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**«Algebraic and geometric methods**  
**of analysis»**

**Book of abstracts**



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## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
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НТБ ОНАФТ

## Some applications of the discriminant and resonance sets of a real polynomial

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Let  $f_n(x)$  be a monic polynomial of degree  $n$  with real coefficients  $f_n(x) \equiv x^n + a_1x^{n-1} + \dots + a_n$ . The  $n$ -dimensional space  $\Pi \equiv \mathbb{R}^n$  of its coefficients  $\mathbf{P} = (a_1, \dots, a_n)$  is called the *coefficient space* of  $f_n(x)$ . A pair of roots  $t_i, t_j, i, j = 1, \dots, n, i \neq j$ , of  $f_n(x)$  is called  $p : q$ -commensurable if  $t_i : t_j = p : q$ .

**Definition 1.** Resonance set  $\mathcal{R}_{p:q}(f_n)$ ,  $p \in \mathbb{Z} \setminus \{0\}$ ,  $q \in \mathbb{N}$  of  $f_n(x)$  is called the set of all points of the coefficient space  $\Pi$  at which  $f_n(x)$  has at least a pair of  $p : q$ -commensurable roots, i.e.

$$\mathcal{R}_{p:q}(f_n) = \{\mathbf{P} \in \Pi : \exists i, j = 1, \dots, n, t_i : t_j = p : q\}. \quad (1)$$

The special case of  $\mathcal{R}_{p:q}(f_n)$  at the  $p = q = 1$  is so called *discriminant set*  $\mathcal{D}(f_n)$ , playing an important role in solution of many problems.

The polynomial  $f_n(x)$  has a pair of  $p : q$ -commensurable roots iff the pair of polynomials  $f_n(px)$  and  $f_n(qx)$  has at least one common root, or in terms of resultant  $\text{Res}_x(f_n(px), f_n(qx)) = 0$ . In the case when  $p = q$  both polynomials  $f_n(px)$  and  $f_n(qx)$  have exactly  $n$  common roots. In case  $a_n = 0$  one of the root is equal to zero, therefore resultant can be written in the form  $\text{Res}_x(f_n(px), f_n(qx)) = a_n(p - q)^n \text{GD}_{p:q}(f_n)$ , where  $\text{GD}_{p:q}(f_n)$  is so called *generalized discriminant* of the polynomial  $f_n(x)$  introduced in [1].

**Definition 2.** The chain  $\text{Ch}_{p:q}^{(k)}(t_i)$  of  $p : q$ -commensurable roots of length  $k$  (shortly *chain of roots*) is called the finite part of geometric progression with common ratio  $p/q$  and scale factor  $t_i$ , each member of which is a root of polynomial  $f_n(x)$ . The value  $t_i$  is called the *generating root*.

The detail structure of the resonance set (1) can be described with the help of so called  *$i$ -th generalized subdiscriminants*  $\text{GD}_{p:q}^{(i)}(f_n)$ , which are nontrivial factors of  $i$ -th subresultants of pair of polynomials  $f_n(px)$  and  $f_n(qx)$ . Such subresultants can be computed as  $i$ -th inners of Sylvester matrix constructed from the coefficients of mentioned above polynomials. For more details see [2].

**Theorem 3.** Polynomial  $f_n(x)$  has exactly  $n - d$  different chains of roots  $\text{Ch}_{p:q}^{(i)}(t_j)$ ,  $j = 1, \dots, n - d$  iff in the sequence  $\{\text{GD}_{p:q}^{(i)}(f_n), i = 0, \dots, n - 1\}$  of  $i$ -th generalized subdiscriminants of  $f_n(x)$  the first nonzero subdiscriminant is  $d$ -th generalized subdiscriminant  $\text{GD}_{p:q}^{(d)}(f_n)$ .

Consider a partition  $\lambda = [1^{n_1} 2^{n_2} \dots i^{n_i} \dots]$  of  $n \in \mathbb{N}$ . Partition functions  $p(n)$  and  $p_l(n)$  return the number of all partitions and the number of all partitions of the length  $l$  of  $n \in \mathbb{N}$  respectively. The value  $i$  in the partition  $\lambda$  defines the length of chain  $\text{Ch}_{p:q}^{(i)}(t_i)$  for a corresponding generating root  $t_i$ , the value  $n_i$  defines the number of different generating roots, which give the chains of root of the length  $i$ . Any partition  $\lambda$  of number  $n$  defines a certain structure of  $p : q$ -commensurable roots of this polynomial and it corresponds to some algebraic variety  $\mathcal{V}_l^i$ ,  $i = 1, \dots, p_l(n)$  of dimension  $l$  in the coefficient space  $\Pi$ . The number of such varieties of dimension  $l$  is equal to  $p_l(n)$  and total number of all varieties consisting the resonance set  $\mathcal{R}_{p:q}(f_n)$  is equal to  $p(n) - 1$ .

Parametrization of variety  $\mathcal{V}_1$  can be expressed in  $q$ -binomial (Gaussian) coefficients:

$$a_i = (-1)^i \begin{bmatrix} n \\ i \end{bmatrix}_q p^{\frac{1}{2}i(i-1)} q^{\frac{1}{2}i(2n-i(i+1))} t_1^i, \quad i = 1, \dots, n, \quad \text{where} \quad \begin{bmatrix} n \\ k \end{bmatrix}_q = \prod_{i=1}^k \frac{q^{n-i+1} - 1}{q^i - 1}.$$

Computation of parametric representation of any variety  $\mathcal{V}_l$ ,  $2 \leq l \leq n - 1$ , from the resonance set  $\mathcal{R}_{p:q}(f_n)$  is based on the following

**Theorem 4** ([3]). Let  $\mathcal{V}_l$ ,  $\dim \mathcal{V}_l = l < n - 1$ , be a variety on which polynomial  $f_n(x)$  has  $l$  different chains of  $p : q$ -commensurable roots and the chain generated by the root  $t_1$  has length  $m > 1$ . Let denote

by  $\mathbf{r}_l(t_1, t_2, \dots, t_l)$  parametrization of variety  $\mathcal{V}_l$ . Therefore the following formula

$$\mathbf{r}_{l+1}(t_1, \dots, t_l, t_{l+1}) = \mathbf{r}_l(t_1, \dots, t_l) + \frac{p(t_{l+1} - p^{m-1}t_1)}{t_1(p^m - q^m)} [\mathbf{r}_l(t_1, \dots, t_l) - \mathbf{r}_l((q/p)t_1, \dots, t_l)] \quad (2)$$

gives parametrization of the part of variety  $\mathcal{V}_{l+1}$ , on which there exists  $\text{Ch}_{p:q}^{(m-1)}(t_1)$ , simple root  $t_{l+1}$  and other chains of roots are the same as on the initial variety  $\mathcal{V}_l$ .

From the geometrical point of view Theorem 4 means that a part of variety  $\mathcal{V}_{l+1}$  is formed as a ruled  $l + 1$ -dimensional surface by the secant lines, which cross its directrix  $\mathcal{V}_l$  at two points defined by such values of parameters  $t_1^1$  and  $t_1^2$  that  $t_1^1 : t_1^2 = q : p$ . At  $p/q \rightarrow 1$  mentioned above ruled surface becomes a tangent ruled surface which parametrization is  $\mathbf{r}_{l+1} = \mathbf{r}_l + m^{-1}(t_{l+1} - t_1)\partial\mathbf{r}_l/\partial t_1$ . If  $f_n(x)$  has on the variety  $\mathcal{V}_{l+1}$  a pair of complex-conjugate roots it is necessary to make continuation of obtained parametrization (2). Finally, it is possible to pass from variety with two chains of roots of length  $k$  to a variety with a chain of roots of length  $2k$ . Thus, combining the mentioned above procedure one can state the following

**Theorem 5** ([3]). *Resonance set  $\mathcal{R}_{p:q}(f_n)$  of a real polynomial  $f_n(x)$  for a certain value of commensurability coefficient  $p : q$  allows polynomial parametrization of each variety  $\mathcal{V}_l \subset \mathcal{R}_{p:q}(f_n)$ ,  $l = 1, \dots, n - 1$ .*

The software library for computation of the resonance set  $\mathcal{R}_{p:q}(f_n)$  was implemented for CAS Maple. The above results were effectively used in solving the following problems.

- (1) The resonance set  $\mathcal{R}_{p:q}(f_3)$  of a cubic was completely described and an outline of investigation of formal stability of a stationary point of a multiparameter Hamiltonian system with three degrees of freedom was proposed [4].
- (2) The discriminant set of a real cubic polynomial was used in computation of global parametrization of one real variety  $\Omega$  that plays an important role in the investigation of the normalized Ricci flow on generalized Wallach spaces related to invariant Einstein metrics [5].
- (3) Parametric representation of the discriminant set  $\mathcal{D}(f_4)$  of a quartic allows to find the set of stability of the linearized multiparameter Hamiltonian system with 4 degrees of freedom [6].

This talk is devoted to the description of the resonance  $\mathcal{R}_{p:q}(f_4)$  and discriminant  $\mathcal{D}(f_4)$  sets of a real quartic  $f_4(x)$  and their application to the problem (3) in nonlinear case.

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