International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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From minimality to maximality via metric reflection

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In 1934 Đuro Kurepa [4] introduced the pseudometric spaces which, unlike metric spaces, allow the zero distance between different points.

Definition 1. Let X be a set and let $d\colon X^2 \to \mathbb{R}$ be a non-negative, symmetric function such that $d(x,x) = 0$ for every $x \in X$. The function d is a *pseudometric* on X if it satisfies the triangle inequality.

If d is a pseudometric on X, we say that (X,d) is a *pseudometric space*.

Definition 2 ([3])**.** Let (X,d) and (Y,ρ) be pseudometric spaces. The spaces (X,d) and (Y,ρ) are *combinatorially similar* if there exist bijections $\Psi\colon Y \to X$ and $f\colon d(X^2) \to \rho(Y^2)$ such that $\rho(x,y) = f\big(d(\Psi(x),\Psi(y))\big)$ for all $x, y \in Y$. In this case, we will say that $\Psi\colon Y \to X$ is a *combinatorial similarity* and that (X,d) and (Y,ρ) are combinatorially similar pseudometric spaces.

Definition 3. Let (X,d) be a pseudometric space. A bijection $f : X \to X$ is a *pseudoidentity* if the equality $d(x, f(x)) = 0$ holds for every $x \in X$.

The groups of all combinatorial self-similarities and all pseudoidentities of a pseudometric space (X,d) will be denoted by $\mathbf{Cs}(X,d)$ and $\mathbf{PI}(X,d)$ respectively. Thus, for every pseudometric space (X,d) we have $\mathbf{PI}(X,d) \subseteq \mathbf{Cs}(X,d) \subseteq \mathbf{Sym}(X)$, where $\mathbf{Sym}(X)$ is the symmetric group of all permutations of the set X.

For every nonempty pseudometric space (X,d), we define a binary relation $\overset{0(d)}{=}$ on X by

$$(x \overset{0(d)}{=} y) \ \Leftrightarrow\ (d(x,y) = 0), \quad \text{for all } x, y \in X.$$

Proposition 4. *Let X be a nonempty set and let $d\colon X^2 \to \mathbb{R}$ be a pseudometric on X. Then $\overset{0(d)}{=}$ is an equivalence relation on X and, in addition, the function δ_d,*

$$\delta_d(\alpha,\beta) := d(x,y), \quad x \in \alpha \in X/\overset{0(d)}{=}, \quad y \in \beta \in X/\overset{0(d)}{=},$$

is a correctly defined metric on the quotient set $X/\overset{0(d)}{=}$.

In what follows we will say that the metric space $(X/\overset{0(d)}{=}, \delta_d)$ is the *metric reflection* of (X,d).

Let us define a class $\mathcal{IP}$ of pseudometric spaces as follows.

Definition 5. A pseudometric space (X,d) belongs $\mathcal{IP}$ if the equalities

$$\mathbf{Cs}(X,d) = \mathbf{PI}(X,d) \quad \text{and} \quad \mathbf{Cs}(X/\overset{0(d)}{=}, \delta_d) = \mathbf{Sym}(X/\overset{0(d)}{=}) \quad \text{hold.}$$

References

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Thurston norm and Euler classes of bounded mean curvature foliations on hyperbolic 3-Manifolds

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Let M be a closed, oriented 3-manifold, and suppose that M contains no non-separating 2 - spheres or tori. For example, M is a closed oriented hyperbolic 3-Manifold.

The Thurston norm on $H_2(M, \mathbb{Z})$ is defined as follows ([1]):

$$||a||_{Th} = inf\{\chi_-(\Sigma)|\ \Sigma \text{ is an embedded oriented surface representing } a \in H_2(M, \mathbb{Z})\}, \quad (1)$$

where $\chi_-(\Sigma) = max\{-\chi(\Sigma), 0\}$. Recall that $\chi(\Sigma) = 2-2g$ denotes the Euler characteristic of a surface Σ of genus g. When Σ is not connected, define $\chi_-(\Sigma)$ to be the sum $\chi_-(\Sigma_1) + \cdots + \chi_-(\Sigma_k)$, where $\Sigma_i,\ i = 1, \ldots, k$ are the connected components of Σ. As Thurston showed, the Thurston norm can be extended in a unique way to the norm in $H_2(M, \mathbb{R})$.

The dual Thurston norm can be defined on $H^2(M, \mathbb{R})$ by the formula

$$||\alpha||^*_{Th} = \sup_{\Sigma} \frac{< \alpha, [\Sigma] >}{2g(\Sigma) - 2}, \quad (2)$$

where $\alpha \in H^2(M, \mathbb{R})$ and the supremum being taken over all connected, oriented surfaces Σ embedded in M whose genus g is at least 2.

Recall that a *taut* foliation is a codimension one foliation of a closed manifold with the property that every leaf meets a transverse circle. Equivalently, by a result of Dennis Sullivan [2], a codimension one foliation is taut if there exists a Riemannian metric that makes each leaf a minimal surface. Thurston proved that the convex hull of the Euler classes of taut foliations on M is the unit ball for the dual Thurston norm. In particular, the Thurston norm $||e(\mathcal{F})||^*_{Th}$ of the Euler class $e(\mathcal{F}) \in H^2(M, \mathbb{R})$ of a taut foliation $\mathcal{F}$ is no more then one.

We represent the following result.

Theorem 1. *Let M be a closed oriented hyperbolic 3-Manifold and $\mathcal{F}$ be a two-dimensional transversely oriented foliation $\mathcal{F}$ whose leaves have the modulus of mean curvature bounded above by the fixed positive constant H_0. Then*

– *If $H_0 \leq 1$, we have $\mathcal{F}$ is taut and $||e(\mathcal{F})||^*_{Th} = 1$.*

– *If $H_0 > 1$, we have*

$$||e(\mathcal{F})||^*_{Th} \leq 2\pi \frac{1600 H_0^2 Vol(M)^2}{C_0^3 inj(M)} + \frac{300 Vol(M)}{inj(M)} + 1,$$

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