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ФІТБ ОНАФТ

Spaces of primitive elements in dual modules over Steenrod algebra 2

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I present a way to generate all primitive elements $PB(n)$ in $B(n) = (A(n-1)/A(n))^*$ modules over A^* dual Steenrod algebra, where $A(n)$ are annihilator modules over Steenrod algebra A . This work began in [7]. For useful notions see also [1, 2, 3, 4] and references summarized in [8, 5, 6, 7]. The filtration described in [6] Theorem 1 property 2 and 3 yields $PB(n) = \bigcup_t PB(n)_t$ and $PB(n)_t = \bigoplus_s PB(n)_t^s$, where s is the number of τ operations and t is the biggest index of such operations. From Theorem 1 [7] property 5 and 1 it is known $\dim PB(n)_t^{s,deg} \leq 1$ and the following diagram is exact

$$0 \rightarrow PC(n)_{k-1} \xrightarrow{\iota_k} PC(n)_k \xrightarrow{\lambda_k} PC(n-1)_{k-1}$$

For given $\alpha \in PB(n-1)_{t-1}$ how to find a primitive $\alpha' \in PB(n)_t$ such that $\pi_t(\alpha') = \alpha$? Properties 2 and 6 state for even n that $PB(n)_{-1} = PB(n)_0 = \langle \xi_1^{n/2} \rangle$; and for odd n : $PB(n)_0 = \langle \xi_1^{\frac{n-1}{2}} \tau_0 \rangle$ and for $s \geq 1$ that $\alpha \tau_0$ is also a primitive. And we can generate new primitives taking products of primitives. Do all primitives in $PB(n)_k^1 \setminus PB(n)_0^1$ also have form $\alpha \tau_0$? So $\alpha \tau_1 \in B(n)$ yields coproduct $\phi^*(\alpha \tau_1) = \phi^*(\alpha)(\xi_1 \otimes \tau_0 + 1 \otimes \tau_1)$ and hence $\alpha \tau_1$ is primitive if and only if $\alpha = \alpha' \tau_0 \in PB(n-1)$. If $\alpha \in PB(n)_{-1}$ then $\alpha \tau_1 = \xi_1^{\frac{n}{2}} \tau_1$ is not primitive. But for example product of not primitive $\alpha = \xi_1^{\frac{n-1}{2}} + \tau_0 \xi_2^{\frac{n-1}{2}} \in B(n)_0^1$ with primitive τ_0 is primitive. The primitivity condition in $B(n)$ leads to the following inductive definition of transformations R_i generating primitives, preserving primitivity.

Definition 1.

$$R_k(\alpha) = \xi^{\frac{p^{k-1}-1}{p-1}} \tau_k \alpha - \sum_{i=1}^{k-1} \xi^{\frac{p^{k-1}-p^i}{p-1}} \xi_{k+1-i}^{p^{i-1}} R_i(\alpha)$$

for $k > 1$ and $R_0(\alpha) = \alpha \tau_0$, $R_1(\alpha) = \alpha \tau_1$

These maps have the following properties.

- Theorem 2.**
- (1) $\forall i, k \in N, \forall \alpha \in B: R_i(\alpha \tau_k) = -R_i(\alpha) \tau_k$
 - (2) $\forall i, k \in N, \forall \alpha \in B: R_i(\alpha \xi_k) = R_i(\alpha) \xi_k$
 - (3) $\forall i, j \in N, \forall \alpha \in B: R_i R_j(\alpha) = -R_j R_i(\alpha)$
 - (4) $\forall \alpha \in PB(n) \cap \text{Im} R_0: R_i(\alpha) \in PB(n+1+2\frac{p^{i-1}-1}{p-1})$

Remark 3. From the definition 1: $R_i(\alpha) = \alpha R_i(1)$. Therefore by induction $R_{i_1} R_{i_2} \dots R_{i_k}(\alpha) = \alpha R_{i_1} R_{i_2} \dots R_{i_k}(1)$. And for example $R_2(1), R_3(1)$ e.t.c. are primitives in $B(n)^1$.

Therefore all primitives have form $\alpha \tau_0$ except $PB(n)_k^1 \setminus PB(n)_0^1$. Induction arguments based on Theorem 1 [7] lead to the general form of primitive elements.

Definition 4. $\alpha_{i_1, i_2, \dots, i_k} = \xi_1^l \tau_{i_1} \tau_{i_2} \dots \tau_{i_k} + \beta$ is a primitive in $PB(n)_{i_k}^k$ associated with (i_1, i_2, \dots, i_t) , $i_k > i_{k-1} > \dots > i_1 = 0$ if it has projection on $J(n)^{k,deg} = B(n)^{k,deg} / (I \cap B(n)^{k,deg})$ equal $a \xi_1^l \tau_{i_1} \tau_{i_2} \dots \tau_{i_k}$, $l = \frac{n-k}{2}$, $a \in Z/p$.

Corollary 5. *There exists the primitive $\alpha_{i_1, i_2, \dots, i_k}$ associated with (i_1, i_2, \dots, i_k) , $i_k > i_{k-1} > \dots > i_1 = 0$ and it is satisfied $\alpha_{i_1, i_2, \dots, i_k} \xi_1^l = R_{i_1} R_{i_2} \dots R_{i_k}(1)$.*

Remark 6. Corollary 5 also presents a way to calculate all associated primitives.

The following theorem is a result of construction of all primitive elements in $B(n)$.

Theorem 7. All $PB(n)^{s,deg}$ in $PB(n) = \cup_k PB(n)_k$ where $PB(n)_k = \oplus PB(n)_k^s$ are zero or one dimensional spaces. $PB(n)^{s,deg}$ has dimension one if and only if there is a sequence (i_1, i_2, \dots, i_t) , $i_s > i_{s-1} > \dots > i_1 = 0$ with conditions

- (1) $n - s$ is even,
- (2) degree of $PB(n)^{s,deg}$ is $deg = (p - 1)(n - s) + \sum_{j=1}^s \dim(\tau_{i_j})$,
- (3) $\frac{n-s}{2} \geq l$, where l is calculated below:
- (4) $l = \sum_{j=2}^s \frac{p^{i_j-1}-1}{p-1} - \sum_{j=2}^{s-1} \frac{p^{i_j-1}}{p-1}$,

When $\frac{n-s}{2} = l$

$$PB(n)^{s,deg} = \langle \alpha_{i_1, i_2, \dots, i_s} \rangle$$

When $\frac{n-s}{2} > l$

$$PB(n)^{s,deg} = \langle \xi_1^{\frac{n-s}{2}-l} \alpha_{i_1, i_2, \dots, i_s} \rangle$$

where $\alpha_{i_1, i_2, \dots, i_s} = \xi_1^{i_1} \tau_{i_1} \tau_{i_2} \dots \tau_{i_s} + \beta$ is the primitive in $B(n)_{i_s}^s$ associated with the sequence (i_1, i_2, \dots, i_t) , $i_s > i_{s-1} > \dots > i_1 = 0$ with conditions 1,2,4 mentioned above and $\frac{n-s}{2} = l$.

Knowledge of primitive elements on $B(n) = (A(n-1)/A(n))^*$ make a feasible to find all indecomposable elements of $(A(n-1)/A(n))$ [8, sec 4] .

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Mokritskaya T. P., Tushev A. V. <i>On some fractal-based estimations of subsidence volume for various types of soils</i>	39
Mukhamadiev F. G. <i>The Shanin number and the predshanin number of N_{τ}^{φ}-kernel of a topological spaces</i>	41
Najmiddinov J. Sh. <i>The effectiveness of the use of computer programs in the teaching of mathematics in academic lyceums</i>	42
Obikhod T. <i>Gromov-Witten invariants and identification of the energy levels of solitonic states</i>	43
Ostrovska O., Yakymiv R. <i>On isometries satisfying deformed commutation relations</i>	45
Prishlyak A., Prus A. <i>Three-color graph of the Morse flow on a compact surface with boundary</i>	46
Pulemotov A. <i>The Ricci Iteration on Homogeneous Spheres</i>	48
Rmuš V. <i>The construction of squaring the circle</i>	49
Samokhvalov S. <i>Riemann-Klein antagonism and problem of energy in general relativity</i>	51
Savchenko A. <i>On generalized spaces of persistence diagrams</i>	52
Sazonova O. <i>Continual approximate solution with acceleration and condensation mode</i>	53
Serdyuk A. S., Sokolenko I. V. <i>Approximation by Fourier sums and interpolation trigonometric polynomials in classes of differentiable functions with high exponents of smoothness</i>	54
Serdyuk A., Stepanyuk T. <i>Lebesgue-type inequalities for the Fourier sums</i>	57
Skuratovskii R. <i>Minimal generating set and structure of wreath product of cyclic groups, comutator of wreath product and the fundamental group of orbit Morse function $\pi_1 O(f)$</i>	59
Vasilchenko A. <i>Spaces of primitive elements in dual modules over Steenrod algebra 2</i>	61
Morrison P. J. <i>A Geometrical Version of the Maxwell-Vlasov Hamiltonian Structure</i>	63
Wojtowicz M. <i>Note on congruent numbers</i>	64
Кадубовський О. А. <i>Про число топологічно нееквівалентних гладких функцій з однією критичною точкою типу сідла на двовимірному торі</i>	65
Ладиненко Л. П. <i>Щодо геометричної характеристики спеціальних майже геодезичних перетворень просторів афінного зв'язку зі скрутом</i>	67
Овчаренко О. О. <i>Життєвий та науковий шлях Марка Григоровича Крейна</i>	68
Подоусова Т. Ю., Вашпанова Н. В. <i>LGT-лінії та A-деформації мінімальних поверхонь</i>	69
Прокіп В. М. <i>Алгоритм побудови унітального дільника для многочленної матриці</i>	70
Синюкова О. <i>Про геодезичні відображення просторів дотичних розшарувань зі спеціальною метрикою</i>	72
Щеглов М. В. <i>Поточкова оцінка відхилення полінома Крякіна від неперервної на відрізку функції</i>	73