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- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
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Kuratowski limits of subsets of real line and their applications to pretangent spaces

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Let (X, d) be an unbounded metric space and $\tilde{r} = (r_n)_{n\in\mathbb{N}}$ be a scaling sequence of positive real numbers tending to infinity. We define the pretangent and tangent spaces $\Omega^X_{\infty,\tilde{r}}$ to (X, d) at infinity as metric spaces whose points are equivalence classes of sequences $(x_n)_{n\in\mathbb{N}} \subset X$ which tend to infinity with the speed of $\tilde{r}$. The detailed description of constructions of these spaces and their basic properties see, e. g., in [2].

Let (Y, δ) be a metric space. For any sequence $(A_n)_{n\in\mathbb{N}}$ of nonempty sets $A_n \subseteq Y$, the *Kuratowski limit inferior* of $(A_n)_{n\in\mathbb{N}}$ is the subset $\operatorname*{Li}_{n\to\infty} A_n$ of Y defined by the rule:

$$\Big(y \in \operatorname*{Li}_{n\to\infty} A_n\Big) \Leftrightarrow (\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N}\ \forall n \geq n_0 \colon B(y, \varepsilon) \cap A_n \neq \varnothing),$$

where $B(y, \varepsilon)$ is the open ball of radius $\varepsilon > 0$ centered at the point $y \in Y$,

$$B(y, \varepsilon) = \{x \in Y : \delta(x, y) < \varepsilon\}.$$

Similarly, the *Kuratowski limit superior* of $(A_n)_{n\in\mathbb{N}}$ can be defined as the subset $\operatorname*{Ls}_{n\to\infty} A_n$ of Y for which

$$\Big(y \in \operatorname*{Ls}_{n\to\infty} A_n\Big) \Leftrightarrow (\forall \varepsilon > 0\ \forall n \in \mathbb{N}\ \exists n_0 \geq n : B(y, \varepsilon) \cap A_{n_0} \neq \varnothing).$$

The Kuratowski limit inferior and limit superior are basic concepts of set-valued analysis in metric spaces and have numerous applications (see, for example, [1]).

We denote $tA := \{tx : x \in A\}$ for any nonempty set $A \subseteq \mathbb{R}$ and $t \in \mathbb{R}$, and, $\nu_0 := \tilde{X}^0_{\infty,\tilde{r}} \in \Omega^X_{\infty,\tilde{r}}$ for any pretangent space $\Omega^X_{\infty,\tilde{r}}$ of an unbounded metric space (X, d). Moreover, for every scaling sequence $\tilde{r}$, we denote by $\boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}}$ the set of all pretangent at infinity spaces to (X, d) with respect to $\tilde{r}$. Write

$$Sp\left(\Omega^X_{\infty,\tilde{r}}\right) := \{\rho(\nu_0, \nu) \colon \nu \in \Omega^X_{\infty,\tilde{r}}\} \text{ and } Sp(X) := \{d(p, x) \colon x \in X\}.$$

Proposition 1. *Let (X, d) be an unbounded metric space, $p \in X$, $\tilde{r} = (r_n)_{n\in\mathbb{N}}$ be a scaling sequence and let $\tilde{\mathbf{R}}$ be the set of all infinite subsequences of $\tilde{r}$. Then the equalities*

$$\bigcup_{\Omega^X_{\infty,\tilde{r}} \in \boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}}} Sp\left(\Omega^X_{\infty,\tilde{r}}\right) = \operatorname*{Li}_{n\to\infty} \left(\frac{1}{r_n} Sp(X)\right),$$

$$\bigcup_{\Omega^X_{\infty,\tilde{r}'} \in \boxtimes^X_{\infty,\tilde{r}'},\ \tilde{r}' \in \tilde{\mathbf{R}}} Sp(\Omega^X_{\infty,\tilde{r}'}) = \operatorname*{Ls}_{n\to\infty} \left(\frac{1}{r_n} Sp(X)\right)$$

hold.

Corollary 2. *Let (X,d) be an unbounded metric space, $\tilde r$ be a scaling sequence and let ${}^1\Omega^X_{\infty,\tilde r}$ be tangent and separable. Then we have*

$$\underset{n\to\infty}{Li}\left(\frac{1}{r_n}Sp(X)\right)=\underset{n\to\infty}{Ls}\left(\frac{1}{r_n}Sp(X)\right)=Sp\left({}^1\Omega^X_{\infty,\tilde r}\right).$$

Corollary 3. *Let (X,d) be an unbounded metric space, $\tilde r$ be a scaling sequence. Then the sets*

$$\bigcup_{\Omega^X_{\infty,\tilde r}\in\boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}}} Sp(\Omega^X_{\infty,\tilde r})\quad\textit{and}\quad\bigcup_{\Omega^X_{\infty,\tilde r'}\in\boxtimes^{\mathbf{X}}_{\infty,\tilde{\mathbf{r}}'},\ \tilde r'\in\tilde{\mathbf{R}}} Sp(\Omega^X_{\infty,\tilde r'})$$

are closed subsets of $[0,\infty)$.

Recall that a metric space (Y,δ) is said to be *strongly rigid* if for all $x,y,z,w\in Y$ the conditions $\delta(x,y)=\delta(w,z)$ and $x\neq y$ imply that $\{x,y\}=\{z,w\}$. Let us consider a strongly rigid metric space (Y,δ) such that:

(i_1) $\delta(x,y)<2$ for all points $x,\,y\in Y$; (i_2) $\sup\{\delta(x,y):x,y\in Y\}=2$;

(i_3) The cardinality of the open ball $B(y^*,r)=\{y\in Y:\delta(y,y^*)<r\}$ is finite for every $r\in(0,2)$ and every $y^*\in Y$.

Corollary 4. *Let (X,d) be an unbounded metric space, $\tilde r$ be a scaling sequence, $\Omega^X_{\infty,\tilde r}$ be tangent and let (Y,δ) be a strongly rigid metric space satisfying conditions (i_1)-(i_3). If $Y_1\subseteq Y$ and $f:\Omega^X_{\infty,\tilde r}\to Y_1$ is an isometry, then $\Omega^X_{\infty,\tilde r}$ is finite.*

Example 5. Let (Y,δ) be a metric space with $Y=\mathbb{N}$ and the metric δ defined such that:

$$\delta(1,2)=1+\frac{1}{2};$$
$$\delta(1,3)=1+\frac{2}{3},\quad\delta(2,3)=1+\frac{3}{4};$$
$$\delta(1,4)=1+\frac{4}{5},\quad\delta(2,4)=1+\frac{5}{6},\quad\delta(3,4)=1+\frac{6}{7};$$
$$\delta(1,5)=1+\frac{7}{8},\quad\delta(2,5)=1+\frac{8}{9},\quad\delta(3,5)=1+\frac{9}{10},\quad\delta(4,5)=1+\frac{10}{11};$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\quad.$$

Then (Y,δ) is a countable, complete and strongly rigid metric space satisfying conditions (i_1)-(i_3). By Corollary 4 no tangent space $\Omega^X_{\infty,\tilde r}$ is isometric to (Y,δ).

Corollary 6. *Let (X,d) be an unbounded metric space and let $\tilde r$ be a scaling sequence. Then the following statements are equivalent:*

(i) *There is a single-point pretangent space $\Omega^X_{\infty,\tilde r}$;*
(ii) *All $\Omega^X_{\infty,\tilde r}$ are single-point;*
(iii) *The equality*

$$\underset{n\to\infty}{Li}\left(\frac{1}{r_n}Sp(X)\right)=\{0\}$$

holds.

References

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