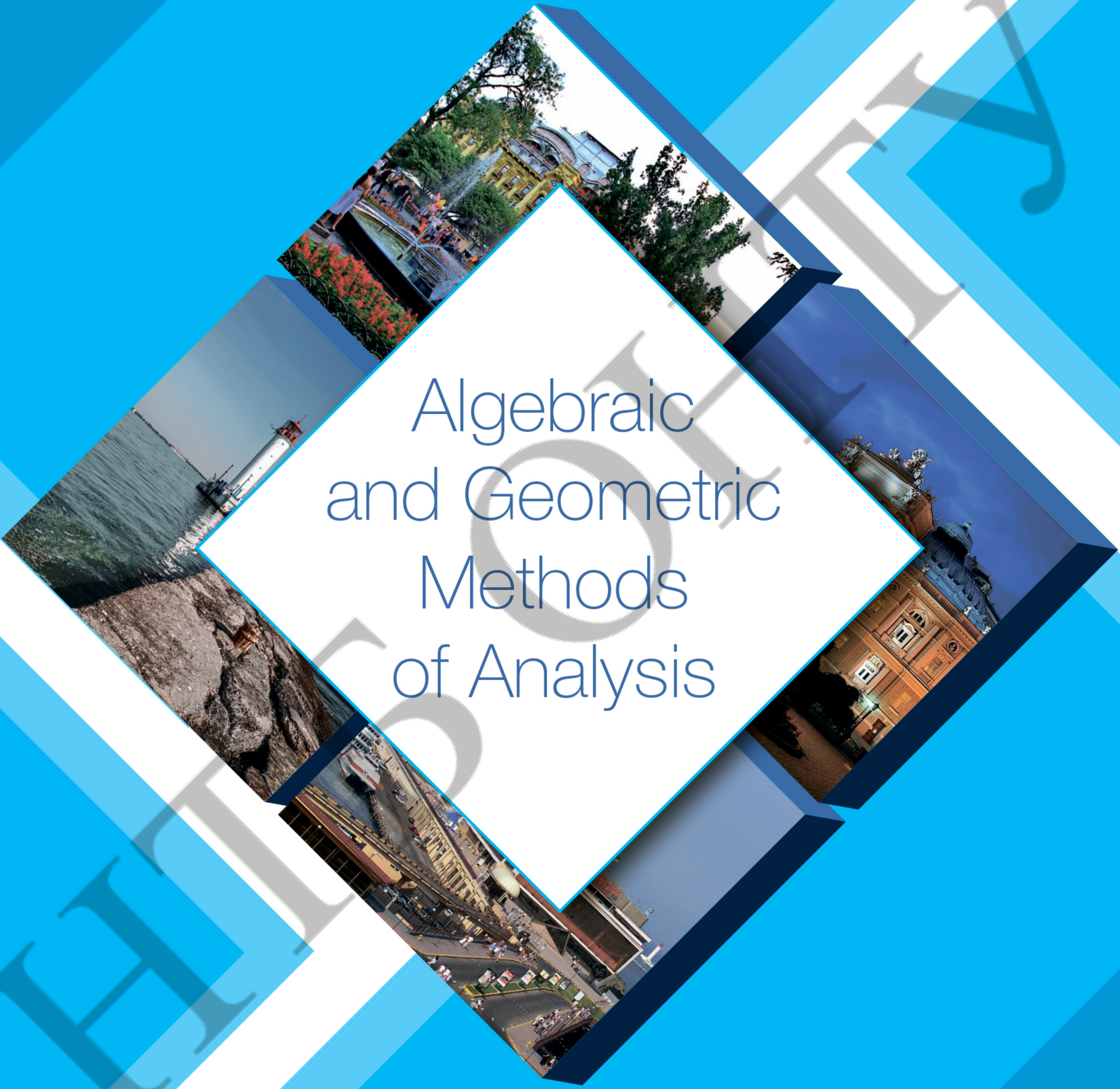
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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(1) $M = S^1 \times [0,1] = \{(z,s) \,|\, z \in \mathbb{C}, |z| = 1,\, 0 \le s \le 1\}$, and $f_0 : S^1 \times [0,1] \to \mathbb{R}$ is given by $f_0(\phi, s) = s$;
(2) $M = D^2 = \{(x,y) \in \mathbb{R}^2 \,|\, x^2 + y^2 \le 1\}$, and $f_0 : D^2 \to \mathbb{R}$ is given by $f_0(x,y) = \pm x^2 \pm y^2$;
(3) $M = S^2 = \{(x,y,z) \,|\, x^2 + y^2 + z^2 = 1\}$, and $f_0(x,y,z) : S^2 \to \mathbb{R}$ is given by $f_0(x,y,z) = z$;
(4) $M = T^2 = \left\{(x,y,z) \in \mathbb{R}^3 \,\middle|\, \left(\sqrt{x^2+y^2} - 2\right)^2 + z^2 = 1\right\}$, and $f_0 : T^2 \to \mathbb{R}$ is given by $f_0(x,y,z) = z$;
(5) $M = T^2 = \{(w,z) \in \mathbb{C}^2 \,|\, |z| = |w| = 1\}$, and $f_0 : T^2 \to S^1$ is given by $f_0(w,z) = z$.

Obviously that functions from (1)–(4) belongs to $\mathcal{F}^0(M,P)$. They are height functions for (1)–(4), and a function from (5) is an angular projection. These functions have a minimum possible number of critical submanifolds, and we will call them *primitive functions*.

Theorem 3. *Let f be a smooth function from $\mathcal{F}^0(M,P)$ and $f_0 \in \mathcal{F}^0(M,P)$ be a primitive function. A function f admits a decomposition*

$$f = \varkappa \circ f_0 \circ h^{-1} \tag{1}$$

for some diffeomorphism $h : M \to M$ and a smooth function $\varkappa : \mathrm{Im}(f_0) \to P$ satisfying the following conditions:

(A) *$\varkappa$ has the only finite number of critical points in which it is not flat, i.e., not all derivatives of $\varkappa$ at each critical point vanish,*
(B) *$\varkappa$ does not have extremes at $f_0(\Sigma^0_{f_0})$ and $f_0(\partial M)$.*

In particular, if $f \in \mathcal{F}^0(T^2, P)$ is null-homotopic, then f_0 is given by (4), and by (5) otherwise. A factorization (1) is not unique and depends on the choice of h.

References

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Nonlinear interpolation of α-Holderian mappings with applications to quasilinear PDEs

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The Marcinkiewicz interpolation theorems for linear operators acting on Lebesgue spaces turned out to be a powerful tool for studying regularity of solutions for linear PDEs in L^p-spaces. The K-method introduced by J. Peetre ([5, 6]) allowed to extend the study of regularity of solutions of linear equations on spaces different from L^p-spaces. The main difficulty to apply Peetre's definition is the identification of the interpolation spaces between two normed spaces embedded in a same topological space. In [2, 3, 4] we did such a study with applications to linear PDEs using new non-standard spaces as grand or small Lebesgue spaces and $G\Gamma$-gamma spaces.

In [7] L. Tartar gave interpolation results on nonlinear Hölderian mappings (which include Lipschitz mappings) and applied them to a variety of boundary value problems as bilinear applications, semilinear PDEs but also on variational inequalities.

In this talk we present some results contained in the recent paper [1], were we extend Tartar's results on nonlinear interpolation of α-Hölderian mappings $\mathcal{T}$ by studying the action of the mappings $\mathcal{T}$ on K-functionals and between interpolation spaces with logarithm functors. Therefore, we identify some interpolation spaces using couples of Lebesgue or Lorentz spaces, recovering spaces as Lorentz–Zygmund spaces or $G\Gamma$-gamma spaces.

We apply these results to obtain regularity on the gradient of the weak or entropic-renormalized solution u to quasilinear equations of the form

$$-\operatorname{div}(\widehat{a}(\nabla u)) + V(x;u) = f, \qquad u = 0 \text{ on } \partial\Omega, \tag{1}$$

associated to the Dirichlet homogeneous condition on the boundary, where Ω is a bounded smooth domain of $\mathbb{R}^n$, $\widehat{a}(\nabla u) = |\nabla u|^{p-2}\nabla u$, V is a nonlinear potential and f belongs to non-standard spaces like Lorentz-Zygmund spaces. We also show that the mapping $\mathcal{T}: \ \mathcal{T}f = \nabla u$ is locally or globally α-Hölderian under suitable values of α and appropriate assumptions on V and $\widehat{a}$.

Furthermore, also the anisotropic version or the variable exponents version of the Laplacian are considered.

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Riemann Integration on a space with a fractal structure

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In this work we start developing a Riemann-type integration theory on spaces which are equipped with a fractal structure (see [1] for more details). The definition of a fractal structure is the next one:

Definition 1. A fractal structure $\mathbf{\Gamma}$ on a set X is a countable family of coverings $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ such that Γ_{n+1} is a strong refinement of Γ_n for each $n \in \mathbb{N}$. Γ_2 is said to be a strong refinement of Γ_1 if Γ_2 is a refinement of Γ_1 (that is, each element of Γ_2 is contained in some element of Γ_1) and for each $B \in \Gamma_1$ it holds that $B = \bigcup\{A \in \Gamma_2 : A \subseteq B\}$. Cover Γ_n is called level n of the fractal structure.