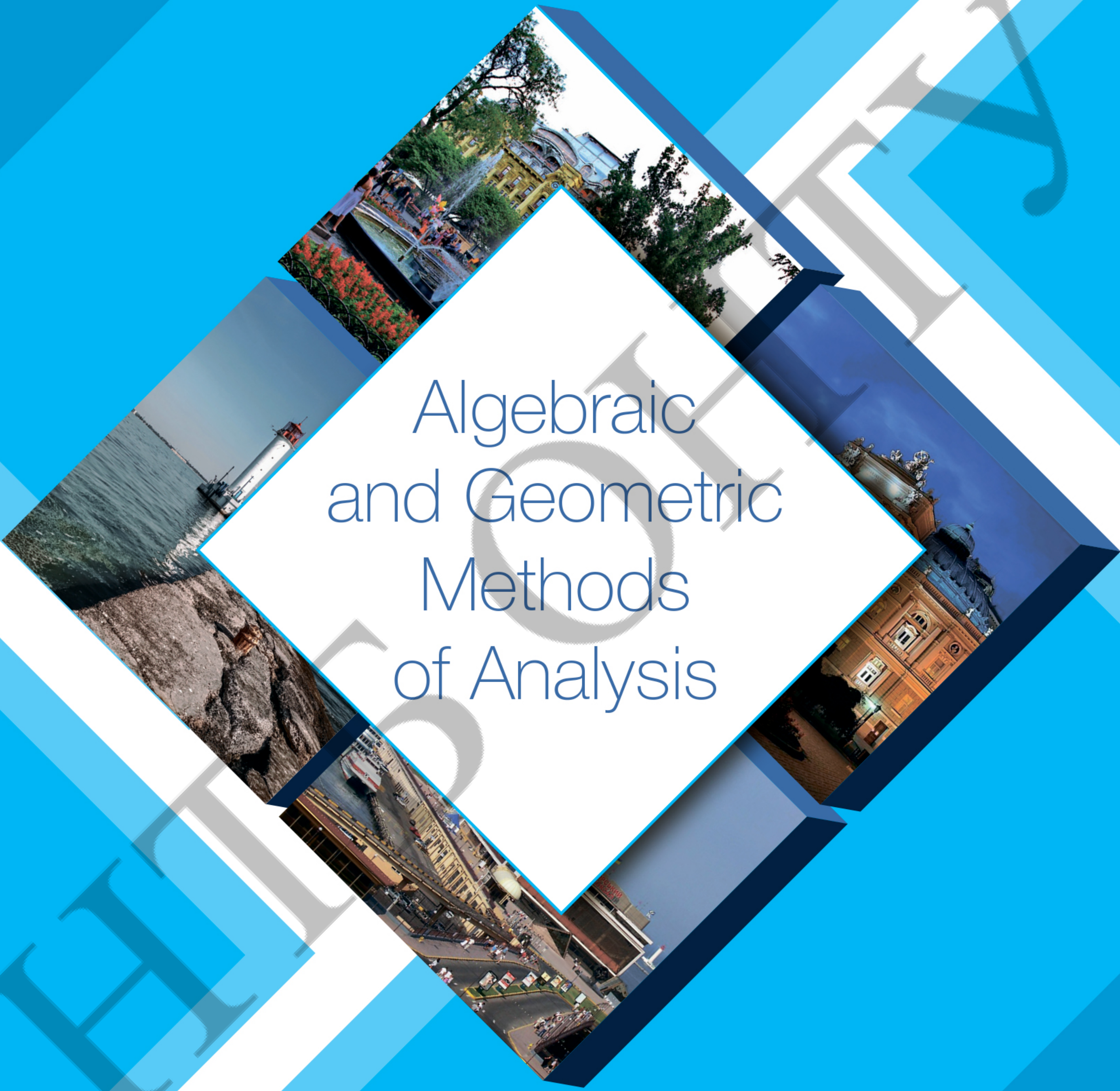
International Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Existence and non-existence of cohomogeneity one Einstein metrics

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A Riemannian metric g is Einstein if $\mathrm{Ric}(g) = \Lambda g$ for some constant Λ. A general existence theorem for homogeneous Einstein metrics was established in [WZ86]. It is natural to turn to the cohomogeneity one Einstein metrics, meaning that the principal orbit G/K is of codimension one. The cohomogeneity one condition reduces the Einstein equation to a system of ODEs. Previously known examples include [Pag78], [BB82], [KS86], [KS88], and [WW98]. Recently, we proved the existence of an Einstein metric on $\mathbb{HP}^{m+1}\sharp\overline{\mathbb{HP}}^{m+1}$ [Chi24], generalizing the result in [Böh98] to all higher dimensions.

We realize that the analytic techniques can be carried over to many other cohomogeneity one spaces. We develop two criteria to check the existence or non-existence of a cohomogeneity one Einstein metrics with a certain fixed principal orbit type. In particular, the principal orbit G/K is the total space of a sphere bundle over a singular orbit G/K, and both the fiber and the base space are irreducible. Each such a principal orbit is associated to a structural triple (d_1, d_2, A), where $d_1 = \dim(\mathsf{H}/\mathsf{K})$, $d_2 = \dim(\mathsf{G}/\mathsf{H})$ and $A > 0$ is a constant obtained from the O'neil tensor in the theory of Riemannian submersion. The corresponding cohomogeneity one space, denoted as M, is a double disk bundle, where G/K collapses to G/H on two ends. The Einstein metric is obtained from the ansatz

$$dt^2 + f_1^2(t)\, b|_{\mathfrak{h}/\mathfrak{k}} + f_2^2(t)\, b|_{\mathfrak{g}/\mathfrak{h}}\,, \tag{1}$$

where t parametrizes the 1-dimensional orbit space and b is a background metric.

Our existence theorem is the following.

Theorem 1. *For any (d_1, d_2) with $d_2 \geq d_1 \geq 2$, there exists a constant $\chi_{d_1,d_2} \in \left(0, \frac{d_2(d_2-1)^2}{d_1^2(d_1d_2-d_2+4)}\right]$ such that if G/K is a principal orbit with $A \in [0, \chi_{d_1,d_2})$, then there is at least one cohomogeneity one Einstein metrics on M.*

The constant χ_{d_1,d_2} is an algebraic function in (d_1, d_2), whose formula is very complicated in general. Nevertheless, we obtain many new examples of inhomogeneous Einstein metrics from previous works on homogeneous Einstein metrics including [DZ79], [WZ85], [Wan92], [DK08], [Nik16], [PZ21], and [LW24].

On the other hand, we also have the following non-existence theorem.

Theorem 2. *Define*

$$\Psi_{d_1,d_2} := \frac{(4(d_1-1)n^2 + d_2^2)(3n + d_1)}{(2n^2 + n + d_1)^2 d_1^2} \frac{d_2(d_2-1)^2}{4(d_1-1)}.$$

If G/K is a principal orbit with $(d_1, d_2) \notin \{(2,2), (2,3), (2,4)\}$ and $A \geq \Psi_{d_1,d_2}$, then there does not exists any G-invariant cohomogeneity one Einstein metrics on M from ansatz (1).

We find some examples of Theorem 2 from the classification in [DK08], including $\mathbb{OP}^2 \sharp \overline{\mathbb{OP}}^2$ with Spin(9)/Spin(7) as its principal orbit.

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