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# Algebraic and Geometric Methods of Analysis

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## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

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ІНТЕРНАЦІОНАЛЬНИЙ ЦЕНТР СПІВРОБОТИ

## Spaces of probability measures and box dimension

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The topology of hyperspaces of sets of given dimension is investigated in numerous publications (see, e.g., [3, 5, 2, 1]). In particular, in [7] the author described the topology of the hyperspace of sets of given Hausdorff dimension.

There are different definitions of dimension for the probability measures (see, e.g., [4, 6, 6]). The present talk is devoted to the spaces of probability measures of given box dimension.

Let us recall some necessary definitions.

Let  $(X, d)$  be a complete metric space. For  $F \subset X$  and  $r > 0$  denote by  $\mathcal{N}_r(F)$  the least number of closed balls of radius  $r$  needed to cover the set  $F$ . The *lower* and *upper box dimensions* of a set  $F$  are defined as follows:

$$\underline{\dim}_{\text{box}}(F) = \underline{\lim}_{r \rightarrow 0} \frac{\log \mathcal{N}_r(F)}{\log(1/r)};$$

$$\overline{\dim}_{\text{box}}(F) = \overline{\lim}_{r \rightarrow 0} \frac{\log \mathcal{N}_r(F)}{\log(1/r)},$$

and the *box dimension* of a set  $F$  is defined as

$$\dim_{\text{box}}(F) = \lim_{r \rightarrow 0} \frac{\log \mathcal{N}_r(F)}{\log(1/r)}$$

whenever the last limit exists.

Note also that the lower (upper) box dimension of a set coincides with the lower (upper) box dimensions of its topological closure.

Additionally, let the quantity  $\lambda_0$  be given by the formula

$$\lambda_0 = \lambda_0(X) = \inf\{\overline{\dim}_{\text{box}}(B_r(x)) \mid x \in X, r > 0\},$$

where  $B_r(x)$  denotes the closed ball of radius  $r$  centered at  $x$ , is called the *smallest local upper box dimension* of  $X$ .

Now, let  $P(X)$  denote the space of probability measures on  $X$  and  $k > 0$ . We endow  $P(X)$  with the weak\* topology. The *lower* and *upper box dimensions* of a measure  $\mu \in P(X)$  are defined, respectively, by the formulae:

$$\underline{\dim}_{\text{box}}(\mu) = \liminf_{k \rightarrow 0} \{\underline{\dim}_{\text{box}}(F) \mid F \in \mathcal{B}(X), \mu(F) \geq 1 - k\};$$

$$\overline{\dim}_{\text{box}}(\mu) = \liminf_{k \rightarrow 0} \{\overline{\dim}_{\text{box}}(F) \mid F \in \mathcal{B}(X), \mu(F) \geq 1 - k\},$$

where  $\mathcal{B}(X)$  denotes the set of Borel subsets in  $X$ .

**Theorem 1.** *Let  $X$  be an infinite complete separable metric space, then the set*

$$\{\mu : \underline{\dim}_{\text{box}}(\mu) = 0\}$$

*is homeomorphic to the separable Hilbert space  $l^2$ .*

**Theorem 2.** *Let  $X$  be an infinite complete separable metric space, then the set*

$$\{\mu: \inf\{\overline{\dim}_{\text{box}}(F): F \in \mathcal{B}(X), \mu(F) > 0\} \geq \lambda_0\}$$

*is homeomorphic to the separable Hilbert space  $l^2$ .*

The proofs of these theorems are based on results by Myjak and Rudnicki [9]. They proved that the mentioned sets are residual in the space of probability measures. We also apply some characterization theorems from infinite-dimensional topology [8].

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