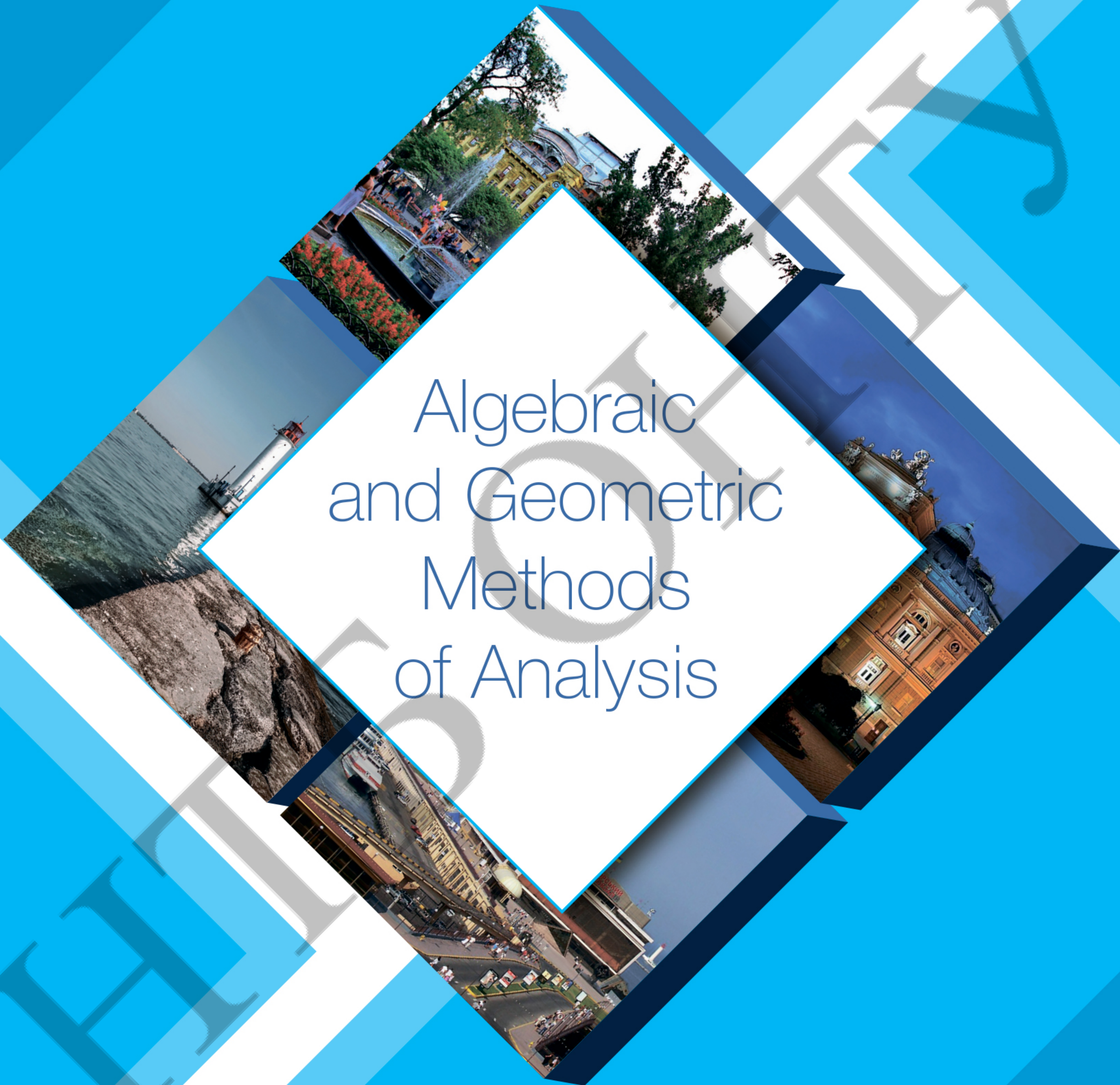
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Theorem 4. *Let (M, g, η, ξ, ϕ) be a K-contact metric manifold, $dim M = 2n + 1$, and let G be a Riemannian g-natural metric on T_1M with $c = 1 - 2a$ and $d = a$. Then the Reeb vector field ξ defining the isometric embedding $\xi : (M, g) \to (T_1M, G)$ is totally geodesic if and only if M is Sasakian manifold.*

Thus totally geodesic property of the Reeb vector fields as isometric embeddings is distinguished Sasakian manifold among K-contact metric manifold with the Riemannian g-natural metrics on the unit tangent bundle.

References

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Classification of smooth structures on line with two origins

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We classified differentiable structures on a line **L** with two origins begin a non-Hausdorff but T_1 one-dimensional manifold obtained by "doubling" 0.

Definition 1. Let τ be the standard topology on $\mathbb{R}$. Then $\mathbf{L} = \mathbb{R} \sqcup \bar{0}$ is a disjoint union of $\mathbb{R}$ with some point $\bar{0}$ endowed with the follwoing topology:

$$\eta = \tau \cup \{(W \setminus 0) \cup \{\bar{0}\} : 0 \in W \in \tau\}$$

whose elements are elements of τ and also open neighborhoods of 0 in which 0 is replaced with $\bar{0}$.

For $k \in \mathbb{N} \cup \{\infty\}$ let H^k be the group of homeomorphisms h of $\mathbb{R}$ such that $h(0) = 0$ and the restriction of h $\mathbb{R} \setminus 0$ is a $\mathcal{C}^k$-diffeomorphism. It contains a subgroup D^k consisting of $\mathcal{C}^k$-diffeomorphisms of $\mathbb{R}$ also fixing 0.

Definition 2. Let H be a group and C, D be two subgroups. Then for each $h \in H$ the following subset of H:

$$ChD = \{chd^{-1} : c \in C, d \in D\}$$

is called the (C, D)-coset of h. If $C = D$, then DhD is also called the D-*double coset* of h. The set

$$Dh^{\pm 1}D := DhD \cup Dh^{-1}D = \{chd^{-1}, ch^{-1}d^{-1} : c, d \in D\}$$

is called the $(D, \pm)$-*double coset of* h.

We are refering to the book by J. Lee [1] and paper of F. Takens [2] for the definition of $\mathcal{C}^k$-structures. So the problem of classification of smooth $\mathcal{C}^k$-structures on $\mathbf{L}$ can be stated as follows:

Problem 3. Describe the orbits of the action of the group $\mathcal{H}(\mathbf{L})$ on the set of $\mathcal{C}^k$-structures on M.

It is shown that there is a natural bijection between $\mathcal{C}^k$-structures on $\mathbf{L}$ up to a $\mathcal{C}^k$-diffeomorphism and double coset classes $D^k \setminus H^k / D^k$ which can be regarded as the orbit space of the action $D^k \times D^k$ on H^k by the rule $(a, b)h = ahb^{-1}$.

Theorem 4. *Let* $k \in \mathbb{N} \cup \{\infty\}$. *Then*

- $\mathcal{C}^k$*-structures on* $\mathbf{L}$ *up to a* $\mathcal{C}^k$ *=diffeomorphism are in one-to-one correspondence with the set* $\mathcal{D}(\mathbb{R}, 0) \setminus \mathcal{H}^k_0(\mathbb{R})^{\pm 1}/\mathcal{D}(\mathbb{R}, 0)$ *of* $(\mathcal{D}(\mathbb{R}, 0), \pm)$*-double coset classes;*
- *while* $\mathcal{C}^k$*-structures on* $\mathbf{L}$ *up to a* $\mathcal{C}^k$*-diffeomorphism fixing* 0 *and* $\bar{0}$ *are in one-to-one correspondence with the set* $\mathcal{D}(\mathbb{R}, 0) \setminus \mathcal{H}^k_0(\mathbb{R})/\mathcal{D}(\mathbb{R}, 0)$ *of* $\mathcal{D}(\mathbb{R}, 0)$*-double coset classes.*

References

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