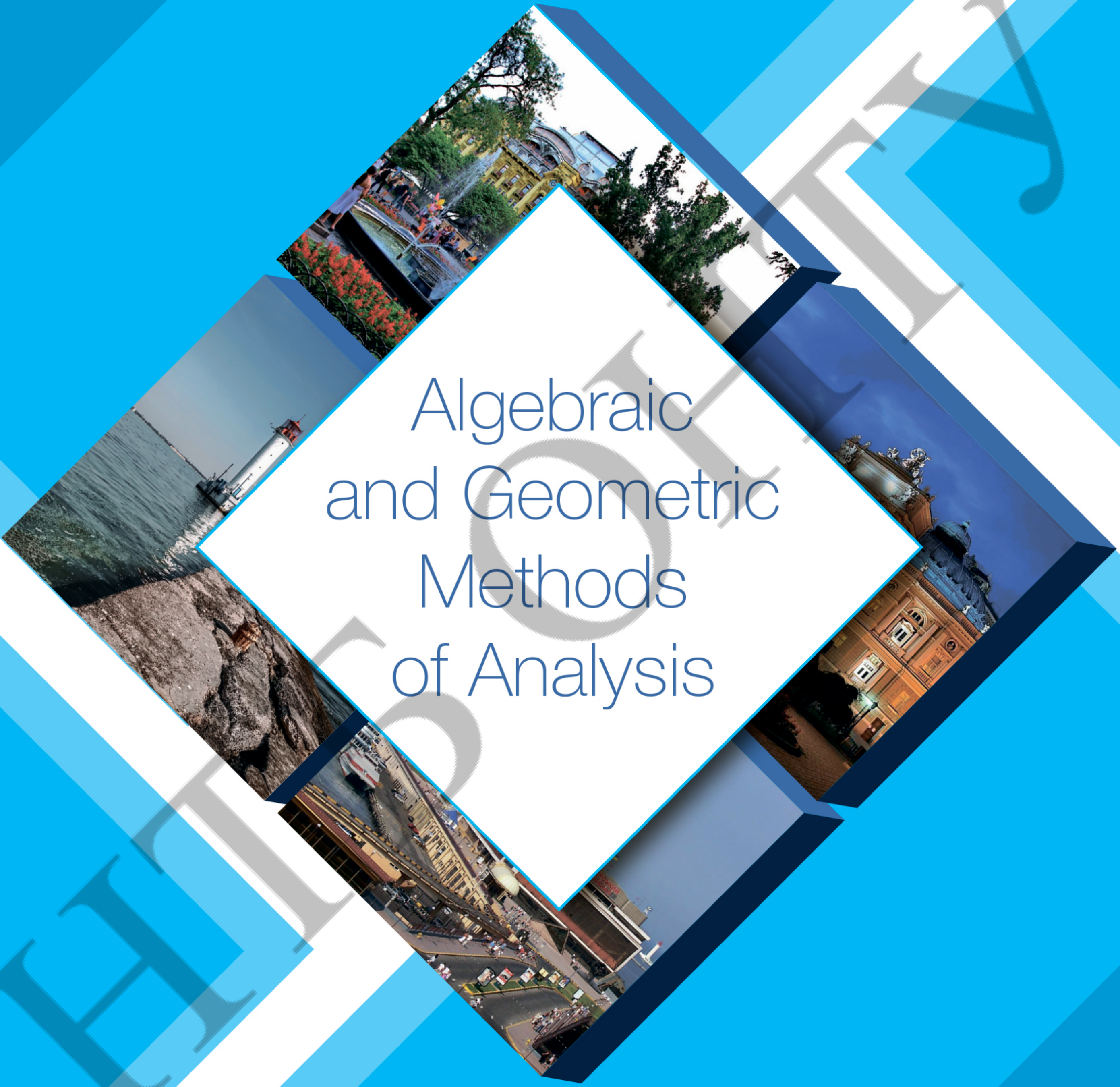
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Nilpotent structures of oriented neutral vector bundles

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Let E be an oriented vector bundle over a manifold M of rank $4n$ and h a neutral metric of E. We call a section N of $\operatorname{End} E$ a *nilpotent structure* of E if on a neighborhood of each point of M, there exists an ordered frame field $e = (e_1, \dots, e_{2n}, e_{2n+1}, \dots, e_{4n})$ of E satisfying

$$h(e_i, e_i) = -h(e_{2n+i}, e_{2n+i}) = 1 \ (i = 1, \dots, 2n), \quad h(e_i, e_j) = 0 \ (i \neq j) \tag{1}$$

and $Ne = e\Lambda_n$, where

$$\Lambda_n := \begin{bmatrix} O_n & -I_n & O_n & I_n \\ I_n & O_n & I_n & O_n \\ O_n & I_n & O_n & -I_n \\ I_n & O_n & I_n & O_n \end{bmatrix},$$

I_n is the $n \times n$ unit matrix and O_n is the $n \times n$ zero matrix. Let N be a nilpotent structure of E. We call N an *ε-nilpotent structure* ($\varepsilon \in \{+, -\}$) if on a neighborhood of each point of M, there exists an ordered frame field e giving the orientation of E and satisfying (1) and $NeI'_{4n,\varepsilon} = eI'_{4n,\varepsilon}\Lambda_n$ with

$$I'_{4n,\varepsilon} := \begin{bmatrix} I_n & O_n & O_n & O_n \\ O_n & I_n & O_n & O_n \\ O_n & O_n & I_n & O_n \\ O_n & O_n & O_n & I_{n,\varepsilon} \end{bmatrix}, \qquad I_{1,\pm} := \pm 1, \quad I_{n,\pm} := \begin{bmatrix} \pm 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \ (n \geq 2).$$

Let N be an ε-nilpotent structure of E. Then such a frame field as e is called an *admissible frame field* of N. For an admissible frame field e of N, we set $\xi = \xi_1 \wedge \cdots \wedge \xi_{2n}$, where

$$\begin{array}{ll} \xi_1 := e_1 - e_{2n+1}, & \xi_i := e_i - e_{2n+i}, \\ \xi_{n+1} := e_{n+1} + \varepsilon e_{3n+1}, & \xi_{n+i} := e_{n+i} + e_{3n+i} \end{array} \quad (i = 2, \dots, n).$$

Then ξ does not depend on the choice of an admissible frame field e of N ([3]). Therefore N gives a section ξ_N of the $2n$-fold exterior power $\bigwedge^{2n} E$ of E. A nilpotent structure is characterized by

(i) $\operatorname{Im} N = \operatorname{Ker} N$, and $\pi_N := \operatorname{Im} N = \operatorname{Ker} N$ is a light-like subbundle of E of rank $2n$,
(ii) $h(\phi, N\phi) = 0$ for any local section ϕ of E

([2], [3]). In particular, N gives a null structure on each fiber of E and h is null-Hermitian with respect to N (see [9]). The subbundle π_N is locally spanned by $\xi_1, \dots, \xi_{2n}$.

Remark Suppose $n = 1$. Then $\bigwedge^2 E$ is a vector bundle over M of rank 6 and h induces a metric $\hat{h}$ of $\bigwedge^2 E$ of signature (2,4). In addition, $\bigwedge^2 E$ is decomposed as $\bigwedge^2 E = \bigwedge^2_+ E \oplus \bigwedge^2_- E$ by two subbundles $\bigwedge^2_+ E$, $\bigwedge^2_- E$ of rank 3 and the restriction of $\hat{h}$ on each of them has signature (1,2). The *light-like twistor spaces* associated with E are fiber bundles $U_0\big(\bigwedge^2_\pm E\big)$ in $\bigwedge^2_\pm E$ respectively such that each fiber is a light cone. Each light-like line subbundle of $\bigwedge^2_+ E$ or $\bigwedge^2_- E$ corresponds to a light-like subbundle of E of rank 2 and each ε-nilpotent structure N of E corresponds to a section of $U_0\big(\bigwedge^2_\varepsilon E\big)$ given by $(1/\sqrt{2})\xi_N$ ([2], [3]). The space-like twistor spaces $U_+\big(\bigwedge^2_\pm E\big)$ associated with E are fiber bundles in $\bigwedge^2_\pm E$ respectively such that each fiber is a hyperboloid of two sheets. A section of $U_+\big(\bigwedge^2_\varepsilon E\big)$ corresponds to a complex structure of E preserving h. See [1], [5] for the space-like twistor spaces. The time-like twistor spaces $U_-\big(\bigwedge^2_\pm E\big)$ associated with E are fiber bundles in $\bigwedge^2_\pm E$ respectively such that each fiber is a hyperboloid of one sheet. A section of $U_-\big(\bigwedge^2_\varepsilon E\big)$ corresponds to a paracomplex structure of E reversing h. See [1], [13], [14] for the time-like twistor spaces. See [7], [10], [11] for the twistor spaces in the case h is a Riemannian (i.e., positive-definite) metric, which are the prototypes of $U_+\big(\bigwedge^2_\pm E\big)$, $U_-\big(\bigwedge^2_\pm E\big)$ and $U_0\big(\bigwedge^2_\pm E\big)$.

Let ∇ be a connection of E satisfying $\nabla h = 0$. Let N be an ε-nilpotent structure of E. We say that N satisfies the *Walker condition* with respect to ∇ if for any local section ψ of π_N, $\nabla\psi$ is a 1-form valued in π_N. See [6], [9], [16] for Walker manifolds. Let $\hat{\nabla}$ be the connection of $\bigwedge^{2n} E$ induced by ∇. Then N satisfies the Walker condition with respect to ∇ if and only if $\hat{\nabla}\xi_N = \alpha \otimes \xi_N$ for a 1-form α. If $\nabla N = 0$, then $\hat{\nabla}\xi_N = 0$ ([4]) and therefore N satisfies the Walker condition ([9]).

The main objects of research in this talk are special nilpotent structures, and they are called *H-nilpotent structures* of (E, h, ∇), where H is a Lie subgroup of $SO(2n, 2n)$ related to neutral hyperKähler structures. There exist a complex structure I and paracomplex structures J_1, J_2 of E such that h, ∇, I, J_1, J_2 form a neutral hyperKähler structure of E if and only if there exists an H-nilpotent structure of (E, h, ∇) ([4]). See [5], [12] for paraquaternionic structures. See [8], [15] for neutral hyperKähler 4-manifolds.

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Multiplicative b-homogeneralized Derivations of Associative Rings

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In this manuscript, we present multiplicative b-homogeneralized derivation on an associative ring R and discuss certain differential (functional) identities having multiplicative b-homogeneralized derivation. Investigating the centralizer of suitable subset over semiprime rings that admit multiplicative b-homogeneralized derivation enhances some outcomes in the literature. We refer the reader to [4] and [2] for more details.

As is well known, the problem of linear mappings preserving fixed products is a very interesting item in the field of operator algebra. Derivations that can be completely determined by the local action on some subsets of algebra have attracted attention of many researchers. The Martindale ring of quotients of a prime ring R was introduced in [6] as a tool for studying rings satisfying a polynomial identity. The concept was extended to semiprime rings in [5]. Historically, the study of derivation was initiated during the 1950s and 1960s. Derivations of rings got a tremendous development in 1957, when [3] established two very striking results in the case of prime rings.

Named that R is a semiprime when R satisfy the expression $r_1Rr_1 = 0$ which yields $r_1 = 0$ and R is prime if $r_1Rr_2 = 0$ which supply two options there either $r_1 = 0$ or $r_2 = 0$. As a factual information about the connection between the previous concepts a prime and semiprime ring mentioned as following: A prime ring forms another kind of ring, which is a semiprime, while the converse, unfortunately, is not always true.

When a ring R admits for all $r_1, r_2 \in R$ satisfying Leibniz's rule, which is $d(r_1r_2) = d(r_1)r_2 + r_1d(r_2)$ then a derivation is that an additive map $d\colon R \to R$. Whenever for all $r_1, r_2 \in R$ there exists an identity $D(r_1r_2) = D(r_1)r_2 + r_1d(r_2)$. Then, D is an additive mapping defined as $D\colon R \to R$ is recorded as a generalized, *i.e.* a generalized derivation, where d worked as an additive mapping derivation over R.

In 2000, a classical definition of homoderivation posted in El Sofy's article [1], where he was described an additive mapping a homoderivation concerning a ring R like ψ from R to R satisfying $\psi(xy) = \psi(x)\psi(y) + \psi(x)y + x\psi(y)$ where x and y belong to R. Moreover, mapping $F\colon R \to Q_{mr}$ associated with derivation (need not be additive) $d\colon R \to R$ such that $F(\sigma\tau) = F(\sigma)\tau + b\tau d(\tau)$ holds for all $\sigma, \tau \in R$ and any fixed $0 \neq b \in Q_s \subset Q_{mr}$. If F is additive (not necessarily additive), then F is called b-generalized derivation (multiplicative b-generalized.

Definition 1. Suppose that R is an associative ring, mapping $F\colon R \to Q_{mr}$ associated with homoderivation $d\colon R \to R$ such that $F(\sigma\tau) = F(\sigma)F(\tau) + F(\sigma)\tau + b\tau d(\tau)$ holds for all $\sigma, \tau \in R$

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