

International  
Scientific Conference



Algebraic  
and Geometric  
Methods  
of Analysis

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Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

#### LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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# Vector bundle construction via monads on multiprojective spaces

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In this paper we construct indecomposable vector bundles associated to monads on multiprojective spaces. Specifically we establish the existence of monads on  $\mathbf{P}^{2n+1} \times \mathbf{P}^{2n+1} \times \dots \times \mathbf{P}^{2n+1}$  and on  $\mathbf{P}^{a_1} \times \dots \times \mathbf{P}^{a_n}$ . We prove stability of the kernel bundle which is a dual of a generalized Schwarzenberger bundle associated to the monads on  $X = \mathbf{P}^{2n+1} \times \mathbf{P}^{2n+1} \times \dots \times \mathbf{P}^{2n+1}$  and prove that the cohomology vector bundle which is simple, a generalization of special instanton bundles. We also prove stability of the kernel bundle and that the cohomology vector bundle associated to the monad on  $\mathbf{P}^{a_1} \times \dots \times \mathbf{P}^{a_n}$  is simple. Lastly, we construct the morphisms that establish the existence of monads on  $\mathbf{P}^1 \times \dots \times \mathbf{P}^1$ .

## REFERENCES

- [1] Ancona V and Ottaviani G: *Stability of special instanton Bundles on  $\mathbf{P}^{2n+1}$* . Transactions of the American Mathematical Society 341 (1994) 677 - 693. doi: [10.2307/2154578](https://doi.org/10.2307/2154578).
- [2] Bohnhorst G and Spindler H (1992). The stability of certain vector bundles on  $\mathbf{P}^n$ . Lecture Notes in Mathematics. Vol 1507, Springer, Berlin, Heidelberg. doi: [10.1007/BFb0094509](https://doi.org/10.1007/BFb0094509)
- [3] Costa L and Miró-Roig R M. Monads and instanton bundles on smooth hyperquadrics. Mathematische Nachrichten, 282 (2009), no 2, 169-179. doi: [10.1002/mana.2000610730](https://doi.org/10.1002/mana.2000610730).
- [4] Dolgachev I and Kapranov M. Arrangements of hyperplanes and vector bundles on  $\mathbf{P}^n$ . Duke Math. J. 71 (1993), 633-664. doi: [10.1215/S0012-7094-93-07125-6](https://doi.org/10.1215/S0012-7094-93-07125-6)
- [5] Fløystad G. Monads on a Projective Space. Communications in Algebra, 28 (2000), 5503 - 5516. doi: [10.1080/00927870008827171](https://doi.org/10.1080/00927870008827171).
- [6] Hartshorne R. Varieties of small codimension in projective space. Bull. Amer. Math. Soc. 80 (1974), 1017-1032. doi [10.1090/S0002-9904-1974-13612-8](https://doi.org/10.1090/S0002-9904-1974-13612-8)
- [7] Hartshorne R. Algebraic vector bundles on projective spaces: A problem list, Topology, 1979, vol 18, 117-128 doi.org/[10.1016/0040-9383\(79\)90030-2](https://doi.org/10.1016/0040-9383(79)90030-2)
- [8] Hoppe H. Generischer Spaltungstyp und zweite Chernklasse stabiler Vektorraumbündel. vom Rang 4 auf  $\mathbf{P}^4$ , Math. Z. 187 (1984), 345-360. eudml.org/doc/[173496](https://eudml.org/doc/173496).
- [9] Horrocks G. Examples of rank three vector bundles on five-dimensional projective space, Journal of London Mathematical Society. (2), 18 (1978), 15-27. doi:[10.1112/jlms/s2-18.1.15](https://doi.org/10.1112/jlms/s2-18.1.15).
- [10] Horrocks G. Vector bundles on the punctured spectrum of a local ring. Proc. London Math. Soc. 14, 689-713 (1964). doi.org/[10.1112/plms/s3-14.4.689](https://doi.org/10.1112/plms/s3-14.4.689).
- [11] Horrocks G and Mumford D. A rank 2 vector bundle on  $\mathbf{P}^4$  with 15,000 symmetries, Topology, 12 (1973), 63-81. doi:[10.1016/0040-9383\(73\)90022-0](https://doi.org/10.1016/0040-9383(73)90022-0).

- [12] Jardim M and Earp HNSá. Monad construction of asymptotically stable Bundles. arXiv (September,2011). [arxiv.org/pdf/1109.2750v1.pdf](https://arxiv.org/pdf/1109.2750v1.pdf)
- [13] Jardim M, Menet M, Prata D and Earp HNSá. Holomorphic bundles for higher dimensional gauge theory. Bulletin London Mathematical society, 49 (2017). doi: [10.1112/blms.12017](https://doi.org/10.1112/blms.12017).
- [14] Kumar, N. Construction of rank two vector bundles on  $\mathbf{P}^4$  in positive characteristic. Invent math 130, 277–286 (1997). doi [10.1007/s002220050185](https://doi.org/10.1007/s002220050185)
- [15] Kumar N, Peterson C and Rao A P. Construction of low rank vector bundles on  $\mathbf{P}^4$  and  $\mathbf{P}^5$ , Journal of Algebraic Geometry 11 (2) (2002), 203–217. doi [10.1090/S1056-3911-01-00309-5](https://doi.org/10.1090/S1056-3911-01-00309-5)
- [16] Maingi D. Vector Bundles of low rank on a multiprojective space. Le Matematiche. Vol. LXIX (2014) - Fasc. II. pp 31-41. doi: [10.4418/2014.69.2.4](https://doi.org/10.4418/2014.69.2.4).
- [17] Maingi D (2021). Indecomposable Vector Bundles associated to Monads on Cartesian products of projective spaces. Turkish Journal of Mathematics. Vol. 45: No. 5. Article 17. Pages 2126-2139. doi: [10.3906/mat-2101-6](https://doi.org/10.3906/mat-2101-6)
- [18] Maingi D. Monads on multiprojective Products of Projective Spaces. to appear in Manuscripta Mathematica (2023),doi: [10.1007/s00229-022-01449-0](https://doi.org/10.1007/s00229-022-01449-0).
- [19] Marchesi S, Marques P M and Soares H. Monads on a Projective Varieties. Pacific Journal of Mathematics, vol 296 (2018), no. 1, 155-180. doi: [10.2140/pjm.2018.296.155](https://doi.org/10.2140/pjm.2018.296.155).
- [20] Okonek C, Schneider M and Spindler H. Vector Bundles on Complex Projective Spaces. Springer, 1980, doi.org/10.1007/978-1-4757-1460-9
- [21] Perrin D. Géométrie algébrique. Une introduction, (1995), EDP Sciences/CNRS édition. ISBN-2-7296-0563-0
- [22] Spindler H and Trautmann G. Special Instanton bundles on  $\mathbf{P}^{2n+1}$  their geometry and their moduli. Mathematische Annalen 286. 1-3 (1990): 559-592. doi: [10.1215/S0012-7094-93-07125-6](https://doi.org/10.1215/S0012-7094-93-07125-6)
- [23] Tango, H. An example of indecomposable vector bundle of rank  $n - 1$  on  $\mathbf{P}^n$ ,  $n \geq 3$ . Journal of Mathematics of Kyoto University, 16, (1976): 137-141, doi: [10.1215/kjm/1250522965](https://doi.org/10.1215/kjm/1250522965).
- [24] Tango H. On morphisms from projective space  $\mathbf{P}^n$  to the Grassmann variety  $\mathbb{G}r(n, d)$ , Journal of Mathematics of Kyoto University, 16 (1976), 201-207. doi: [10.1215/kjm/1250522969](https://doi.org/10.1215/kjm/1250522969).

## About one problem of the Gauss-Kuzmin type

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Let  $[a_0; a_1, \dots, a_n, \dots]$  — continued fraction, where  $a_0 \in \mathbb{Z}_+$ ,  $a_j > 0$  for all  $j \in \mathbb{N}$ . Consider left shift operator

$$T([0; a_1, a_2, \dots, a_n, \dots]) = [0; a_2, a_3, \dots, a_{n+1}, \dots].$$

Let  $f_n(x) = \lambda(T^{-n}([0; x]))$ , where  $x \in (0; 1]$ ,  $\lambda(\cdot)$  — Lebesgue measure. The problem of finding

$$f(x) = \lim_{n \rightarrow +\infty} f_n(x)$$

for classical continued fractions was posed by Gauss. Kuzmin [1] showed that  $f(x) = \log_2(x + 1)$  and

$$|f_n(x) - \log_2(1 + x)| \leq C\beta^{(n^\eta)} \quad \forall x \in (0; 1]$$

for  $\eta = 0,5$  some  $C > 0$  and  $\beta \in (0; 1)$ . Levy [2] showed that it is possible to take  $\beta = 0,7$  and  $\eta = 1$ . Wirsing [4] showed that, for the constant  $\gamma \approx 0,3037$

$$\psi(x) = \lim_{n \rightarrow +\infty} \frac{f_n(x) - \log_2(1 + x)}{(-\gamma)^n} \quad \forall x \in (0; 1],$$

where  $\psi(x)$  — analytic function.

It is known [3] that for each  $t \in [0, 5; 1]$  there exists a sequence  $(b_n)$  such that  $b_n \in \{0, 5; 1\}$  for all  $n \in \mathbb{N}$  and  $t = [0; b_1, \dots, b_n, \dots]$ . The last image is called  $A_2$ -image. A countable set of numbers  $t \in [0, 5; 1]$  has two  $A_2$ -images.

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