration of different configurations of solitonic type in Calabi-Yau space.

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НТБ ОНАХТ

On isometries satisfying deformed commutation relations

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We consider certain perturbation of family of pairwise orthogonal isometries. Namely, we study properties and representation theory of C^*-algebra $\mathcal{E}^q_{1,n}$ generated by isometries t, s_j, $j = \overline{1,n}$, subject to the relations

$$s_i^* s_j = 0,\ i \neq j, \quad t^* s_j = q s_j t^*.$$

In recent paper [1] was studied the C^*-algebra $\mathcal{E}^q_{n,m}$ with $n, m \geq 2$, generated by families $\{t_j\}_{j=1}^m$ and $\{s_i\}_{i=1}^n$. In particular, it was shown that for $|q| < 1$ one has $\mathcal{E}^q_{n,m} \simeq \mathcal{E}^0_{n,m}$ and for $|q| = 1$ the C^*-isomoprphism class of quotient of $\mathcal{E}^q_{n,m}$ by the unique largest ideal is independent of q and isomorphic to the tensor product of Cuntz algebras $\mathcal{O}_n \otimes \mathcal{O}_m$.

We show that the result for $|q| < 1$ remains true for $\mathcal{E}^q_{1,n}$.

Theorem 1. *For any $q \in \mathbb{C}$, $|q| < 1$, one has an isomorphism $\mathcal{E}^q_{1,n} \simeq \mathcal{E}^0_{1,n}$.*

Notice that the proof contains an explicit construction of the required isomorphism, which is similar to the one given in [1].

For the case $|q| = 1$ we obtain the following facts.

Definition 2. The Fock representation, π^q_F, of $\mathcal{E}^q_{1,n}$, is the unique up to unitary equivalence irreducible $*$-representation having the vacuum vector Ω, $||\Omega|| = 1$, such that

$$\pi^q_F(s_j^*)\Omega = 0, \quad \pi^q_F(t^*)\Omega = 0, \quad j = \overline{1,n}.$$

Theorem 3. *The Fock representation of $\mathcal{E}^q_{1,n}$ exists and is faithful.*

Theorem 4. *The C^*-algebra $\mathcal{E}^q_{1,n}$ is nuclear.*

Also we prove an analog of Wold decomposition Theorem for such family of isometries, and study irreducible representations corresponding to each of its components.

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Three-color graph of the Morse flow on a compact surface with boundary

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We consider the Morse flows [1] (Morse-Smale flows without closed orbits) on the compact surfaces with boundary. There was constructed a complete topological invariant of these flows – an equipped three-colored graph.

The graph T will be called *three-color graph*, if all its vertices have a degree not bigger 3, and edges are painted in three colors, so that edges of different colors converge at each vertex. Colors are denoted by the letters s, t, u.[2, 3] The vertices of three-colored graph correspond to the standard areas on the surface, that look like a curvilinear triangle or quadrilateral. There were found conditions in which a three-colored graph generates a flow.

Theorem 1. *For a connected tricolor graph having properties*

1) each edge of the graph is marked with one of the three letters: s, t, u, and each vertex is white or black;

2) two edges of the same type can not come out from each vertex;

3) for each black inner vertex there is a su -cycle of length 4 that contains it;

4) if two black vertices are connected by a $u-$ or $s-$ edge and one of them is bounded, then the other will be bound;

5) each white vertex is internal. And if it is connected to the black vertex $u-$ edge ($s-$ edge), then this black vertex will be the limit.

there exists a Morse flow on a connected surface with a boundary, the three-color graph of which is a given graph.[1]

The number of topologically non-equivalent flows with 2, 3, 4, and 5 standard areas was calculated. For each of them, the surface on which this flow is set is determined. The distribution of the number of flows on the surfaces is shown in the table.

	2 stand.areas	3 stand.areas	4 stand.areas	5 stand.areas
Disk (D^2)	5	3	18	22
Myobius leaf		1	5	15
Myobius with a hole			2	2
Ring($S^1 \times I$)	1		10	10
Ring with a hole			1	1
Klein bottle				3
Torus with a hole				1

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НТБ ОНАХТ

The Ricci Iteration on Homogeneous Spheres

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The Ricci iteration is a discrete analogue of the Ricci flow. Introduced in 2007, it has been studied extensively as a new approach to uniformisation. In this talk, we will discuss the Ricci iteration on spheres that are equipped with transitive Lie group actions. Joint work with Timothy Buttsworth (Queensland), Yanir Rubinstein (Maryland) and Wolfgang Ziller (Pennsylvania).

НТБ ОНАХТ

The construction of squaring the circle

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The paper contains the original method for the construction of squaring the circle, one of the famous Greek problems more than 25 centuries old, known to be unsolvable by using only a ruler and compass. The solution of the problem is possible if the diameter of the given circle is divided by a point using the Thales theorem on proportional length in and the ratio of large real numbers. The process of solving the above-mentioned problem relies on the Euclidean geometry and contains a description of the construction, construction, proof, and discussion. The construction leading to the solution of the problem is based on the assumption that the tools (instruments) are perfectly precise and that the solution is completed if used a finite number of times.

The proof contains two derived formulas in accordance with the rules of Numerical analysis, combined into a single (universal) formula which can be used in practice. In discussion the conditions on which the problem can be solvable, as well as number of solutions are given.

1. Squaring the circle using only a straightedge and compass is possible

Description of construction:

A given circle with a central point O and radius r are denoted by $k(O, r)$. The length AB is diameter of an arbitrary circle k. (Fig.1) As shown by the previous method, when constructing of the length $X = \sqrt{2}$, we divide diameter AB by the point C in the ratio of integers 11000000 and 3005681, i.e. $AC : CB = 11000000 : 3005681$,

in the following way:

On the arbitrary ray A_q we determine point M by "transferring" 11000000 arbitrary unit lengths. Then we determine the point N so that the length MN equals 3005681 arbitrary unit lengths.

Then we construct a length NB. Through the point M we draw a line s parallel to the length NB. The intersection of the line s and length AB is denoted by C. Through the point C we construct the line l so that it is parallel to the ray A_q and its intersection with the length NB we denote by the point L. (Fig. 1) The length AB is divided in the above mentioned ration by the point C.

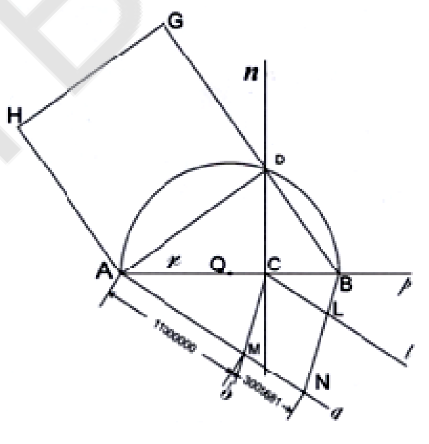

Fig.1

Triangles AMC and CLB are similar, so we can form the proportion:

$$AC : AM = CB : CL ...(3)$$

Based on relation (3), we replace:

$$AM = 11{\cdot}106 \text{ and } MN = CL = 3005681 = 3,005681{\cdot}106$$

It follows that $AC : CB = 11 \cdot 10^6 : 3,005681 \cdot 10^6$, Q.E.D. (Quod erat demonstrandum)

After having it shortened with 106, we get:

$$AC : CB = 11 : 3,005681 = t ...(4)$$

Based on relation (4) $AC : 11 = t => AC = 11t$ and

$$CB : 3,005681 = t => CB = 3,005681t....(5),$$

where t is a non-negative real number, i.e. $t > 0$ and $t \in R$.

Let us construct a line n through the point C to be perpendicular to the diameter AB, and denote its (one) intersection with the periphery of the circle by D. Then we draw lengths AD and BD. AD represents the side of the square whose area is equal to the area of the given circle. Then we construct the square $ADGH$ (Figure 1).

НТБ ОНАХТ

Riemann-Klein antagonism and problem of energy in general relativity

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B. Riemann and F. Klein had laid different principles at the foundation of geometry: the length principle which requires the possibility to measure the lengths of arbitrary lines no matter how they are situated, and the equality principle which is established by coincidence of figures in the space by miens of transformations belonging to a group of transformations of the space - the principal group of the geometry under consideration. According to E. Cartan, there is an antagonism between these principles owing to the absence of any homogeneity in an arbitrary curved Riemannian space.

In the work [1] was constructed the group of parallel translations DP, which realizes for a Riemannian space the equality principle and has as a subgroup the group of Riemannian translations RT which realizes the length principle. Therefore, the group DP unites the two approaches laid at the foundation of geometry by B. Riemann and F. Klein, thus overcoming the Riemann - Klein antagonism.

In the present work the group of parallel translations DP is used for determination of energy-momentum tensor of gravitational field.

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НТБ ОНАХТ

On generalized spaces of persistence diagrams

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Topological Data Analysis (TDA) is a relative new branch of applied mathematics. It provides some metric and topological structures for analyzing big data represented as point set in the euclidean spaces or more general metric (topological) spaces.

The persistent homology is one of the most important topological tools in TDA (see [1]). In order to visualize the persistent homology one uses the so-called persistence diagrams (see, e.g., [2] and the references therein). The sets of persistence diagrams bear natural metrics or topology and the studying of the metric and topological properties of the obtained spaces is important for understanding properties of the data sets.

Following [3] we consider the generalized spaces of persistence diagrams. Let X be a set. A diagram on X is a function $D\colon X^2 \to \mathbb{Z} = 0$ such that $D(p) = 0$ for all but finitely many $p \in X^2$, and $D(p) = 0$ for all $p \in \Delta_X = \{(x, x) \mid x \in X\} \subset X^2$. In [4] it is remarked that the set $\mathcal{D} = \mathcal{D}(X)$ of all persistence diagrams can be naturally identified with the infinite symmetric product $SP^\infty(X^2/\Delta_X)$ (with the base point $* = \Delta_X$).

If X is a topological space and $X = \varinjlim X_n$, where $X_1 \subset X_2 \subset X_3 \subset \dots$, then one can topologize $\mathcal{D}$ as $\varinjlim SP^n(X_n^2/(\Delta_X \cap X_n^2)$ having in mind a natural identification of $[x_1, \dots, x_n] \in SP^n(X_n^2/(\Delta_X \cap X_n^2)$ with $[x_1, \dots, x_n, *] \in SP^{n+1}(X_{n+1}^2/(\Delta_X \cap X_{n+1}^2)$.

Our result is a generalization of one of the main results of [4]. Recall that $\mathbb{R}^\infty$ is the direct limit of the sequence $\mathbb{R} \subset \mathbb{R}^2 \subset \mathbb{R}^3 \subset \dots$. An $\mathbb{R}^\infty$-manifold is a k_ω-space, which is locally homeomorphic to $\mathbb{R}^\infty$.

An ANR-space is an absolute neighborhood retract in the class of metrizable spaces.

Theorem 1. *Let $X = \varinjlim X_n$, where (X_n) is a sequence of finite-dimensional compact metrizable ANR-spaces. If $\dim X > 0$, then the space $\mathcal{D}(X)$ is an $\mathbb{R}^\infty$-manifold.*

One can also find some sufficient conditions on X such that the space $\mathcal{D}(X)$ is a Q^∞-manifold, where Q is the Hilbert cube and Q^∞ is the direct limit of the sequence

$$Q = Q \times \{*\} \hookrightarrow Q \times Q = Q \times Q \times \{*\} \hookrightarrow Q \times Q \times Q = \dots,$$

for an arbitrary $* \in Q$.

The proofs are based on Sakai's Characterization Theorem for $\mathbb{R}^\infty$ and Q^∞ (see [5]).

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НТБ ОНАХТ

Continual approximate solution with acceleration and condensation mode

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The kinetic equation Boltzmann is the main instrument to study the complicated phenomena in the multiple-particle systems, in particular, rarefied gas. This kinetic integro-differential equation for the model of hard spheres has a form [1, 2]:

$$D(f) = Q(f, f), \tag{1}$$

$$D(f) = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x}, \tag{2}$$

$$\begin{aligned} Q(f, f) = \frac{d^2}{2} \int\limits_{\mathbb{R}^3} dv_1 \int\limits_{\Sigma} d\alpha |(v - v_1, \alpha)|[f(t, v_1', x)f(t, v', x)- \\ - f(t, v_1, x)f(t, v, x)], \end{aligned} \tag{3}$$

We will consider the continual distribution [3]:

$$f = \int\limits_{\mathbb{R}^3} \varphi(t, x, u)M(v, u, x, t)du, \tag{4}$$

which contains the local Maxwellian of special form describing the acceleration and condensation flows of a gas (is an analogue of vortices) [4]. They have the form:

$$M(v, u, x, t) = \rho_0 e^{\beta\left((u-[\omega\times t])^2+2[\omega\times x]\right)} \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} e^{-\beta(v-u-[\omega\times t])^2}. \tag{5}$$

The purpose is to find such a form of the function $\varphi(t, x, u)$ and such a behavior of all hydrodynamical parameters so that the uniform-integral remainder [3, 4]

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)|dv, \tag{6}$$

or its modification "with a weight":

$$\widetilde{\Delta} = \sup_{(t,x)\in\mathbb{R}^4} \frac{1}{1 + |t|} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)|dv, \tag{7}$$

tends to zero.

Also some sufficient conditions to minimization of remainder Δ or $\widetilde{\Delta}$ are found. The obtained results are new and may be used with the study of evolution of screw and whirlwind streams.

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НТВ ОНАХТ

Approximation by Fourier sums and interpolation trigonometric polynomials in classes of differentiable functions with high exponents of smoothness

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Let C and $L_p,\ 1 \le p \le \infty$, be the spaces of 2π–periodic functions with the standart norms $\|\cdot\|_C$ and $\|\cdot\|_p$. Further, let $W^r_{\beta,p}, 1 \le p \le \infty$, be the sets of all 2π-periodic functions f, representable as convolutions of the form

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \varphi(x-t)B_{r,\beta}(t)dt, \quad a_0 \in \mathbb{R},\ \varphi \perp 1,\ \|\varphi\|_p \le 1, \tag{1}$$

where $B_{r,\beta}(\cdot)$ are Weyl-Nagy kernels of the form

$$B_{r,\beta}(t) = \sum_{k=1}^{\infty} k^{-r}\cos\left(kt - \frac{\beta\pi}{2}\right), \quad r > 0, \quad \beta \in \mathbb{R}. \tag{2}$$

The classes $W^r_{\beta,p}$ are called as Weyl-Nagy classes (see, e.g., [1]). If $r \in \mathbb{N}$ and $\beta = r$, then the functions of the form (8) are the well-known Bernoulli kernels and the classes $W^r_{\beta,p}$ coincide with the well-known classes W^r_p, which consist of 2π-periodic functions with absolutely continuous derivatives up to $(r-1)$-th order inclusive and such that $\|f^{(r)}\|_p \le 1$ and $f^{(r)}(x) = \varphi(x)$ for almost everywhere $x \in \mathbb{R}$, where φ is the function from (10).

For arbitrary $\mathfrak{N} \subset X$, where $X = C$ or $L_p,\ 1 \le p \le \infty$, we consider the quantity

$$\varepsilon_n(\mathfrak{N})_X = \sup_{f\in\mathfrak{N}} \|f(\cdot) - S_{n-1}(f;\cdot)\|_X, \tag{3}$$

where $S_{n-1}(f;x)$ is the partial Fourier sum of order $n-1$ of the function f.

In the case of Weyl-Nagy classes $W^r_{\beta,\infty}$ and $X = C$ for the exact upper bounds (3) the following asymptotic estimate holds

$$\varepsilon_n(W^r_{\beta,\infty})_C = \frac{4}{\pi^2}\frac{\ln n}{n^r} + O\left(\frac{1}{n^r}\right), \quad r > 0, \quad \beta \in \mathbb{R}. \tag{4}$$

For $r \in \mathbb{N}$ and $\beta = r$ this estimate was obtained by A.N. Kolmogorov (1935), for arbitrary $r > 0$ by V.T. Pinkevich (1940) and S.M. Nikol'skii (1941). In the general case the estimate (4) follows from results, which were obtained in the works of A.V.Efimov (1960) and S.A. Telyakovskii (1961). It should be also noticed, that a similar asymptotic equality holds for the classes $W^r_{\beta,1}$ in the metric of the space L_1. In these works the parameters r and β of the Weyl-Nagy classes were assumed to be fixed, and the question about the dependence of the remainder term in the estimate (4) on these parameters was not considered.

The character of the dependence on r and β of the remainder term in estimate (4) was investigated by I.G. Sokolov (1955), S.G. Selivanova (1955), G.I. Natanson (1961), S.A. Telyakovskii (1968, 1989) and S.B. Stechkin (1980). In the work of S.B. Stechkin [2] the asymptotic behavior, as $n \to \infty$ and $r \to \infty$, of the quantities $\varepsilon_n(W^r_{\beta,\infty})_C$ was completely investigated. Besides, S.B. Stechkin [2, theorem 4] proved that for rapidly growing r the remainder can be improved. Namely, for arbitrary $r \ge n+1$

and $\beta \in \mathbb{R}$ the following equality holds:

$$\varepsilon_n(W^r_{\beta,\infty})_C = \frac{1}{n^r}\left(\frac{4}{\pi} + O(1)\left(1+\frac{1}{n}\right)^{-r}\right), \tag{5}$$

where $O(1)$ is a quantity uniformly bounded with respect to n, r and β. If $r/n \to \infty$, then the estimate (5) becomes the asymptotic equality. It also follows from [2] that for the quantity $\varepsilon_n(W^r_{\beta,1})_{L_1}$ the analogous estimate to (5) takes place. S.A. Telyakovskii (1989) showed that the remainder in formulas (5) can be replaced by a smaller one, namely, write $O(1)(1+2/n)^{-r}$ instead of $O(1)(1+1/n)^{-r}$.

We establish generalized analogs of estimates (5) for quantities $\varepsilon_n(W^r_{\beta,p})_C$ and $\varepsilon_n(W^r_{\beta,1})_{L_p}$, respectively, for arbitrary values $1 \le p \le \infty$.

Theorem 1. *Let* $1 \le p \le \infty$, $n \in \mathbb{N}$ *and* $\beta \in \mathbb{R}$. *Then for* $r \ge n+1$ *the following estimates hold:*

$$\varepsilon_n(W^r_{\beta,p})_C = \frac{1}{n^r}\left(\frac{\|\cos t\|_{p'}}{\pi} + O(1)\left(1+\frac{1}{n}\right)^{-r}\right), \tag{6}$$

$$\varepsilon_n(W^r_{\beta,1})_{L_p} = \frac{1}{n^r}\left(\frac{\|\cos t\|_{p}}{\pi} + O(1)\left(1+\frac{1}{n}\right)^{-r}\right), \tag{7}$$

where $1/p + 1/p' = 1$ *and* $O(1)$ *are quantities uniformly bounded in all analyzed parameters. The estimates (6) and (7) are the asymptotic equalities, as* $r/n \to \infty$.

Let $f \in C$. By $\tilde{S}_{n-1}(f;x)$ we denote a trigonometric polynomial of degree $n-1$, that interpolates $f(x)$ at the equidistant nodes $x_k^{(n-1)} = \frac{2k\pi}{2n-1}$, $k \in \mathbb{Z}$, i.e., such that $\tilde{S}_{n-1}(f;x_k^{(n-1)}) = f(x_k^{(n-1)})$, $k \in \mathbb{Z}$.

For $\mathfrak{N} \subset C$ and $X = C$ or $X = L_p, 1 \le p \le \infty$, consider the following approximative characteristic

$$\tilde{\varepsilon}_n(\mathfrak{N})_X = \sup_{f\in\mathfrak{N}} \|f(\cdot) - \tilde{S}_{n-1}(f;\cdot)\|_X. \tag{8}$$

The problems of finding of asymptotic behavior for quantity of the form (8) in important functional classes $\mathfrak{N}$ was investigated by S.M. Nikol'skii, V.P. Motornyi, A.I. Stepanets, A.S. Serdyuk, and others.

The following statement is true [3].

Theorem 2. *Let* $1 \le p \le \infty$, $n \in \mathbb{N}$ *and* $\beta \in \mathbb{R}$. *Then for* $r \ge n+1$ *the following estimates hold:*

$$\tilde{\varepsilon}_n(W^r_{\beta,p})_C = \frac{1}{n^r}\left(\frac{2}{\pi}\|\cos t\|_{p'} + O(1)\left(1+\frac{1}{n}\right)^{-r}\right), \tag{9}$$

$$\tilde{\varepsilon}_n(W^r_{\beta,1})_{L_p} = \frac{1}{n^r}\left(\frac{2^{1-\frac{1}{p}}}{\pi^{1+\frac{1}{p}}}\|\cos t\|_p^2 + O(1)\left(\frac{1}{n} + \left(1+\frac{1}{n}\right)^{-r}\right)\right), \tag{10}$$

where$1/p + 1/p' = 1$ *and* $O(1)$ *are quantities uniformly bounded in all analyzed parameters. The estimates (9) and (10) are the asymptotic equalities, as* $r/n \to \infty$, $n \to \infty$.

Comparing formulas (6), (7), (9) and (10) we see that

$$\tilde{\varepsilon}_n(W^r_{\beta,p})_C \sim 2\varepsilon_n(W^r_{\beta,p})_C,$$

$$\tilde{\varepsilon}_n(W^r_{\beta,1})_{L_p} \sim \frac{2^{1-\frac{1}{p}}}{\pi^{\frac{1}{p}}}\|\cos t\|_p \; \varepsilon_n(W^r_{\beta,1})_{L_p},$$

as $r/n \to \infty$, $n \to \infty$.

НТБ ОНАХТ

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НТБ ОНАХТ

Lebesque-type inequalities for the Fourier sums

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Let L_p, $1 \le p < \infty$, be the space of 2π–periodic functions f summable to the power p on $[0, 2\pi)$, in which the norm is given by the formula $\|f\|_p = \Big(\int\limits_0^{2\pi} |f(t)|^p dt\Big)^{\frac{1}{p}}$; and C be the space of continuous 2π–periodic functions f, in which the norm is specified by the equality $\|f\|_C = \max\limits_t |f(t)|$.

Denote by $C^{\alpha,r}_\beta L_p$, $\alpha > 0$, $r > 0$, $\beta \in \mathbb{R}$, $1 \le p \le \infty$, the set of all 2π–periodic functions, representable for all $x \in \mathbb{R}$ as convolutions of the form (see, e.g., [1, p. 133])

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int\limits_{-\pi}^{\pi} P_{\alpha,r,\beta}(x-t)\varphi(t)dt,\ a_0 \in \mathbb{R},\quad \varphi \perp 1,\quad \varphi \in L_p, \tag{1}$$

where $P_{\alpha,r,\beta}(t)$ are generalized Poisson kernels

$$P_{\alpha,r,\beta}(t) = \sum_{k=1}^{\infty} e^{-\alpha k^r} \cos\big(kt - \frac{\beta\pi}{2}\big),\quad \alpha, r > 0,\quad \beta \in \mathbb{R}.$$

If the functions f and φ are related by the equality (1), then function f in this equality is called generalized Poisson integral of the function φ. The function φ in equality (1) is called as generalixed derivative of the function f and is denoted by $f^{\alpha,r}_\beta$.

Let $E_n(f)_{L_p}$ be the best approximation of the function $f \in L_p$ in the metric of space L_p, $1 \le p \le \infty$ by the trigonometric polynomials t_{n-1} of degree $n-1$, i.e.,

$$E_n(f)_{L_p} = \inf_{t_{n-1}} \|f - t_{n-1}\|_{L_p}.$$

Our aim is to obtain of asymptotically best possible Lebesque-type inequalities, for functions from the class $C^{\alpha,r}_\beta L_p$, where norms $\|f(\cdot) - S_{n-1}(f;\cdot)\|_C$ are estimated via best approximations $E_n(f^{\alpha,r}_\beta)_{L_p}$ for $0 < r < 1$ and $1 \le p < \infty$. Here $S_{n-1}(f;\cdot)$ is the partial Fourier sums of order $n-1$ for a function f. For $r \ge 1$ such inequalities were established in [1]–[3].

For arbitrary $\alpha > 0$, $r \in (0,1)$ and $1 \le p < \infty$ we denote by $n_0 = n_0(\alpha, r, p)$ the smallest integer n such that

$$\frac{1}{\alpha r}\frac{1}{n^r} + \frac{\alpha r p}{n^{1-r}} \le \begin{cases} \frac{1}{14}, & p = 1,\\ \frac{1}{(3\pi)^3}\cdot\frac{p-1}{p}, & 1 < p < \infty.\end{cases}$$

We use Gauss hypergeometric function $F(a, b; c; d)$ of the form

$$F(a, b; c; z) = 1 + \sum_{k=1}^{\infty} \frac{(a)_k (b)_k}{(c)_k}\frac{z^k}{k!},$$

where $(x)_k$ is the Pochhammer symbol, defined by $(x)_k := x(x+1)...(x+k-1)$.

We showed that the following theorems hold.

Theorem 1. *Let $0 < r < 1$, $\alpha > 0$, $\beta \in \mathbb{R}$, $1 < p < \infty$, and $n \in \mathbb{N}$. Then for any function $f \in C^{\alpha,r}_{\beta} L_p$ and $n \geq n_0(\alpha, r, p)$, fthe following inequality is true:*

$$\|f(\cdot) - S_{n-1}(f;\cdot)\|_C \leq e^{-\alpha n^r} n^{\frac{1-r}{p}} \Big(\frac{\|\cos t\|_{p'}}{\pi^{1+\frac{1}{p'}} (\alpha r)^{\frac{1}{p}}} F^{\frac{1}{p'}} \Big(\frac{1}{2}, \frac{3-p'}{2}; \frac{3}{2}; 1\Big) +$$

$$+ \gamma_{n,p} \Big(\Big(1 + \frac{(\alpha r)^{\frac{p'-1}{p}}}{p'-1}\Big) \frac{1}{n^{\frac{1-r}{p}}} + \frac{(p)^{\frac{1}{p'}}}{(\alpha r)^{1+\frac{1}{p}}} \frac{1}{n^r} \Big) \Big) E_n(f^{\alpha,r}_{\beta})_{L_p}, \quad \frac{1}{p} + \frac{1}{p'} = 1, \tag{2}$$

where $F(a, b; c; d)$ is Gauss hypergeometric function.

Moreover for any function $f \in C^{\alpha,r}_{\beta} L_p$ one can find a function $F(x) = F(f; p; n; x)$, such that $E_n(F^{\alpha,r}_{\beta})_{L_p} = E_n(f^{\alpha,r}_{\beta})_{L_p}$ and for $n \geq n_0(\alpha, r, p)$ the following equality is true

$$\|F(\cdot) - S_{n-1}(F;\cdot)\|_C = e^{-\alpha n^r} n^{\frac{1-r}{p}} \Big(\frac{\|\cos t\|_{p'}}{\pi^{1+\frac{1}{p'}} (\alpha r)^{\frac{1}{p}}} F^{\frac{1}{p'}} \Big(\frac{1}{2}, \frac{3-p'}{2}; \frac{3}{2}; 1\Big) +$$

$$+ \gamma_{n,p} \Big(\Big(1 + \frac{(\alpha r)^{\frac{p'-1}{p}}}{p'-1}\Big) \frac{1}{n^{\frac{1-r}{p}}} + \frac{(p)^{\frac{1}{p'}}}{(\alpha r)^{1+\frac{1}{p}}} \frac{1}{n^r} \Big) \Big) E_n(f^{\alpha,r}_{\beta})_{L_p}, \quad \frac{1}{p} + \frac{1}{p'} = 1. \tag{3}$$

In (2) and (3) the quantity $\gamma_{n,p} = \gamma_{n,p}(\alpha, r, \beta)$ is such that $|\gamma_{n,p}| \leq (14\pi)^2$.

Theorem 2. *Let $0 < r < 1$, $\alpha > 0$, $\beta \in \mathbb{R}$, $n \in \mathbb{N}$. Then, for any $f \in C^{\alpha,r}_{\beta} L_1$ and $n \geq n_0(\alpha, r, 1)$, the following inequality holds:*

$$\|f(\cdot) - S_{n-1}(f;\cdot)\|_C \leq e^{-\alpha n^r} n^{1-r} \Big(\frac{1}{\pi \alpha r} + \gamma_{n,1} \Big(\frac{1}{(\alpha r)^2} \frac{1}{n^r} + \frac{1}{n^{1-r}} \Big) \Big) E_n(f^{\alpha,r}_{\beta})_{L_1}. \tag{4}$$

Moreover for any function $f \in C^{\alpha,r}_{\beta} L_1$ one can find a function $F(x) = F(f; n, x)$ in the set $C^{\alpha,r}_{\beta} L_1$, such that $E_n(F^{\alpha,r}_{\beta})_{L_1} = E_n(f^{\alpha,r}_{\beta})_{L_1}$ and for $n \geq n_0(\alpha, r, 1)$ the following equality holds

$$\|F(\cdot) - S_{n-1}(F;\cdot)\|_C = e^{-\alpha n^r} n^{1-r} \Big(\frac{1}{\pi \alpha r} + \gamma_{n,1} \Big(\frac{1}{(\alpha r)^2} \frac{1}{n^r} + \frac{1}{n^{1-r}} \Big) \Big) E_n(f^{\alpha,r}_{\beta})_{L_1}. \tag{5}$$

In (4) and (5), the quantity $\gamma_{n,p} = \gamma_{n,p}(\alpha, r, \beta)$ is such that $|\gamma_{n,p}| \leq (14\pi)^2$.

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Minimal generating set and structure of wreath product of cyclic groups, comutator of wreath product and the fundamental group of orbit Morse function $\pi_1 O(f)$

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Let i_j be the orders of C_{i_j}. In this work the previous result of the author [1] is strengthen also there is considered new class of *wreathcyclic* groups $\Im$ (let $G \in \Im$) which constructed by formula:

$$G = (\mathop{\wr}_{j_0=0}^{n_0} C_{k_{j_0}}) \times (\mathop{\wr}_{j_1=0}^{n_1} C_{k_{j_1}}) \times \ldots \times (\mathop{\wr}_{j_l=0}^{n_l} C_{k_{j_l}}), 1 \leq k_{j_i} < \infty, n_i < \infty.$$

Theorem 1. *If orders of cyclic groups $\mathbb{C}_{n_i}$, $\mathbb{C}_{n_j}$ is mutually coprime $i \neq j$ then the group $G = C_{i_1} \wr C_{i_2} \wr \ \ldots \ \wr C_{i_m}$ admits two generators β_0, β_1.*

The subtree of X^* induced by the set of vertices $\cup_{i=0}^{k} X^i$ is denoted by $X^{[k]}$.

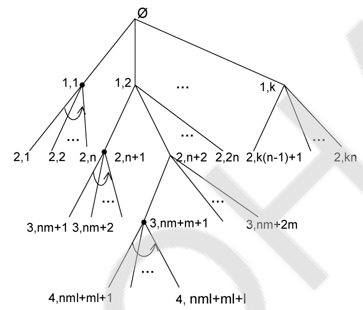

FIGURE 1.1. Directed automorhism

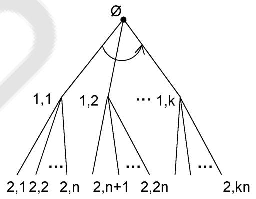

FIGURE 1.2. Rooted automorhism

We construct the generators of $\mathop{\wr}_{j=0}^{n} C_{i_j}$ as a rooted automorphism β_0 in Figure 2 and a directed β_1 along a path l in Figure 1.1 on a rooted labeled truncated tree $X^{[k]}$.

Let $l = x_1 x_2 x_3 ... x_k$ be an finite ray in $X^{[k]}$.

Definition 2. We say that the automorphism g of $\mathbb{X}$ is directed along l and we call l the spine of g if all vertex permutations along the ray l and all vertex permutations corresponding to vertices whose distance to the ray l is at least 2 are trivial (Figure 1).

Definition 3. An automorphism of X is rooted if all of its vertex permutations that correspond to non-empty words are trivial.

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Corollary 4. *A center of the group $\mathbb{Z}\ltimes_{\phi}(\mathbb{Z})^{n}\simeq(\mathbb{Z},X)\wr\mathbb{Z}$ consists of normal closure of diagonal of Z^{n}, trivial an element, and kernel of action by conjugation that is $n\mathrm{Z}$. Other words*

$$Z(H)=\langle 1;\ \underbrace{h,h,...,h}_{n}),\ e,\ (n\mathbb{Z},X)\wr\mathbb{E}\rangle\simeq n\mathbb{Z}\times\mathbb{Z},$$

where $h,\ g\in\mathrm{Z}$, $Z(H)\simeq n\mathbb{Z}\times\mathbb{Z}$.

Corollary 5. *A center of a group of form $\mathbb{Z}\ltimes_{\phi}(\mathcal{B})^{n}\simeq(\mathbb{Z},X)\wr\mathcal{B}$ generates by normal closure of: diagonal of $\mathcal{B}^{n}$, trivial an element, and $n\mathrm{Z}\underset{X}{\wr}\mathcal{E}$.*

In our case the Morse function [2] f on M that has the following properties:

(1) f is constant on the bound M,
(2) it has 2 points of maximum at a saddle point,
(3) at these 2 points of maximum, the values of the function are equal; in every critical point of f the germ of f is C^{∞} equivalent to some homogeneous polynomial of 2 real variables without multiple factors.

Consider a group H of automorphisms of M which are induced by the action of diffeomorphisms h of a group $D(M)$ such that preserving the Mebius function f, that is, such h are from the stabilizer $S(f)\lhd D(M)$. Generators of their stabilizers by right action by diffeomorphisms $\pi_0 S(f|_{X_i},\partial X_i)$ are τ_i.

The first generator ρ of cyclic group Z realizes shift of Mebius band and second τ realize rotation of domains X_i of simple connectedness on Mebius band when passing through the twisting point of Mebius band (M).

Proposition 6. *The group $H\simeq\mathbb{Z}\ltimes(\mathbb{Z})^{n}=\langle\rho,\ \tau\rangle$ with defined above homomorphism in $AutZ^{n}$ has two generators and non trivial relations*

$$\rho^{n}\tau\rho^{-n}=\tau^{-1},\ \rho^{i}\tau\rho^{-i}\rho^{j}\tau\rho^{-j}=\rho^{j}\tau\rho^{-j}\rho^{i}\tau\rho^{-i},\ 0<i,j<n.$$

Also this group admits another presentation in generators and relations

$$\left\langle\rho,\tau_{1},...,\tau_{n}\left|\rho\tau_{i(\bmod n)}\rho^{-1}=\tau_{i+1(\bmod n)}\right.,\ \tau_{i}\tau_{j}=\tau_{j}\tau_{i},\ i,\ j\leq n\right\rangle. \tag{1}$$

Proposition 7. *The commutator of Sylow 2-subgroup $(Syl_2 A_{2^k})'$ has order $2^{2^{k}-k-2}$.*

Proposition 8. *The second commutator of Sylow 2-subgroup $(Syl_2 A_{2^k})$ has the order $2^{2^{k}-3k+1}$.*

Corollary 9. *The Frattini factor of $(Syl_2 A_{2^k})'$ is isomorphic to elementary abelian subgroup $(C_2)^{2k-3}$. Any minimal generator set of $(Syl_2 A_{2^k})'$. has $2k-3$ generators.*

Example 10. The minimal generating set of $Syl_2'(A_8)$ consists of 3 generators: $(1,3)(2,4)(5,7)(6,8)$, $(1,2)(3,4), (1,3)(2,4)(5,8)(6,7)$. The commutator $Syl_2'(A_8)\simeq C_2^3$ that is an elementary abelian 2-group of order 8. Minimal generating set of $Syl_2'(A_{16})$ consist of 5 (that is $2\cdot4-3$) generators:
$(1,4,2,3)(5,6)(9,12)(10,11), (1,4)(2,3)(5,8)(6,7), (1,2)(5,6)$,
$(1,7,3,5)(2,8,4,6)(9,14,12,16)(10,13,11,15), (1,7)(2,8)(3,6)(4,5)(9,16,10,15)\times$
$\times(11,14,12,13)$.

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Spaces of primitive elements in dual modules over Steenrod algebra 2

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I present a way to generate all primitive elements $PB(n)$ in $B(n) = (A(n-1)/A(n))^*$ modules over A^* dual Steenrod algebra, where $A(n)$ are annihilator modules over Steenrod algebra A. This work began in [7]. For useful notions see also [1, 2, 3, 4] and references summarized in [8, 5, 6, 7]. The filtration described in [6] Theorem 1 property 2 and 3 yields $PB(n) = \bigcup_t PB(n)_t$ and $PB(n)_t = \bigoplus_s PB(n)_t^s$, where s is the number of τ operations and t is the biggest index of such operations. From Theorem 1 [7] property 5 and 1 it is known $dimPB(n)_t^{s,deg} \leq 1$ and the following diagram is exact

$$0 \longrightarrow PC(n)_{k-1} \xrightarrow{\iota_k} PC(n)_k \xrightarrow{\lambda_k} PC(n-1)_{k-1}$$

For given $\alpha \in PB(n-1)_{t-1}$ how to find a primitive $\alpha' \in PB(n)_t$ such that $\pi_t(\alpha') = \alpha$? Properties 2 and 6 state for even n that $PB(n)_{-1} = PB(n)_0 = \langle \xi_1^{n/2} \rangle$; and for odd n: $PB(n)_0 = \langle \xi_1^{\frac{n-1}{2}} \tau_0 \rangle$ and for $s \geq 1$ that $\alpha\tau_0$ is also a primitive. And we can generate new primitives taking products of primitives. Do all primitives in $PB(n)_k^1 \backslash PB(n)_0^1$ also have form $\alpha\tau_0$? So $\alpha\tau_1 \in B(n)$ yields coproduct $\phi^*(\alpha\tau_1) = \phi^*(\alpha)(\xi_1 \otimes \tau_0 + 1 \otimes \tau_1)$ and hence $\alpha\tau_1$ is primitive if and only if $\alpha = \alpha'\tau_0 \in PB(n-1)$. If $\alpha \in PB(n)_{-1}$ then $\alpha\tau_1 = \xi_1^{\frac{n}{2}}\tau_1$ is not primitive. But for example product of not primitive $\alpha = \xi_1^{\frac{n-1}{2}} + \tau_0\xi_2^{\frac{n-1}{2}} \in B(n)_0^1$ with primitive τ_0 is primitive. The primitivity condition in $B(n)$ leads to the following inductive definition of transformations R_i generating primitives, preserving primitivity.

Definition 1.

$$R_k(\alpha) = \xi^{\frac{p^{k-1}-1}{p-1}} \tau_k \alpha - \sum_{i=1}^{k-1} \xi^{\frac{p^{k-1}-p^i}{p-1}} \xi_{k+1-i}^{p^{i-1}} R_i(\alpha)$$

for $k > 1$ and $R_0(\alpha) = \alpha\tau_0$, $R_1(\alpha) = \alpha\tau_1$

These maps have the following properties.

Theorem 2. *(1)* $\forall i,k \in N$, $\forall \alpha \in B$: $R_i(\alpha\tau_k) = -R_i(\alpha)\tau_k$
(2) $\forall i,k \in N$, $\forall \alpha \in B$: $R_i(\alpha\xi_k) = R_i(\alpha)\xi_k$
(3) $\forall i,j \in N$, $\forall \alpha \in B$: $R_iR_j(\alpha) = -R_jR_i(\alpha)$
(4) $\forall \alpha \in PB(n) \cap ImR_0$: $R_i(\alpha) \in PB(n+1+2\frac{p^{i-1}-1}{p-1})$

Remark 3. From the definition 1: $R_i(\alpha) = \alpha R_i(1)$. Therefore by induction $R_{i_1}R_{i_2}\cdots R_{i_k}(\alpha) = \alpha R_{i_1}R_{i_2}\cdots R_{i_k}(1)$. And for example $R_2(1)$, $R_3(1)$ e.t.c. are primitives in $B(n)^1$.

Therefore all primitives have form $\alpha\tau_0$ except $PB(n)_k^1 \backslash PB(n)_0^1$. Induction arguments based on Theorem 1 [7] lead to the general form of primitive elements.

Definition 4. $\alpha_{i_1,i_2,\ldots i_k} = \xi_1^l \tau_{i_1}\tau_{i_2}\ldots\tau_{i_k} + \beta$ is a primitive in $PB(n)_{i_k}^k$ associated with $(i_1, i_2, \ldots i_t)$, $i_k > i_{k-1} > \ldots > i_1 = 0$ if it has projection on $J(n)^{k,deg} = B(n)^{k,deg}/(I \cap B(n)^{k,deg})$ equal $a\xi_1^l\tau_{i_1}\tau_{i_2}\ldots\tau_{i_k}$, $l = \frac{n-k}{2}$, $a \in Z/p$.

Corollary 5. *There exists the primitive* $\alpha_{i_1,i_2,\ldots i_k}$ *associated with* $(i_1, i_2, \ldots i_k)$, $i_k > i_{k-1} > \ldots > i_1 = 0$ *and it is satisfied* $\alpha_{i_1,i_2,\ldots i_k}\xi_1^l = R_{i_1}R_{i_2}\cdots R_{i_k}(1)$.

Remark 6. Corollary 5 also presents a way to calculate all associated primitives.

The following theorem is a result of construction of all primitive elements in $B(n)$.

Theorem 7. *All $PB(n)^{s,deg}$ in $PB(n) = \cup_k PB(n)_k$ where $PB(n)_k = \oplus PB(n)_k^s$ are zero or one dimensional spaces. $PB(n)^{s,deg}$ has dimension one if and only if there is a sequence $(i_1, i_2, \dots i_t)$, $i_s > i_{s-1} > \dots > i_1 = 0$ with conditions*

(1) $n - s$ is even,
(2) degree of $PB(n)^{s,deg}$ is $deg = (p-1)(n-s) + \sum_{j=1}^{s} dim(\tau_{i_j})$,
(3) $\frac{n-s}{2} \geq l$, where l is calculated below:
(4) $l = \sum_{j=2}^{s} \frac{p^{i_j - 1}-1}{p-1} - \sum_{j=2}^{s-1} \frac{p^{i_j}-1}{p-1}$,

When $\frac{n-s}{2} = l$

$$PB(n)^{s,deg} = \langle \alpha_{i_1, i_2, \dots i_s} \rangle$$

When $\frac{n-s}{2} > l$

$$PB(n)^{s,deg} = \langle \xi_1^{\frac{n-s}{2} - l} \alpha_{i_1, i_2, \dots i_s} \rangle$$

where $\alpha_{i_1, i_2, \dots i_s} = \xi_1^l \tau_{i_1} \tau_{i_2} \cdots \tau_{i_s} + \beta$ is the primitive in $B(n)_{i_s}^s$ associated with the sequence $(i_1, i_2, \dots i_t)$, $i_s > i_{s-1} > \dots > i_1 = 0$ with conditions 1,2,4 mentioned above and $\frac{n-s}{2} = l$.

Knowledge of primitive elements on $B(n) = (A(n-1)/A(n))^*$ make a feasible to find all indecomposable elements of $(A(n-1)/A(n)$ [8, sec 4] .

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НТБ ОНАХТ

A Geometrical Version of the Maxwell-Vlasov Hamiltonian Structure

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We present a geometrization of the Hamiltonian approach of classical electrodynamics, via (non-canonical) Poisson structures. This relativistic Hamiltonian framework (introduced by Morrison, Marsden, Weinstein) is a field theory written in terms of differential forms, independently of the gauge potentials. This algebraic and geometric description of the Vlasov kinetics is well suited for a perturbation theory, in a strong inhomogeneous magnetic field (expansion in $1/|B|$, with all the curvature terms...), like in magnetically confined plasmas, and in any coordinates, for instance adapted to a Tokamak (toroidal coordinates, or else...).

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Note on congruent numbers

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A positive integer A is said to be a *congruent number* if A is the area of a right triangle with rational sides. One may consider only A square-free.

In 1998, F. R. Nemenzo [3] listed all congruent numbers less than 40 000, and non-congruent numbers were studied by F. Lemmermeyer [2] and W. Cheng and X. Guo [1] among others.

I will present short proof of the following

Theorem 1. *Every positive integer A fulfilling the Diophantine equation*

$$A^2 = x^2 + y^4$$

is a congruent number.

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Про число топологічно нееквівалентних гладких функцій з однією критичною точкою типу сідла на двовимірному торі

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Нехай M_g – замкнена гладка орієнтовна поверхня роду $g \geq 0$, а $C_{k,l}(M_g)$ – клас гладких функцій на M_g (з трьома критичними значеннями), які мають точно k локальних мінімумів (максимумів), l локальних максимумів (мінімумів) та одну (в загальному випадку *вироджену*) критичну точку типу сідла, індекс Пуанкаре якої становить $1-n = 2-2g-k-l$ (напр. [6], [7]).

Через $C_n(M_g)$ позначимо клас гладких функцій на M_g (з трьома критичними значеннями), які окрім локальних мінімумів та локальних максимумів мають лише одну (в загальному випадку *вироджену*) критичну точку типу сідла, індекс Пуанкаре якої становить $1-n = 2-2g-\lambda$, де $\lambda \geq 2$ — сумарне число локальних мінімумів та максимумів.

Функції f_1 і f_2 з класу $C_n(M_g)$ називають топологічно еквівалентними, якщо існують гомеоморфізми $h : M_g \to M_g$ і $h' : R^1 \to R^1$ (h' зберігає орієнтацію), такі що $f_2 = h' \circ f_1 \circ h^{-1}$.

Якщо h зберігає орієнтацію, то функції f_1 та f_2 називають топологічно спряженими (напр. [6]) або ж O-топологічно еквівалентними (напр. [7]).

В загальному випадку, для довільних натуральних g, k, l (або ж k, l і $n = 2g+k+l-1$, тобто для функцій з фіксованим сингулярним типом), задача про підрахунок числа топологічно нееквівалентних функцій з класу $C_{k,l}(M_g)$ виявилась досить важкою та нерозв'язаною до сьогодні проблемою.

Серед найбільш суттєвих просувань в цьому напрямі слід відзначити наступні.

Задачу про підрахунок числа топологічно нееквівалентних функцій з класу $C_{1,1}(M_g)$ (для довільного роду $g \geq 1$) повністю розв'язано в роботі [7].

Одержані в роботі [2] точні формули цілком вирішують питання про підрахунок числа як O-топологічно нееквівалентних, так і числа топологічно нееквівалентних функцій з класу $C_n(M_0)$.

В роботі [8] для довільних натуральних k і l повністю розв'язані задачі про підрахунок числа O-топологічно та топологічно нееквівалентних функцій з класу $C_{k,l}(M_0)$.

Як з'ясувалося ([1] з посиланням на роботу [4]), задача про перерахування одноклітинкових двокольорових карт з n ребрами (одне з яких є поміченим), k білими та l чорними вершинами тісно пов'язана із задачею про підрахунок числа топологічно нееквівалентних функцій з класу $C_{k,l}(M_g)$. Відомості про карти можна знайти, наприклад, в огляді [4] та роботі [1].

Так, наприклад, з урахуванням результатів роботи [1], для довільного роду $g \geq 0$ та натуральних k і l, при яких $n = 2g+k+l-1$ є **простим** числом, в [10] наведено точні формули для підрахунку числа O-топологічно нееквівалентних функцій з класу $C_{k,l}(M_g)$.

Для двовимірного тору $T^2 = M_1$ задачі про підрахунок числа O-топологічно та топологічно нееквівалентних функцій повністю розв'язані лише на класах $C_{1,l}(T^2)$ ($C_{k,1}(T^2)$) в роботі [9] та $C_{2,l}(T^2)$ ($C_{k,2}(T^2)$) в роботі [11].

В загальному випадку — для фіксованих натуральних k і l — задача про підрахунок числа топологічно нееквівалентних функцій з класу $C_{k,l}(T^2)$ також залишається нерозв'язаною.

Якщо ж розглянути (більш ємний) клас функцій $C_n(T^2)$, то, з урахуванням результатів робіт [1], [5] і [3], можна встановити справедливість наступного твердження

Theorem 1 (основна). *Число O-топологічно нееквівалентних функцій з класу $C_n(T^2)$ можна обчислити за формулою*

$$t^*(n) = \frac{1}{n}\left(\frac{1}{6}C_{n-1}^2 C_{2(n-1)}^{n-1} + a(n) + 2b\left(\tfrac{2n}{3}\right) + 2c\left(\tfrac{n}{2}\right) + 2d\left(\tfrac{n}{3}\right)\right), \qquad (1)$$

де $n \geq 3$, $u(p) = \frac{(2p)!}{p!p!} = C_{2p}^p$,

$a(2p+1) = 0, \qquad a(2p) = \frac{p(p-1)}{6} \cdot C_{2p}^p = \frac{p(p-1)}{6} \cdot u(p);$

$c(2p+1) = 0, \qquad c(2p) = p \cdot C_{2p}^p = p \cdot u(p);$

$d(2p+1) = 0, \qquad d(2p) = p \cdot C_{2p}^p = p \cdot u(p);$

$$b(2p+1) = 0, \qquad b(2p) = (2p-1) \cdot C_{2(p-1)}^{p-1} = (2p-1) \cdot u(p-1).$$

n	3	4	5	6	7	8	9	10	11	12	13	14
$t^*(n)$	1	4	14	76	330	1 522	6 680	29 256	125 970	539 292	2 288 132	9 659 416

Table 1.1. Початкові значення числа O-топологічно нееквівалентних функцій з класу $C_n(T^2)$

На думку автора, цілком досяжним є одержання точних формул для підрахунку й числа топологічно нееквівалентних функцій з класу $C_n(T^2)$.

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Щодо геометричної характеристики спеціальних майже геодезичних перетворень просторів афінного зв'язку зі скрутом

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Розглядаються простори A^n класу C^r ($n > 2$, $r > 1$) афінного зв'язку зі скрутом. Крива L називається майже геодезичною лінією простору A^n, якщо існує такий компланарний вздовж L двовимірний розподіл, якому у кожній точці належить дотичний вектор цієї кривої, [1]. З точки зору теорії кривини кривих у просторах афінного зв'язку, майже геодезичні лінії характеризуються як криві, перша кривина яких є довільною, а друга і всі наступні кривини тотожно дорівнюют нулю.

Нескінченно мале перетворення

$$\widetilde{x}^h = x^h + \varepsilon\xi^h(x^1; x^2; \ldots; x^n)$$

простору A^n називається майже геодезичним перетворенням, якщо у наслідок такого перетворення кожна геодезична лінія простору A^n переходить у криву, яка, нехтуючи доданками другого і більш високих порядків малості відносно параметру ε, є майже геодезичною лінією простору A^n.

Існують три типи майже геодезичних перетворень просторів афінного зв'язку зі скрутом¸ [2]. Перетворення другого типу Π_2 характеризується тим, що у результаті таких перетворень геодезичні лінії переходять у криві, які, у головному, є майже геодезичними лініями спеціального виду, так званими F-кривими, визначеними спеціальним аффінором F, [2].

Досліджено перетворення типу Π_2, які задовольняють умову взаємності, у тому розумінні, що обернені для них перетвореня також є майже геодезичними перетвореннями типу Π_2, що відповідають тому ж самому аффінору. Для спеціальних перетворень типу Π_2 знайдені диференціально-алгебраїчного характеру обмеження на аффінор F, які визначають такі перетворення, як перетворення, що, у головному, зберігають клас відповідних F-кривих. Наведені необхідні приклади.

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НТБ ОНАХТ

Життєвий та науковий шлях Марка Григоровича Крейна

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Народився М.Г. Крейн 3 квітня 1907 року в Києві в багатодітній (мав трьох братів та двох сестер) ремісничій родині. Зацікавлення математикою проявилось з ранніх років. В тринадцятирічному віці почав слухати лекції одного з молодих тоді вихованців знаменитої Київської алгебраїчної школи – професора Бориса Миколайовича Делоне (згодом члена-кореспондента академії наук СРСР). З 14 років починає брати участь у роботі Київського алгебраїчного семінару професора Дмитра Олександровича Граве. В сімнадцятирічному віці переїздить до Одеси. За однією з версій причиною переїду було бажання почати самостійне доросле життя і вступ до Одеського циркового училища. Але тут чекала відмова і юнак почав замислюватися над кар'єрою моряка. Більш ймовірною причиною переїзду до Одеси було те, що за два роки до цього у 1922 році до Одеси переїхав працювати вихованець Київської алгебраїчної школи – Микола Григорович Чеботарьов. Після виступу М.Г. Крейна на семінарі М.Г. Чеботарьова з доповіддю про свої наукові результати було прийнято рішення відразу зарахувати юного дослідника до аспірантури, без диплому про вищу освіту. Науковим керівником Крейна став професор М.Г. Чеботарьов. З цього часу розпочинається стрімка наукова кар'єра молодого математика: після закінчення аспірантури, в 22 роки, він стає доцентом, а в 26 років – професором. В 32-річному віці його обирають членом – кореспондентом Академії наук України. Працював в навчальних та науково-дослідних закладах Одеси, Харкова, Києва. На початку 50-х років минулого століття, після так званої справи лікарів, В Радянському Союзі розпочалася антисемітська компанія. Це не могло не відбитися на долі Марка Григоровича. В 1954 році він зайняв посаду завідувача кафедри теоретичної механіки в Одеському інженерно-будівельному інституті, що очевидним чином не відповідало Значимості М.Г. Крейна як математика. Працював він на цій посаді аж 20 років. В 1974 році керівництво вирішило відправити М.Г. Крейна на пенсію, але втрутилася математична громадськість Країни і він був зарахований на посаду провідного наукового співробітника Південного відділення АН України. На цій посаді працював до самої смерті 17 жовтня 1989 року. Наукова спадщина М.Г. Крейна велика та значима. Він є автором близько 300 наукових праць, в тому числі 8 монографій, які перекладалися і видавалися в найбільш престижних видавництвах світу. Спектр наукових інтересів Крейна дуже широкий: алгебра, функціональний аналіз, теорія функцій, теорія інтегральних та диференціальних рівнянь, математична фізика, аналітична механіка. Особливо відомий своїм вкладом у розвиток методів функціонального аналізу, теорії операторів в функціональних просторах, пов'язаних з конкретними проблемами математичної фізики. Напевне, не буде перебільшенням сказати, що значна частина сучасної математики має свої корені в дослідженнях Крейна.

НТБ ОНАХТ

LGT-лінії та А-деформації мінімальних поверхонь

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Відомо [1], що будь-який ненульовий розв'язок T^α наступної системи диференціальних рівнянь

$$\left(\frac{HT^{\gamma}_{,\gamma}}{2H^2-K}\right)_{,\alpha} g^{\alpha\beta} - \left(\frac{K\left(d^{\alpha i}T^{\beta}_{,i}+d^{\beta i}T^{\alpha}_{,i}\right)}{2(2H^2-K)}\right)_{,\alpha} + b^{\beta}_{\alpha}T^{\alpha}=0 \qquad (1)$$

визначатиме нетривіальну ареальну нескінченно малу (н.м.) деформацію (А-деформацію) однозв'язної регулярної поверхні S класу C^4 ненульової гаусової кривини без омбілічних точок, гомеоморфної області $\overline{G}$ площини у E_3- просторі, зі стаціонарними лініями геодезичного скруту (LGT-лініями).

Будемо шукати розв'язок (1) у випадку, коли S-мінімальна поверхня $(2H=0)$.

Справедлива

Теорема 1. *Кожна мінімальна поверхня допускає нетривіальну А-деформацію зі збереженням LGT-ліній в достатньо малій області G. Тензори деформації мають представлення*

$$T^{\alpha\beta}=\frac{1}{2}\left(d^{\alpha i}T^{\beta}_{,i}+d^{\beta i}T^{\alpha}_{,i}\right), \qquad T^{\alpha}=-g^{\alpha i}u_i,$$

де функція $u(x^1,x^2)\in C^3$ є розв'язком диференціального рівняння

$$u_{11}+u_{22}+pu_1+qu_2+eu=0$$

і залежить від довільної функції $\nu(x^1,x^2)\in C^3, u_\alpha=\frac{\partial u}{\partial x^\alpha}, \quad u_{\alpha\beta}=\frac{\partial^2 u}{\partial x^\alpha \partial x^\beta}, p,q,e\in C^1(\overline{G})$ - відомі функції точки S.

Нехай Q- частина поверхні S, яка гомеоморфна області $D\subset\overline{G}$. Позначимо границю Q через L, а її образ на площині - через Γ. Враховуючи знайдений геометричний зміст функції $u(x^1,x^2)$, отримано наступний результат:

Теорема 2. *Будь-яка мінімальна поверхня при кожній із наступних граничних умов*

$$1) c^{\alpha\beta}\mathbf{r}_\beta\left(\delta\mathbf{n}\right)_{,\alpha}=2K\omega(x^1,x^2), \quad (x^1,x^2)\in\Gamma$$

$$2) \epsilon_{\alpha\beta}\rho^{\alpha\beta}=-K\omega(x^1,x^2), \qquad (x^1,x^2)\in\Gamma$$

де $\omega(x^1,x^2)\neq 0$ - наперед задана функція класу $C^1(\Gamma)$, допускає єдину нетривіальну А-деформацію зі збереженням LGT-ліній.

Слід відзначити, що у випадку $\omega(x^1,x^2)\equiv 0$ на Γ поверхня Q буде жорсткою відносно вказаних А-деформацій.

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НТБ ОНАХТ

Алгоритм побудови унітального дільника для многочленної матриці

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Нехай $\mathbb{F}_{n\times n}$ та $\mathbb{F}_{n\times n}[x]$ – кільця $(n\times n)$-матриць над полем $\mathbb{F}$ та кільцем многочленів $\mathbb{F}[x]$ відповідно. Позначимо: I_n – одинична $(n\times n)$-матриця і O – нульова $(n\times n)$-матриця.

Для неособливої нижньої трикутної матриці $A(x)=\begin{bmatrix} a_1(x) & 0 & \dots & \dots & 0\\ a_{21}(x) & a_2(x) & 0 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ a_{n1}(x) & a_{n2}(x) & a_{n3}(x) & \dots & a_n(x)\end{bmatrix}$, де $a_i(x)\in\mathbb{F}[x]$ – унітальні многочлени і $\deg a_{ij}(x)<\deg a_i(x)$ для всіх $i>j$, вкажемо умови її зображення у вигляді добутку $A(x)=B(x)C(x)$, де $B(x)=I_nx^r+\sum_{i=1}^{r}B_ix^{r-i}\in\mathbb{F}_{n,n}[x]$ – унітальна многочленна матриця степеня $r\geq 1$ із визначником $\det B(x)=b(x)$.

Якщо матриця $B(x)=I_nx^r+\sum_{i=1}^{r}B_ix^{r-i}\in\mathbb{F}_{n,n}[x]$ є лівим дільником трикутної матриці $A(x)$, то з рівності $A(x)=B(x)C(x)$ отримуємо $A(x)=D(x)G(x)$, де $D(x)=\left[d_{ij}(x)\right]$ – нижня трикутна матриця з наступними властивостями: $d_{ii}(x)\in\mathbb{F}[x]$ – унітальні многочлени, $\deg d_{ij}(x)<\deg d_{ii}(x)$ для всіх $i>j$, $\deg\prod_{i=1}^{k}d_{ii}(x)\leq kr$, $\prod_{i=1}^{n}d_{ii}(x)=\det B(x)=b(x)$. Отже, $a_i(x)=d_{ii}(x)g_{ii}(x)$ для всіх $1\leq i\leq n$. Нижче вкажемо алгоритм побудови унітального дільника $B(x)$ із неособливої трикутної матриці $A(x)\in\mathbb{F}_{n,n}[x]$.

1). Нехай визначник неособливої нижньої трикутної матриці $A(x)\in\mathbb{F}_{n,n}[x]$ зображений у вигляді добутку $\prod_{i=1}^{n}a_i(x)=b(x)c(x)$, де $b(x)\in\mathbb{F}[x]$ – унітальний многочлен степеня nr, $r<\deg A(x)$.

2). Діагональні елементи $a_i(x)$ матриці $A(x)$ запишемо у вигляді $a_i(x)=d_{i,m_i}^{(l)}(x)g_{i,m_i}^{(l)}(x)$, де $d_{i,m_i}^{(l)}(x)\in\mathbb{F}[x]$ – унітальні многочлени або елементи поля $\mathbb{F}$ для всіх $m_i=1,2,\ldots;\ l=1,2,\ldots$. За елементами $d_{i,m_i}^{(l)}(x)$ побудуємо множину діагональних $(n\times n)$-матриць наступним чином:

$$\mathbf{D}_b=$$

$$\left\{D^{(l)}(x)=\operatorname{diag}(d_{1,m_1}^{(l)}(x),d_{2,m_2}^{(l)}(x),\ldots,d_{n,m_n}^{(l)}(x)),\ \text{де}\ \deg\prod_{i=1}^{k}d_{i,m_i}^{(l)}(x)\leq kr\ \text{і}\ \prod_{i=1}^{n}d_{i,m_i}^{(l)}(x)=b(x)\right\}.$$

3). Для кожної матриці $D^{(l)}(x)\in\mathbf{D}_b$ для $A(x)$ будуємо факторизації $A(x)=T_b^{(l)}(x)G(x)$, де $T_b(x)=\begin{bmatrix} d_{1,m_1}^{(l)}(x) & 0 & \dots & \dots & 0\\ t_{m_2,1}^{(l)}(x) & d_{2,m_2}^{(l)}(x) & 0 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ t_{m_n,1}^{(l)}(x) & t_{m_n,2}^{(l)}(x) & \dots & \dots & d_{n,m_n}^{(l)}(x)\end{bmatrix}$ – трикутна матриця така, що $\deg t_{m_i,j}^{(l)}(x)<\deg d_{i,m_i}^{(l)}(x)$ для всіх $j<i$.

Очевидно, що для деяких матриць $D^{(l)}(x)$ факторизацій матриці $A(x)=T_b^{(l)}(x)G(x)$ може і не існувати. Пошук елементів $t_{m_i,j}^{(l)}(x)$ базується на знаходженні розв'язків $\{u_{ij}(x),v_{ij}(x)\}$ рівняння $b_i(x)u_{ij}(x)+c_j(x)v_{ij}(x)=\widetilde{a}_{ij}(x)$ таких, що $\deg v_{ij}(x)<b_i(x)$. Якщо ж ці розв'язки існують, то остання нерівність гарантує їхню єдиність. Зауважимо, що коефіцієнтами многочленів $t_{m_i,j}^{(l)}(x)$ можуть бути параметри із поля $\mathbb{F}$. Множину таких трикутних матриць позначимо через

$$\mathbf{Tr}_b=\left\{T_b^{(l)}(x)=\left[t_{i,j}^{(l)}(x)\right],\ \text{де}\ \left[\begin{array}{lll} t_{i,j}^{(l)}(x)=0, & \text{якщо} & j>i;\\ t_{i,k}^{(l)}(x)=d_{i,m_k}^{(l)}(x), & \text{якщо} & k=m_i;\\ \deg t_{m_i,j}^{(l)}(x)<\deg d_{i,m_i}^{(l)}(x) & \text{для всіх} & j<m_i.\end{array}\right.\right\}.$$

Зрозуміло, якщо одна з наведених вище умов не виконується, то для $A(x)$ не існує лівих унітальних дільників із визначником $\det B(x)=b(x)$. Враховуючи теорему 2 із [1] та наведені вище міркування отримуємо.

Теорема 1. *Для трикутної неособливої матриці $A(x)$ існує факторизація $A(x) = B(x)C(x)$, де $B(x) \in \mathbb{F}_{n,n}[x]$ – унітальна многочленна матриця степеня $r \geq 1$ із визначником* $\det B(x) = b(x)$, *тоді і тільки тоді, коли в множині* $\mathbf{Tr}_b$ *існує матриця $T(x) = \sum_{i=0}^{s} T_i x^{s-i}$, для якої*

$$\operatorname{rank}\begin{bmatrix} T_0 & O & \cdots & O \\ T_1 & T_0 & & \vdots \\ T_2 & T_1 & \vdots & O \\ \vdots & \vdots & & T_0 \\ \vdots & \vdots & \vdots & \vdots \\ T_s & T_{s-1} & \vdots & T_{s-r} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} T_0 & O & \dots & \dots & O \\ T_1 & T_0 & & \vdots & \vdots \\ T_2 & T_1 & \vdots & O & O \\ \vdots & \vdots & & T_0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & O \\ T_s & T_{s-1} & \vdots & T_{s-r} & I_n \end{bmatrix}.$$

Приклад 2. Для матриці $A(x) = \begin{bmatrix} x^2+x & 0 \\ x^2+1 & x(x^2+x+1) \end{bmatrix} \in \mathbb{Q}_{2,2}[x]$ вкажемо дільники $I_2x + B_0$ із визначниками $x^2 + x$, x^2 і $x^2 + x + 1$ відповідно та дільники $I_nx^2 + B_1x + B_2$ із визначниками $x^2(x^2 + x + 1)$ і $(x^2 + x)(x^2 + x + 1)$ відповідно. Результати обчислень наведено у таблиці.

$b(x)$	x^2+x	x^2+x	x^2	x^2+x+1
$\operatorname{diag}(d_1(x), d_2(x))$	$\operatorname{diag}(x^2+x, 1)$	$\operatorname{diag}(x+1, x)$	$\operatorname{diag}(x, x)$	$\operatorname{diag}(1, x^2+x+1)$
$\mathbf{Tr}_b$	$\begin{bmatrix} x^2+x & 0 \\ ax+b & 1 \end{bmatrix}; a, b \in \mathbb{Q}$	–	$\begin{bmatrix} x & 0 \\ 1 & x \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ x & x^2+x+1 \end{bmatrix}$
$I_2x + B_0$	–	–	$\begin{bmatrix} x & 0 \\ 1 & x \end{bmatrix}$	$\begin{bmatrix} x+1 & 1 \\ -1 & x \end{bmatrix}$

$b(x)$	$x^2(x^2+x+1)$	$(x^2+x)(x^2+x+1)$	$(x^2+x)(x^2+x+1)$
$\operatorname{diag}(d_1(x), d_2(x))$	$\operatorname{diag}(x, x^3+x^2+x)$	$\operatorname{diag}(x^2+x, x^2+x+1)$	$\operatorname{diag}(x+1, x^3+x^2+x)$
$\mathbf{Tr}_b$	$\begin{bmatrix} x & 0 \\ 2x^2+x+1 & x^3+x^2+x \end{bmatrix}$	$\begin{bmatrix} x^2+x & 0 \\ x^2+1 & x^2+x+1 \end{bmatrix}$	-
$I_2x^2 + B_1x + B_2$	$\begin{bmatrix} x^2+0,5x & 0,5x \\ 0,5(1-x) & x^2+0,5(x+1) \end{bmatrix}$	$\begin{bmatrix} x^2+x & 0 \\ -x & x^2+x+1 \end{bmatrix}$	-

Література

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НТБ ОНАХТ

Про геодезичні відображення посторів дотичних розшарувань зі спеціальною метрикою

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Дослідження у межах інваріантної теорії наближень геометричних об'єктів ріманова простору V^n, $n \in N$, дозволяють побудувати на дотичному розшаруванні $T(V^n)$ різні метрики та різні об'єкти афінного зв'язку [1]. Кожна з таких метрик породжує на $T(V^n)$ певну геометрію, схожу на фінслерову, але відмінну від неї [2].

У роботі розглянуто простір $T(V^n)$ з метрикою

$$ds^2 = 3g_{\alpha\beta}(x)dx^\alpha dx^\beta - \widetilde{g}_{\alpha\beta}(x;y)dx^\alpha \tilde{D}y^\beta, \tag{1}$$

де $g_{\alpha\beta}(x)$ — компоненти метричного тензору базового ріманова простору V^n,

$$\widetilde{g}_{\alpha\beta}(x;y) = g_{\alpha\beta}(x) + \frac{1}{3}R_{i\alpha\beta j}(x)y^i y^j;$$

$$\widetilde{D}y^\alpha = dy^\alpha + \widetilde{\Gamma}^\alpha_{\beta\gamma}(x;y)y^\beta dx^\gamma;$$

$$\widetilde{\Gamma}^\alpha_{\beta\gamma}(x;y) = \Gamma^\alpha_{\beta\gamma}(x) - \frac{1}{3}R^\alpha_{(\beta\gamma)\sigma}(x)y^\sigma,$$

де $\Gamma^\alpha_{\beta\gamma}(x)$, $R^\alpha_{\beta\gamma\sigma}(x)$, $R_{i\alpha\beta j}(x)$ — відповідно, компоненти афінного зв'язку, тензора Рімана і тензора кривини базового ріманова простору V^n.

Компоненти $g_{ij}(x;y)$ метричного тензору метрики (1) підраховані у явному вигліді. Спираючись на них, за формулами, аналогічними до стандартних формул ріманової геометрії, побудовані символи Кристофеля другого роду, отримані рівняння, що визначають криві, які називаються геодезичними лініями простору $T(V^n)$.

Далі природним чином введено поняття геодезичного відображення простору $T(V^n)$, у локальному аспекті проаналізовано проблему існування таких відобажень, знайдено критерій існування таких відображень у випадку, коли базовий простір V^n є простором Ейнштейна.

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НТБ ОНАХТ

Поточкова оцінка відхилення полінома Крякіна від неперервної на відрізку функції

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Нехай C - простір неперервних функцій на відрізку $I := [0,1]$ зі стандартною нормою

$$\|f\| := \max_{x\in I} |f(x)|$$

Нехай $k \in N$, визначимо

$$\triangle_h^k f(x) := \sum_{j=0}^{k} (-1)^{k-j} C_n^k f(x+ih)$$

$k-$ий модуль неперервності функції f в точці $1/k$ визначається наступним чином:

$$\omega_k(f, 1/k) := \sup_{x,x+kh\in I} |\triangle_h^k f(x)|$$

Розглянемо многочлени, які інтегрально наближають функцію f на I, тобто

$$\int_0^{i/k} (f(t) - Q_{k-1}(f,t))dt = 0,\ i = 0, 1...k$$

де $degQ_{k-1}(f,t) \le (k-1)$

З [1] відомо, що

$$||f - Q_{k-1}|| \le \widetilde{W}(k)\omega_k(f, 1/k)$$

де $\widetilde{W}(k) = 2$ при $k \le 82000$ і $\widetilde{W}(k) = 2 + exp(-2)$ при $k > 82000$.

Однак така оцінка "досягається"лише на кінцях відрізка I (якщо точно, то на відрізках $[0; 1/k]$ і $[(k-1)/k, 1]$. Тому постає запитання, чи можна цю оцінку покращити всередині відрізка, тобто отримати, що для $x \in [1/k; (k-1)/k]$ виконується нерівність

$$|f(x) - Q_{k-1}(x)| \le p(x)\omega_k(f, 1/k),$$

де $p(x)$ - функція, яка залежить від x (можливо, є константою), але значення якої менші за 2, оцінки, яка вже є відомою.

Основним результатом є наступна теорема:

Теорема 1.

$$|g(x)| \le \frac{4mlnk}{C_k^m}$$

де $x \in [m/k, (m+1)/k], m < k/2,$ *а* $g := f - Q.$

Оцінка на відрізку симетрична ($Q(f(1-x), t) = Q(f, 1-t)$), тому для тих x, у яких $m > k/2$, отримаємо аналогічну формулу, помінявши в ній m на $k - m$.

Цей вираз уже при $m \ge 1$ буде малим за рахунок того, що $C_k^m \ge k$ при $m \ge 1$ і є набагато меншим за 2 (відому рівномірну оцінку). Крім того, чим більше m, тим ця оцінка краща за попередню отриману оцінку.

Отже, таким чином на відрізку $[1/k; (k-1)/k]$ покращено раніше отриману оцінку для многочлена інтегрального наближення. Отримана в роботі оцінка приблизно дорівнює $O(\frac{m\ln m}{C_k^m})$ на відрізку $[m/k, (m+1)/k]$. Це ще раз підкреслює, що многочлен інтегрального наближення "найгірше"поводить себе близько до кінців відрізка, а всередині наближує його набагато краще. Оскільки це не є досить природним, то дає підстави для подальшого дослідження неперервної на відрізку функції та поліномів, які їх наближають.

Крім того, за умови, що максимум і мінімум на відрізках $[0, 1/k]$ і $[(k-1)/k, 1]$, то за рахунок зміни полінома на сталу або лінійну функцію, можна покращити оцінку найкращого наближення на всьому відрізку. Це покращення буде тим більше, чим більша буде різниця абсолютних величин максимума і мінімума на кінцях відрізка.

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НТБ ОНАХТ

Про Р-деформації поверхонь обертання

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Раніше у роботах [1], [2] вивчалися нескінчено малі геодезичні деформації (Р-деформації) поверхонь в евклідовому просторі E^3. Для таких деформацій знайдено нову форму основних рівнянь, яку представлено через тензорні поля $\overset{\circ}{T}^{\alpha\beta}$, T^α та функцію ψ похідної вектора зміщення $\overline{U}_i = c_{i\alpha}\left(\overset{\circ}{T}^{\alpha\beta} - \frac{3}{2}\psi c^{\alpha\beta} + c_1 c^{\alpha\beta}\right)\overline{r}_\beta + c_{i\alpha}T^\alpha\overline{n}$, виписано ознаки афінних деформацій. У результаті дослідження основних рівнянь отримано наступні результати.

Теорема 1. *Для того, щоб нескінченно мала деформація поверхні S (ненульової повної кривини $K \neq 0$) класу C^3 була геодезичною, необхідно і достатньо, щоб на поверхні існували функції ψ, φ, які задовольняють наступні рівняння:*

$$\frac{K_i}{3K^2}\left(3\nabla_h\psi_m + \lambda g_{mh}\right) - \frac{1}{3K}\left(3\nabla_{hi}\psi_m + \lambda_i g_{mh}\right) = \psi_h g_{im} - \psi_i g_{hm} + \psi_m g_{hi},$$
$$-\frac{\psi_\alpha c^{\alpha\beta}}{K^2}(K_\gamma b^\gamma_\beta - 2K_\beta H) + \nabla_\beta(\varphi_\alpha d^{\alpha\beta}) + 2H\varphi = 0,$$

де $\lambda = -\frac{3}{2}\nabla_\beta\psi_\alpha g^{\alpha\beta}$, $\lambda_i = \partial_i\lambda$, $\nabla_{hi} = \nabla_i\nabla_h$.

Тоді тензорні поля $\overset{\circ}{T}^{\alpha\beta}$, T^α, що представляють похідну вектора зміщення, мають вигляд

$$\overset{\circ}{T}^{\alpha\beta} = \frac{1}{K}\left(\frac{\varphi K}{2}g^{\alpha\beta} - \frac{1}{4}\nabla_h\psi_k g^{h\beta}c^{k\alpha} - \frac{1}{4}\nabla_h\psi_k g^{h\alpha}c^{k\beta}\right), \tag{1}$$

$$T^\alpha = \frac{1}{2}\left(-\psi_h c^{hk} d^\alpha_k + \varphi_k d^{k\alpha}\right). \tag{2}$$

Тут $K_i = \partial_i K$, H - середня кривина поверхні, $d^{ij} = \frac{1}{K}c^{i\alpha}c^{j\beta}b_{\alpha\beta}$, $d^{i\alpha}b_{j\alpha} = \delta^i_j$.

Теорема 2. *Поверхні обертання $\overline{r} = (ucosv, usinv, f(u))(K \neq const)$ допускають нетривіальні Р-деформації при $\varphi = 0$. При цьому*

$$\overset{\circ}{T}^{11} = \overset{\circ}{T}^{22} = 0, \overset{\circ}{T}^{12} = \frac{Cu}{4\sqrt{1+f'^2}},$$
$$T^1 = 0, T^2 = -\frac{Cu}{2f'}, \psi = C\frac{u^2}{2} + C_2, C, C_2 - const.$$

Теорема 3. *Для того, щоб поверхня S класу C^3 сталої повної кривини ($K = const \neq 0$) допускала Р-деформацію, необхідно і достатньо, щоб існували функції ψ, φ, що задовольняють рівняння*

$$\nabla_{hi}\psi_m = -K(2\psi_i g_{mh} + \psi_h g_{im} + \psi_m g_{hi}),$$
$$\nabla_\beta(\varphi_\alpha d^{\alpha\beta}) + 2H\varphi = 0.$$

Тоді тензорні поля $\overset{\circ}{T}^{\alpha\beta}$, T^s похідної вектора зміщення $\overline{U}_i$ мають вигляд (1), (2).

Серед поверхонь $K = const \neq 0$ вибрано сферу та розглянуто випадок $\varphi = 0$.

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НТБ ОНАХТ

QA-деформація зі стаціонарним ортом нормалі еліптичного параболоїда

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Нехай векторно-параметричне рівняння еліптичного параболоїда задано у вигляді

$$\overline{r}(u,v) = \{u\cos v, u\sin v, \frac{u^2}{2}\}.$$

В роботі досліджується його квазіареальна нескінченно мала деформація (або коротко QA-деформація) вигляду

$$\overline{r}^*(u,v,t) = \overline{r}(u,v) + t\overline{U}(u,v),$$

де $\overline{U}(u,v)$ – поле зміщення, $t \longrightarrow 0$, при якій залишається стаціонарним орт нормалі поверхні.

Задача про QA-деформацію зі стаціонарним ортом нормалі поверхні від'ємної гауссової кривина K розглянута в статті [1]. В даній роботі досліджується така деформація поверхні додатної гауссової кривини, зазначимо, що поверхня еліптичного параболоїда задовольняє цій умові.

Представимо поле зміщення через його компоненти

$$\overline{U}(u,v) = U^\alpha \overline{r}_\alpha + U^\circ \overline{n}.$$

Розглядувана задача звелась до відшукування розв'язків неоднорідного диференціального рівняння з частинними похідними другого порядку відносно невідомої функції U° :

$$U^\circ_{\alpha,\beta} d^{\alpha\beta} - \frac{K_\alpha}{K} d^{\alpha\beta} U^\circ_\beta + 2HU^\circ = 2\mu, \;\; K \neq 0.$$

Це рівняння узагальнює відоме однорідне характеристичне рівняння Вейнгартена для нескінченно малих згинань [2]. Закон змінювання елемента площі поверхні при її нескінченно малій деформації виражається через функцію μ [1].

Має місце теорема.

Теорема 1. *Поверхня еліптичного параболоїда допускає QA-деформацію зі стаціонарним ортом нормалі, при якій координати поля зміщення мають вигляд*

$$\overline{U}(u,v) = \{\frac{c((1+u^2)\sin v - u^2 v\cos v)}{u\sqrt{1+u^2}}, \; \frac{-c((1+u^2)\cos v + u^2 v\sin v)}{u\sqrt{1+u^2}}, \; \frac{cv}{\sqrt{1+u^2}}\},$$

де стала $c \neq 0$. При цьому функція $\mu = \frac{cv(2+u^2)}{2\sqrt{(1+u^2)^3}}$.

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Конформные и геодезические отображения на Риччи-симметрические пространства

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Конформные отображения римановых пространств рассматривались во многих работах. Эти отображения имеют существенное приложение в общей теории относительности.

Вопрос о том, допускает или не допускает риманово пространство конформное отображение на некоторое пространство Эйнштейна был сведен Г. Бринкманом [1] к проблеме существования решения некоторой нелинейной системы дифференциальных уравнений типа Коши относительно неизвестных функций. Эта задача детально изложена в монографии А. З. Петрова [2].

В работах [3, 4] основные уравнения указанных отображений сведены к линейной системе дифференциальных уравнений в ковариантных производных.

Теория геодезических отображений идейно восходит к работе Т. Леви-Чивита [5], в которой он поставил и решил в специальной системе координат задачу о нахождении собственно римановых пространств с общими геодезическими. Примечательно, что она была связана с изучением уравнений динамики механических систем. Затем теория геодезических отображений развивалась в работах Томаса, Вейля, Широкова, Солодовникова, Синюкова, Микеша и других.

Самыми известными уравнениями геодезических отображений являются уравнения Леви-Чивита. Затем Г. Вейль получил эти уравнения и для геодезических отображений аффинносвязных пространств.

Н. С. Синюков [6] доказал, что основные уравнения геодезических отображений (псевдо)-римановых пространств эквивалентны некоторой линейной системе уравнений типа Коши в ковариантных производных.

В работе [7] эти результаты обобщены на случай геодезических отображений эквиаффинных пространств на (псевдо)-римановы пространства.

Аффинносвязное или риманово пространство называют Риччи-симметрическим, если тензор Риччи в нем абсолютно параллелен. Таким образом, Риччи-симметрические пространства $\overline{A}_n(\overline{V}_n)$ характеризуются условием

$$\overline{R}_{ij|k} = 0,$$

где символ $''|''$ обозначает ковариантную производную в $\overline{A}_n(\overline{V}_n)$, $\overline{R}_{ij}$ – тензор Риччи пространства $\overline{A}_n(\overline{V}_n)$.

В работе [8] рассмотрены конформные отображения римановых пространств V_n на Риччи-симметрические римановы пространства $\overline{V}_n$. Основные уравнения таких отображений получены в виде замкнутой системы уравнений типа Коши в ковариантных производных. Установлено количество существенных параметров, от которых зависит общее решение полученной системы уравнений типа Коши в ковариантных производных.

В работе [9] изучены геодезические отображения пространств аффинной связности A_n на Риччи-симметрические пространства $\overline{A}_n$. Основные уравнения указанных отображений получены в виде замкнутой системы уравнений типа Коши в ковариантных производных. Установлено количество существенных параметров, от которых зависит общее решение полученной системы уравнений типа Коши в ковариантных производных.

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НТБ ОНАХТ

Компьютерное моделирование упрочняющего фазового перехода в дисперсно-армированных материалах

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Существенная особенность дисперсно-армированных гетерогенных материалов – наличие в их структуре кластеров фибры, которые при критической концентрации образуют связную область перколяционного типа [1], [2], провоцируя структурный фазовый переход, и, следовательно, скачкообразно изменяя свойства образца.

В предложенной модели упрочняющего структурного фазового перехода реализована возможность исследования в компьютерных экспериментах результатов армирования материала смесью фибры и порошка. Для этого сформулирована новая континуальная перколяционная задача, описывающая кластерную систему с квазиточечными и квазилинейными элементами.

Задача решается методом Монте-Карло в кубе размером 10^6 условных единиц длины. Элементы, из которых формируется модельный кластер, создаются генератором случайных чисел (ГСЧ) с равномерным распределением. В каждом эксперименте фибра имеет фиксированную длину, и ее положение определяет ГСЧ: он задаёт координату ее начала и выбирает угол поворота относительно ребер куба. Единичные фибры считаются соединёнными, если у них либо есть общая точка, либо расстояние между ними не превышает некоторое заданное, играющее в модели, как и длина фибры, роль управляющего параметра.

Этот феномен обнаруживает интересную особенность перколяционной задачи с необычным составом элементов, а значит, и самой технологии – невозможность обеспечить статистическую устойчивость явления. Потеря стабильности при фиксированном значении максимального угла поворота фибры и ее длины связана с зависимостью интервалов ряда значений параметров задачи, в частности, порога протекания и фрактальной размерности от степени вариативности параметров конкретных реализаций перколяционного кластера, которая, как оказалось, излишне велика. Обсуждение ситуации возможно в рамках представления о гиперслучайных величинах, для которых «статистические оценки в общем случае не являются состоятельными, т.е. при увеличении объема выборки их погрешность не стремится к нулю» [3]. При этом каждый из результатов, полученный при фиксированных значениях управляющих параметров, имеет стандартную для таких задач погрешность, равную, примерно, $12 \div 15$ процентов.

Идея одновременного использования фибры и порошка для упрочнения материала принадлежит авторам работы [4], которые успешно использовали ее при создании бетона, обладающего повышенной прочностью при растяжении.

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НТБ ОНАХТ

Проективная классификация рациональных функций

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Рассматриваем группу Галуа $Aut(\mathbb{C}(z)/\mathbb{C})$, которая изоморфна группе Ли $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$ и преобразования Мёбиуса имеют вид

$$f(z) \mapsto f\left(\frac{az+b}{cz+d}\right),$$

где $a, b, c, d \in \mathbb{C}$ и $ad - bc = 1$.

Представление алгебры Ли $\mathfrak{sl}_2(\mathbb{C})$ в векторных полях на $\mathbb{C}\mathbf{P}^1$ имеет вид: $\mathfrak{sl}_2(\mathbb{C}) = \langle \partial_z, z\partial_z, z^2\partial_z \rangle$.

Пусть $X \in \mathfrak{sl}_2(\mathbb{C})$ - векторное поле из алгебры. Обозначим через $X^{(k)}$ продолжение этого векторного поля на многообразии k-джетов $\mathbf{J}^k$ функций. Тогда соответствие $X \mapsto X^{(k)}$ даёт представление алгебры Ли $\mathfrak{sl}_2(\mathbb{C})$ в векторных полях на $\mathbf{J}^k$.

Например, взяв $k = 3$, мы получаем следующую реализацию алгебры Ли $\mathfrak{sl}_2(\mathbb{C})$:

$$\partial_z, z\partial_z - u_1\partial_{u_1} - 2u_2\partial_{u_2} - 3u_3\partial_{u_3},$$

$$z^2\partial_z - 2zu_1\partial_{u_1} - (4zu_2 + 2u_1)\partial_{u_2} - (6zu_3 + 6u_2)\partial_{u_3},$$

в стандартных координатах.

Мы говорим, что рациональная функция F на многообразии $\mathbf{J}^k$ является проективным дифференциальным инвариантом порядка $\leq k$ ([1], [2], [3]), если $X^{(k)}(F) = 0$, для всех векторных полей $X \in \mathfrak{sl}_2(\mathbb{C})$.

Теорема 1. (1) *Существует два независимых проективных дифференциальных инварианта порядка* ≤ 3

$$J_0 = u, J_3 = u_1^{-3}u_3 - \frac{3}{2}u_1^{-4}u_2^2,$$

и все остальные инварианты порядка ≤ 3 *являются рациональными функциями* J_0 *и* J_3.

(2) *Эти инварианты разделяют регулярные орбиты, т.е.* $\mathfrak{sl}_2(\mathbb{C})$ *- орбиты точек, где* $u_1 \neq 0$.

Отметим, что

(1) Значение инварианта J_3 на рациональной функции $f(z)$ равно производной Шварца функции, обратной к $f(z)$.

(2) Регулярные $\mathfrak{sl}_2(\mathbb{C})$ - орбиты в $\mathbf{J}^3$ имеют размерность 3. Есть также особые орбиты размерности 3, они $\mathfrak{sl}_2(\mathbb{C})$ - орбиты в области $u_1 = 0, u_2 \neq 0$.

(3) Сингулярные орбиты размерности 2 являются $\mathfrak{sl}_2(\mathbb{C})$ - орбитами в области $u_1 = u_2 = 0$, $u_3 \neq 0$.

(4) Сингулярные орбиты размерности 1 являются $\mathfrak{sl}_2(\mathbb{C})$ - орбитами в области $u_1 = u_2 = u_3 = 0$.

(5) Сингулярные орбиты размерностей 3, 2 и 1 соответственно задаются уравнениями:

$$u = c, \quad u_1 = 0, \quad u_2 \neq 0,$$

$$u = c, \quad u_1 = 0, \quad u_2 = 0, \quad u_3 \neq 0,$$

$$u = c, \quad u_1 = 0, \quad u_2 = 0, \quad u_3 = 0,$$

где c константа.

Теорема 2. (1) *Поле проективных дифференциальных инвариантов порождается инвариантами* J_0, J_3 *и производной Трессе* $\nabla = \frac{1}{u_1}\frac{d}{dz}$, *т.е. любой проективный дифференциальный инвариант является рациональной функцией инвариантов* J_0, J_3 *и их инвариантных производных.*

(2) *Это поле разделяет регулярные* $\mathfrak{sl}_2(\mathbb{C})$ *- орбиты в пространствах джетов, где регулярность орбиты в* $\mathbf{J}^k$, $k > 3$, *означает, что ее проекция в* $\mathbf{J}^3$ *регулярная орбита.*

Чтобы описать $\mathfrak{sl}_2(\mathbb{C})$ - орбиты в $\mathbb{C}(z)$ отметим, что значения $J_0(f)$ и $J_3(f)$ базисных дифференциальных инвариантов J_0, J_3 на рациональной функции $f(z)$ также рациональные функции.

Степень трансцендентности поля $\mathbb{C}(z)$ равна 1, и поэтому идеал полиномиальных соотношений

$$P\left(J_0(f), J_3(f)\right) = 0 \tag{1}$$

между ними порождается неприводимым полиномом.

Коэффициенты последнего полинома зависят от f, поэтому мы будем обозначать соответствующий неприводимый многочлен $P_f(X,Y)$. Тогда соотношение (1) можно рассматривать как утверждение, что f является решением обыкновенного дифференциального уравнения 3-го порядка:

$$\varepsilon_f = \left\{ P_f\left(u, u_1^{-3}u_3 - \frac{3}{2}u_1^{-4}u_2^2\right) = 0 \right\} \subset J^3.$$

Проективная группа $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$ является группой симметрий дифференциального уравнения $\mathcal{E}_f$ и, следовательно, действует на его пространстве решений.

Теорема 3. *Стабилизатор рациональной функции $h(z)$ для $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$ - действия дискретен, если $h \neq const$.*

Мы говорим, что рациональная функция $h(z)$ является *регулярной*, если $h \neq const$.

Например, стабилизатор функции Жуковского $f(z) = \frac{z^2+1}{2z}$ является группой $\mathbb{Z}_2$, порождённой инверсией $z \mapsto z^{-1}$.

Теорема 4. *Две регулярные рациональные функции f и g являются $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$ - эквивалентными тогда и только тогда, когда $P_f = \lambda P_g$.*

Отметим, что для трансцендентных расширений поля $\mathbb{C}$ дифференциальные инварианты, а также соответствующие дифференциальные уравнения играют роль неприводимых многочленов для конечных расширений Галуа.

Так например для функции Жуковского $f(z) = \frac{z^2+1}{2z}$

$$P_f(X,Y) = \left(X^2-1\right)^2 Y + \frac{3}{8},$$

а соответствующее обыкновенное дифференциальное уравнение имеет вид

$$\left(u^2-1\right)^2\left(u_1u_3 - \frac{3}{2}u_2^2\right) + \frac{3}{8}u_1^4 = 0.$$

Все регулярные решения этого уравнения являются рациональными функциями, которые $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$-эквивалентны функции Жуковского.

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Инфинитезимальные преобразования в симметрическом римановом пространстве 1-го класса V_n

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П. А. Широковым ([1]) было изучено симметрическое риманово пространство 1-го класса V_n. Для $n = 4$ метрический тензор такого пространства V_n имеет вид:

$$g_{ij}(x) = \underset{\circ}{g}_{ij} + \frac{1}{3}\underset{\circ}{R}_{i\alpha\beta j} x^\alpha x^\beta$$

$$R_{i\alpha\beta j} = b_{\alpha j} b_{i\beta} - b_{ij} b_{\alpha\beta}$$

$$\left(\underset{\circ}{g}_{ij}\right) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \ (b_{ij}) = \begin{pmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \ \epsilon_i = \pm 1 \tag{1}$$

Из (10) следует, что пространство 2-го приближения $\widetilde{V}_n^2$ ([2], [3]) для симметричного V_n 1-го класса изометрично исходному V_n. Поэтому группа Ли $\widetilde{G}_r$ инфинитезимальных преобразований в $\widetilde{V}_n^2$ изоморфна группе Ли G_r исходного V_n. Используя это обстоятельство, доказаны следующие утверждения.

Теорема 1. *Симметрическое риманово пространство V_n 1-го класса допускает группу Ли движений G_8.*

Найден базис этой группы и её структурные константы.

Теорема 2. *Инфинитезимальные конформные преобразования 2-ой степени в симметрическом римановом пространстве V_n первого класса являются гомотетическими преобразованиями.*

Найдены $\xi^h_{p|}$, $(p = 10)$ - линейно независимые с постоянными коэффициентами гомотетические векторы Киллинга.

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О некоторых диффеоморфизмах псевдоримановых пространств со структурой Яно-Хоу-Чена

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Структурой Яно-Хоу-Чена мы называем аффинорную структуру, структурный аффинор F которой удовлетворяет уравнению 4-й степени $F^4 \pm F^2 = 0$. К понятию такой квадриструктуры К.Яно,С.Хоу и В.Чен пришли при изучении подмногообразий в почти контактных многообразиях [2].

Структура Яно-Хоу-Чена является естественным обобщением *e-структуры* [3], которая определяется наличием на многообразии X_n тензорного поля типа (1,1) F_i^h, удовлетворяющего условиям

$$F_\alpha^h F_i^\alpha = e\delta_i^h, \quad e = \pm 1, 0, \quad i, h, \alpha, \beta, \ldots = 1, 2, \ldots, n,$$

а также *f-структуры* [2], для которой

$$F_\alpha^h F_\beta^\alpha F_i^\beta + F_i^h = 0.$$

Мы рассматриваем псевдориманово пространство (V_n, g_{ij}), на котором определена структура Яно-Хоу-Чена, согласованная с метрикой в виде

$$F_{ij} + F_{ji} = 0, \quad F_{ij} = g_{i\alpha} F_j^\alpha,$$

и ковариантно постоянная, то есть

$$F_{i,j}^h = 0.$$

Здесь «,» - знак ковариантной производной в V_n.

Изучались различные диффеоморфизмы таких пространств. В частности, доказано, что такие V_n не допускают нетривиальных геодезических [3], F-планарных и $2F$-планарных отображений [1] с сохранением аффинорной структуры на $(\overline{V}_n, \overline{g}_{ij})$, в котором

$$\overline{F}_{ij} + \overline{F}_{ji} = 0, \quad \overline{F}_{ij} = \overline{g}_{i\alpha} F_j^\alpha.$$

Этот факт представляет собой обобщенный аналог известной в теории геодезических отображений келеровых пространств теоремы Яно-Вестлейка.

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НТБОНАХТ

Закономерности теории квази-геодезических отображений рекуррентно-параболических пространств

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Рассмотрим римановы пространства (V_n, g_{ij}) и $(\overline{V}_n, \overline{g}_{ij})$, находящиеся в квази-геодезическом отображении, основные уравнения которого в общей по отображению системе координат (x^i) имеют вид [1]:

$$\overline{\Gamma}^h_{ij}(x) = \Gamma^h_{ij}(x) + \psi_{(i}(x)\delta^h_{j)} + \varphi_{(i}(x)F^h_{j)}(x)$$

$$\overline{F}_{(ij)}(x) = 0, \qquad \overline{F}_{ij}(x) = F^\alpha_j(x)\overline{g}_{\alpha i}(x),$$

где $\overline{\Gamma}^h_{ij}, \Gamma^h_{ij}$ - компоненты объектов связности пространств $\overline{V}_n$ и V_n, соответственно; ψ_i, φ_i - ковекторы; F^h_i - аффинор.

В [2] было введено понятие *рекуррентно-параболической* структуры $F^h_i(x)$ на (V_n, g_{ij}) , для которой

$$F^\alpha_i F^h_\alpha = 0, \quad F_{ij} + F_{ji} = 0, \quad F_{ij} = F^\alpha_j g_{\alpha i},$$

$$F^h_{i,j} = \rho_j(x)F^h_i(x), \quad i, h, j, \alpha, \beta, \ldots = 1, 2, \ldots, n,$$

где ρ_j - ковектор, «,» - знак ковариантной производной в V_n. Само V_n при этом также названо *рекуррентно-параболическим.*

Мы построили по аналогии с тем, как это сделано в теории геодезических отображений римановых пространств [3], инвариантное преобразование, которое из пары рекуррентно-параболических пространств, состоящих в квази-геодезическом соответствии, позволяет получить новую пару также рекуррентно-параболических квази-геодезически соответствующих пространств. Более того, применение этого инвариантного преобразования многократно дает возможность построить бесконечную последовательность пар рекуррентно-параболических пространств, находящихся в квази-геодезическом отображении.

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Инвариантеые решения двумерного уравнения теплопроводности

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Методы группового анализа широко используются для исследования уравнений в частных производных и для интегрирования обыкновенных дифференциальных уравнений В работах [1], [2],[3], [4],[5],[6]. рассматривается вопросы интегрирование обыкновенных дифференциальных уравнений и линейных дифференциальных уравнений в частных производных, на основе известных инфинитезимальных симметрий. В работе [3] разработан вычислительный метод, явно определяющий полную группу симметрий произвольного дифференциального уравнения в частных производных. В работе [4] рассматриваются вопросы групповой классификации дифференциальных уравнений и их решений. В работе [2] найдена алгебра Ли инфинитезимальных образующих группы симметрий для двумерного и трехмерного уравнения теплопроводности. Алгебра Ли инфинитезимальных образующих группы симметрий для одномерного уравнения теплопроводности использована в работе [6].

Рассмотрим двумерное уравнение теплопроводности

$$u_t = \sum_{i=1}^{2} \frac{\partial}{\partial x_i}(k_i(u)\frac{\partial u}{\partial x_i}) + Q(u) \tag{1}$$

где $u = u(x_1, x_2, t) \geq 0$ — температурная функция, $k_i(u) \geq 0, Q(u)$ — функции от температуры u. Функция $Q(u)$ описывает процесс тепловыделения, если $Q(u) > 0$ и процесс теплопоглощения, если $Q(u) < 0$.

Исследования показывают, коэффициенты теплопроводности $k_1(u), k_2(u)$ в достаточно широком диапазоне изменения параметров может быть описан степенной функцией температуры, т. е. имеет вид $k(u) = u^\sigma$.

Рассмотрим случай $k_1(u) = k_2(u) = u^\sigma$, $Q(u) = u$. В этом случае уравнение (1.1) имеет следующий вид:

$$u_t = u^\sigma \Delta u + \sigma u^{\sigma-1}(\nabla u)^2 + u \tag{2}$$

где $\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2}$ — оператор Лапласа,$\nabla u = \{\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}\}$ —градиент функции u.

Как показано в работе [2] следующие векторные поля являются инфинитезимальными образующими группы симметрий для уравнения (1.2):

$$X_1 = \sigma x_1 \frac{\partial}{\partial x_1} + \sigma x_2 \frac{\partial}{\partial x_2} + 2u\frac{\partial}{\partial u}, X_2 = \exp(-\sigma t)\frac{\partial}{\partial t} + \exp(-\sigma t)u\frac{\partial}{\partial u}. \tag{3}$$

Потоки векторных полей X_1, X_2 порождают следующие группы преобразований соответственно

$$(t, x_1, x_2, u) \to (t, x_1 e^s, x_2 e^s, u e^{2s}), s \in \mathbb{R} \tag{4}$$

$$(t, x_1, x_2, u) \to (\frac{1}{\sigma}\ln(e^{\sigma t} + \sigma s), x_1, x_2, u(e^{\sigma t} + \sigma s)^{\frac{1}{\sigma}}, s \in \mathbb{R} \tag{5}$$

Мы найдем решения уравнения (1.2), инвариантные относительно групп преобразований (1.4), (1.5). Для этого сначала находим инвариантые функции этих преобразований.

Известно, что [3, с. 117] гладкая функция $f : M \to R$ является инвариантной функцией группы преобразований G, действующей на многообразии M тогда и только тогда, когда $Xf = 0$ для каждой инфинитезимальной образующей X группы G.

Используя этот критерий мы находим, что функции $I_1 = \frac{(x_1+x_2)\exp(\sigma t/2)}{\sqrt{u^\sigma}}$, $I_2 = \frac{x_1}{x_2}$ являются инвариантными фунциями группы преобразований (1.4),(1.5), что вытекает из следущих равенств $X_1(I_1) = 0$, $X_1(I_2) = 0$, $X_2(I_1) = 0$, $X_2(I_2) = 0$.

Теорема 1. *Решения уравнения (2), инвариантные относительно групп преобразований (4),(5) имеют вид*

$$u(t,x_1,x_2)=\frac{\sigma}{2}e^t\frac{(x_1+x_2)^{2/\sigma}}{2}V(\xi) \quad (6)$$

где $V(\xi)-$ общее решение дифференциальное уравнение второго порядка:

$$f(\xi)VV''+f(\xi)V'^2+4\sigma(\xi+1)[\frac{\sigma}{2}(\xi^2+\xi)-2\xi+2]VV'+4[2+2(\frac{2}{\sigma}-1)]V^2=0, \quad (7)$$

где $f(\xi)=(\xi+1)^2(\xi^2+1)$, $g(\xi)=\sigma(\xi+1)[\frac{\sigma}{2}(\xi^2+\xi)-2\xi+2]$.

Теперь рассмотрим случай, когда есть поглощение тепла: $k_1(u)=k_2(u)=u^\sigma$, $Q(u)=-u$. В этом случае уравнение (1.1) имеет следующий вид:

$$u_t=u^\sigma\Delta u+\sigma u^{\sigma-1}(\nabla u)^2-u \quad (8)$$

Как показано в работе [2] следующие векторные поля являются инфинитезимальными образующими группы симметрий для уравнения (8):

$$X_1=\sigma x_1\frac{\partial}{\partial x_1}+\sigma x_2\frac{\partial}{\partial x_2}+2u\frac{\partial}{\partial u}, X_2=\exp(\sigma t)\frac{\partial}{\partial t}+\exp(\sigma t)u\frac{\partial}{\partial u}. \quad (9)$$

Используя вышеприведенный критерий мы находим, что функции $I_1=\frac{(x_1+x_2)\exp(-\sigma t/2)}{\sqrt{u^\sigma}}$, $I_2=\frac{x_1}{x_2}$ являются инвариантными фунциями группы преобразований (1.5),(1.6), что вытекает из следущих равенств $X_1(I_1)=0$, $X_1(I_2)=0$, $X_2(I_1)=0$, $X_2(I_2)=0$.

Теорема 2. *Решения уравнения (2), инвариантные относительно групп преобразований (4),(5) имеют вид*

$$u(t,x_1,x_2)=\frac{\sigma}{2}e^{-t}\frac{(x_1+x_2)^{2/\sigma}}{2}V(\xi) \quad (10)$$

где $V(\xi)$ общее решение дифференциальное уравнение второго порядка (7).

Выводы. В уравнении (2) есть источник тепловыделения, поэтому в каждой точке области переменных (x_1,x_2), отличных от точек $(0,0)$, температурная функция (6) возрастает экспоненциально при возрастающем t. В уравнении (1.12) есть источник поглощения, в каждой точке области переменных (x_1,x_2), отличных от точек $(0,0)$, температурная функция (10) убывает экспоненциально при возрастающем t.

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Новый вид условий нежесткости многогранников

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Известно, что нежесткость симплициального многогранника рода 0 с n вершинами определяется тем фактом, что ранг матрицы линейной системы

$$AX = 0 \qquad (1)$$

для компонент векторов поля бесконечно малого изгибания должен быть меньше $3n-6$. Элементы матрицы A выражаются через координаты соединенных ребрами вершин многогранника и поэтому при изменении системы координат или при движении многогранника как твердого тела ее элементы должны измениться. Но если мы умножим уравнение (1) слева на транспонированную матрицу A^T, получим уравнение вида $BX = 0$, котором элементы матрицы B уже будут зависеть только от квадратов длин ребер и диагоналей, и тем самым формально подтверждается, что, во-первых, жесткость/нежесткость многогранника не зависит от выбора системы координат, во-вторых, она зависит не только от длин ребер, но также и от длин диагоналей.

НТБ ОНАХТ

Заузленные сферы с постоянным отношением

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В полупространстве $E^3_+(0)$, определяемом условием $x_4 = 0$, $x_3 \geq 0$, возьмем дугу γ с концами на плоскости $\pi : x_4 = 0$, $c_3 = 0$. Полупространство $E^3_+(0)$ будем вращать вокруг плоскости π. Пространство $E^3_+(0)$, повернутое на угол φ, будем обозначать $E^3_+(\varphi)$. При вращении на 360° точки дуги γ, находящиеся в $E^3_+(\varphi)$, заметут множество S, гомеоморфное сфере S^2. Полученная поверхность заузлена (см [1]). Поэтому эта гладкая поверхность называется *заузленной сферой.* Радиус-вектор заузленной сферы можно записать в виде

$$\mathbf{X}(u,v) = \begin{pmatrix} x_1(u) \\ x_2(u) \\ x_3(u)\cos v - x_4(u)\sin v \\ x_3(u)\sin v + x_4(u)\cos v \end{pmatrix}, \tag{1}$$

где кривая $\mathbf{X}(u, 0)$ есть профильная кривая этой поверхности.

В настоящей работе рассматривается случай заузленной сферы у которой профильная кривая плоская, радиус-вектор которой имеет вид

$$\mathbf{X}(u,0) = (\rho(u)\cos u, \rho(u)\sin u, \rho(u)\cos u, \rho(u)\sin u). \tag{2}$$

Несложно подсчитать касательную $\mathbf{X}^T$ и нормальную $\mathbf{X}^\perp$ компоненты радиус-вектора заузленной сферы. Имеем

$$\|\mathbf{X}^T\| : \|\mathbf{X}^\perp\| = \frac{\rho'}{\rho}.$$

Если отношение длины касательной компоненты к длине нормальной компоненты постоянно на подмногобразии $F^n \subset E^{n+m}$, то говорят о подмногообразии *постоянного отношения.* Таким образом имеет место следующая теорема.

Теорема 1. *Пусть $F^2 \subset E^4$ есть заузленная сфера, заданная радиус-вектором (1). Тогда поверхность F^2 есть поверхность постоянного отношения если и только если профильная кривая является кривой постоянного отношения и $\rho(u) = c_1 e^{c_2 u}$, где c_1 и c_2 есть действительные постоянные.*

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НТБ ОНАХТ

О построении псевдоримановых пространств с f-структурой, находящихся в каноническом $2F$-планарном отображении II типа

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Рассмотрим римановы пространства (V_n, g_{ij}, F_i^h) и $(\overline{V}_n, \overline{g}_{ij}, \overline{F}_i^h)$, на которых определены аффинорные структуры. В [1] показано, что $2F$-планарное отображение ($2F$ПО) (V_n, g_{ij}, F_i^h) на $(\overline{V}_n, \overline{g}_{ij}, \overline{F}_i^h)$ по необходимости сохраняет структуру, то есть в общей по отображению системе координат (x^i)

$$F_i^h(x) = \overline{F}_i^h(x),$$

и основные уравнения $2F$ПО имеют вид

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \psi_{(i}\delta_{j)}^h + \phi_{(i}\overset{1}{F}{}_{j)}^h + \sigma_{(i}\overset{2}{F}{}_{j)}^h,$$

где $\Gamma_{ij}^h, \overline{\Gamma}_{ij}^h$ - компоненты объектов связности $V_n, \overline{V}_n$; $\psi_i(x), \phi_i(x), \sigma_i(x)$ - некоторые ковекторы, а круглыми скобками обозначена операция симметрирования. $2F$ПО считается тривиальным при $\psi_i = \phi_i = \sigma_i = 0$.

Здесь обозначено

$$\overset{1}{F}{}_i^h = F_i^h, \quad \overset{2}{F}{}_i^h = F_\alpha^h F_i^\alpha.$$

Мы показали, что нетривиальные $2F$ПО могут быть лишь одного из трех типов:

$$I \quad \psi_i = 0, \quad \phi_i \neq 0, \quad \sigma_i \neq 0;$$

$$II \quad \psi_i \neq 0, \quad \phi_i = 0, \quad \sigma_i \neq 0;$$

$$III \quad \psi_i \neq 0, \quad \phi_i \neq 0, \quad \sigma_i \neq 0.$$

При этом $2F$-планарное отображение названо *каноническим I(II) типа* (обозначается $2F$ПО(I)($2F$ПО(II)) в случае I(II)) и просто $2F$ПО в случае III.

Говорят, что F_i^h определяет *f-структуру* [2] на псевдоримановом пространстве (V_n, g_{ij}), если имеют место условия

$$F_\alpha^h F_\beta^\alpha F_i^\beta + F_i^h = 0, \quad i, h, \alpha, \beta, \ldots = 1, 2, \ldots, n,$$

$$Rg\|F_i^h\| = 2k \quad (2k < n).$$

Полагаем f-структуру согласованной с метрикой в виде

$$F_{ij} + F_{ji} = 0, \quad F_{ij} = g_{i\alpha}F_j^\alpha$$

В дальнейшем полагаем аффинор ковариантно постоянным:

$$\overset{1}{F}{}_{i,j}^h = 0,$$

где « ,» - знак ковариантной производной в V_n.

Мы рассмотрели $2F$ПО(II) псевдоримановых пространств с абсолютно параллельной f-структурой и построили преобразование, которое дает возможность из одной пары таких пространств, находящихся в $2F$ПО(II), получить новую пару псевдоримановых пространств с абсолютно параллельной f-структурой, принципиально отличающихся от исходной пары и при этом также находящихся в $2F$ПО(II).

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Зміст

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