International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of

Dvytro Grave

(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Moreover, for $n > 2$ even and for any $s \in [1, n-1]$ there exists $K, C \in \mathcal{K}^n$ such that K is complete w.r.t. C with $s(K) = s$, such that $\frac{D(K,C)}{w(K,C)} = \frac{s+1}{2}$, while for $n > 2$ odd and any $s \in [1, n]$ there exists $K \in \mathcal{K}^n$ which is complete w.r.t. C with $s(K) = s$, such that $\frac{D(K,C)}{w(K,C)} = \frac{s+1}{2}$.

On the possibility of joining two pairs of points in convex domains using paths

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Recall, that a set C is *convex* if any pair of points $x, y \in C$ may be joined by some segment which belongs to C, as well. We define the Euclidean distance between sets and the Euclidean diameter by the formulae

$$d(A,B) = \inf_{x\in A, y\in B} |x-y|\,, \quad d(A) = \sup_{x,y\in A} |x-y|\,.$$

Sometimes we also write $\mathrm{dist}\,(A,B)$ instead $d(A,B)$ and $\mathrm{diam}\,E$ instead $d(E)$, as well. As usually, we set

$$B(x_0,r) = \{x \in \mathbb{R}^n : |x-x_0| < r\}\,,$$
$$S(x_0,r) = \{x \in \mathbb{R}^n : |x-x_0| = r\}\,.$$

We emphasize that, the results established here have already been obtained in particular case, when a domain is the unit ball [1]. Concerning some applications of modulus inequalities in the mapping theory, see [2], cf. [3]–[4].

Theorem 1. *Let D' be a bounded convex domain in $\mathbb{R}^n$, $n \geqslant 2$, and let $E := B(y_*, \delta_*/2)$ be a ball centered at the point $y_* \in D'$, where $\delta_* := d(y_*, \partial D')$. Let $z_0 \in \partial D'$. Then for any points $A, B \in B(z_0, \delta_*/8) \cap D'$ there are points $C, D \in \overline{B(y_*, \delta_*/2)}$, for which the segments $[A, C]$ and $[B, D]$ are such that*

$$\mathrm{dist}\,([A,C],[B,D]) \geqslant C_0 \cdot |A-B|\,, \tag{1}$$

where $C_0 > 0$ is some constant depending only on δ_* and $d(D')$.

Recall that, a Borel function $\rho : \mathbb{R}^n \to [0,\infty]$ is called *an admissible* for a family Γ of paths γ in $\mathbb{R}^n$, if the relation

$$\int\limits_{\gamma} \rho(x)\,|dx| \geqslant 1 \tag{2}$$

holds for any locally rectifiable path $\gamma \in \Gamma$. *A modulus* of Γ is defined as follows:

$$M(\Gamma) = \inf_{\rho \in \,\mathrm{adm}\,\Gamma} \int\limits_{\mathbb{R}^n} \rho^n(x)\,dm(x)\,. \tag{3}$$

The following statements hold.

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Corollary 2. *Let, under conditions of Theorem 1,* Γ *denotes the family of all paths joining the segments* $[A,C]$ *and* $[B,D]$ *in* D'*. Then*

$$M(\Gamma) \leqslant \frac{m(D')}{C_0^n} \cdot \frac{1}{|A-B|^n}\,, \tag{4}$$

where M *is the modulus of families of paths defined in (3),* $m(D')$ *denotes the Lebesgue measure of* D'*, and* C_0 *is a constant in (1).*

Corollary 3. *Let, under conditions of Theorem 1,* Γ *denotes the family of all paths joining the segments* $[A,C]$ *and* $[B,D]$ *in* D'*. Then*

$$M(\Gamma) \geqslant \widetilde{c_n} \cdot \log\left(1+\frac{3\delta_*}{8|A-B|}\right)\,, \tag{5}$$

where M *is the modulus of families of paths defined in (3),* $\widetilde{c_n} > 0$ *is some constant depending only on* n *and* D'*.*

References

[1] Sevost'yanov E.A. (2022). On logarithmic Hölder continuity of mappings on the boundary. *Annales Fennici Mathematici, 47,* 251–259.

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[3] Martio, O., Ryazanov, V., Srebro, U., Yakubov, E. (2004). Mappings with finite length distortion. *J. d'Anal. Math., 93,* 215–236.

[4] Martio, O., Ryazanov, V., Srebro, U., Yakubov, E. (2005). On Q-homeomorphisms. *Ann. Acad. Sci. Fenn. Math., 30*(1), 49–69 (2005).

Backström curves

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Recall some definitions.

Definition 1. (1) A *non-commutative curve* is a pair $(X,\mathcal{A})$, where X is an algeraic curve over a field $\Bbbk$ and $\mathcal{A}$ is a sheaf of $\mathcal{O}_X$-algebras coherent as a sheaf of $\mathcal{O}_X$-modules.

(2) A non-commutative curve $(X,\mathcal{H})$ is called *hereditary* if for every point $x \in X$ the localization $\mathcal{H}_x$ is hereditary (equivalently, $\operatorname{gl.dim}\mathcal{H}=1$).

(3) A non-commutative curve $(X,\mathcal{A})$ is called *Backström* if there is a hereditary non-commutative curve $(X,\mathcal{H})$ such that $\mathcal{H}\supset\mathcal{A}$ and $\operatorname{rad}\mathcal{H}_x = \operatorname{rad}\mathcal{A}_x$ for all points $x \in X$.

(4) The *Auslander envelope* of a Backström non-commutative curve $(X,\mathcal{A})$ is defined as the non-commutative curve $(X,\tilde{\mathcal{A}})$, where $\tilde{\mathcal{A}} = \mathcal{E}nd_{\mathcal{A}}(\mathcal{A}\oplus\mathcal{H})$.

For instance, every (usual) algebraic curve such that all its singularities are simple nodes is a Backström curve, as well as the union of the coordinate axes in the affine space of any dimension.

We study the structure of Backström curves and their Auslander envelopes and prove the following results.

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