

International scientific conference «Algebraic and geometric methods of analysis»

Book of abstracts



May 31 - June 5, 2017
Odessa
Ukraine

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
- History and methodology of teaching in mathematics

ORGANIZERS

- The Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- The Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- The International Geometry Center

PROGRAM COMMITTEE

Chairman: Prishlyak A.

(Kyiv, Ukraine)

Balan V.

(Bucharest, Romania)

Banakh T.

(Lviv, Ukraine)

Fedchenko Yu.

(Odesa, Ukraine)

Fomenko A.

(Moscow, Russia)

Fomenko V.

(Taganrog, Russia)

Glushkov A.

(Odesa, Ukraine)

Haddad M.

(Wadi al-Nasara, Syria)

Herega A.

(Odesa, Ukraine)

Khruslov E.

(Kharkiv, Ukraine)

Kirichenko V.

(Moscow, Russia)

Kirillov V.

(Odesa, Ukraine)

Konovenko N.

(Odesa, Ukraine)

Maksymenko S.

(Kyiv, Ukraine)

Matsumoto K.

(Yamagata, Japan)

Mashkov O.

(Kyiv, Ukraine)

Mykytyuk I.

(Lviv, Ukraine)

Milka A.

(Kharkiv, Ukraine)

Mikesh J.

(Olomouc, Czech Republic)

Mormul P.

(Warsaw, Poland)

Moskaliuk S.

(Wien, Austria)

Panzhenskiy V.

(Penza, Russia)

Pastur L.

(Kharkiv, Ukraine)

Plachta L.

(Krakov, Poland)

Pokas S.

(Odesa, Ukraine)

Polulyakh E.

(Kyiv, Ukraine)

Rahula M.

(Tartu, Estonia)

Sabitov I.

(Moscow, Russia)

Savchenko A.

(Kherson, Ukraine)

Sergeeva A.

(Odesa, Ukraine)

Strikha M.

(Kyiv, Ukraine)

Shvets V.

(Odesa, Ukraine)

Shelekhov A.

(Tver, Russia)

Shurygin V.

(Kazan, Russia)

Vlasenko I.

(Kyiv, Ukraine)

Zadorozhnyj V.

(Odesa, Ukraine)

Zarichnyi M.

(Lviv, Ukraine)

Zelinskiy Y.

(Kyiv, Ukraine)

ADMINISTRATIVE COMMITTEE

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Volkov V., Director of the Educational Research Institute of Mechanics, Automation and Computer Systems named after P. M. Platonov;
- Bukaros A., Dean of the Faculty of automation, mechatronics and robotics

ORGANIZING COMMITTEE

Kirillov V.
Konovenko N.
Fedchenko Yu.

Hladysh B.
Nuzhnaya N.
Osadchuk E.

Maksymenko S.
Khudenko N.
Cherevko E.

On projective classes of rational functions

Konovenko Nadiia

(Department of Mathematics, ONAFT, Odessa, Ukraine)

E-mail: konovenko@ukr.net

Lychagin Valentin

(Department of Mathematics, University of Tromsø, Norway)

E-mail: valentin.lychagin@uit.no

We study equivalence classes of rational functions on Riemann's sphere $\overline{\mathbb{C}}$ with respect to Möbius transformations (cf. [1]). The Lie group $\mathbf{SL}_2(\mathbb{C})$ acts on $\overline{\mathbb{C}}$ and the corresponding representations of Lie algebra $\mathfrak{sl}_2(\mathbb{C})$ is given by the vector fields

$$\mathfrak{sl}_2(\mathbb{C}) = \left\langle \frac{\partial}{\partial z}, \quad z \frac{\partial}{\partial z}, \quad z^2 \frac{\partial}{\partial z} \right\rangle.$$

Denote by J^k the manifold of k -jets of analytical functions on $\overline{\mathbb{C}}$, and let $(z, u, u_1, \dots, u_k)$ be the natural (local) coordinates on J^k .

Then a rational function I on the manifold J^k we call a projective invariant of order $\leq k$, if $X^{(k)}(I) = 0$, for all vector fields $X \in \mathfrak{sl}_2(\mathbb{C})$. Here we denoted by $X^{(k)}$ the k -th prolongations of the vector field X .

Theorem 1. *The field of rational differential invariants of order $\leq k$ is generated by invariants*

$$J_0 = u, \quad J_3 = u_1^{-3}u_3 - \frac{3}{2}u_2^2u_1^{-4},$$

and invariant derivatives

$$J_4 = \nabla(J_3), \quad \dots, \quad J_k = \nabla^{k-3}(J_3),$$

where $\nabla = u_1^{-1} \frac{d}{dz}$.

This field separates regular $\mathbf{SL}_2(\mathbb{C})$ orbit.

For given rational function $f = f(z)$ we denote $J_3(f)$ the value of the projective invariant J_3 on f .

The transcendent degree of the field of rational functions on the Riemann sphere equals one, and therefore between functions f and $J_3(f)$ there is an algebraic relation

$$R(f, J_3(f)) = 0. \tag{1}$$

The polynomial R_f , which satisfies (1) and having the smallest degree, we call generating polynomial for f .

Theorem 2. (1) *The projective class of a rational function $f(z)$ is defined by the generating polynomial $R_f(f, J_3(f))$.*

(2) *Two rational functions $f(z), g(z)$ are projectively equivalent if and only if the function $g(z)$ is a solution of the following ordinary differential equation:*

$$R_f\left(u, u_1^{-3}u_3 - \frac{3}{2}u_2^2u_1^{-4}\right) = 0. \tag{2}$$

Remark that this differential equation is so-called automorphic equation in the sense that the Möbius group $\mathbf{PSL}_2(\mathbb{C})$ acts in a transitive way on the solution space of differential equation (2).

In particular, all solutions of differential equation (2) are rational functions.

Example 3. (1) Solutions of the following differential equation

$$(D + 4au)^2(2u_1u_3 - 3u_2^2) + 12a^2u_1^4 = 0$$

are rational functions that are projectively equivalent to polynomials of the second degree $f(z) = az^2 + bz + c$ with given discriminant D and leading coefficient a .

Remark that $\frac{D}{a}$ is a projective invariant.

(2) Solutions of differential equation

$$(u^2 - 4)(2u_1u_3 - 3u_2^2) + 12u_1^4 = 0$$

are rational functions which are projectively invariant to the Jukowski function $f(z) = z + z^{-1}$.

REFERENCES

- [1] N. Kononenko. Differential invariants and $\mathfrak{sl}_2$ - geometries. *Kiev: "Naukova Dumka" NAS of Ukraine*, 2013, 192 p.

НТБ ОНАХТ

Konovenko N., Lychagin V. <i>On projective classes of rational functions</i>	71
Kozerenko S. <i>Orientations of trees and signed Markov graphs</i>	73
Kuzmenko T. <i>Constructive description of G-monogenic mappings in the algebra of complex quaternions</i>	74
Lyubashenko V. <i>Moyal and Rankin-Cohen deformations of algebras</i>	76
Markitan V. <i>Fractal properties of sets associated with Markov representation of real numbers defined by a double stochastic matrix</i>	78
Matsumoto K. <i>Warped product semi-slant submanifolds in locally conformal Kaehler manifolds</i>	79
Mormul P. <i>Weak and strong nilpotentizability in the monster towers hosting flag distributions</i>	80
Mukhamadiev F. G. <i>The local density and the local weak density of N_τ^φ-kernel of a topological space X and superextensions</i>	82
Muradoglu Z., Gunduz Aras C. <i>A study for decision making problems by using interval soft sets</i>	84
Muradov R. S. <i>Archimedean copula functions and their some algebraic properties with applications</i>	85
Obikhod T. V. <i>BPS states of Fourfolds as candidates for Kaluza-Klein modes</i>	87
Parasyuk I. O. <i>Landau-type inequalities for curves on Riemannian manifolds</i>	88
Prislyak A., Prus A. <i>Morse-Smale flows on torus with hole</i>	90
Reinov O. <i>On nuclear operators with trace $V = 1$ and $V^2 = 0$</i>	91
Sabitov I. Kh. <i>Multiple roots of the volume polynomials for polyhedra</i>	92
Samokhvalov S. <i>Theory of gravity in the affine frame</i>	93
Shamolin M. V. <i>Integrable systems with dissipation on the tangent bundle of two-dimensional manifold</i>	94
Turhan T., Ayyildiz N. <i>On geometry of spatial kinematics in Lorentzian space</i>	96
Turhan T., Ayyildiz N. <i>A study on the integral invariants of a closed spacelike ruled surface</i>	97
Vasilchenko A. N. <i>Dual modules over Steenrod algebra 2</i>	98
Vlasenko I. <i>Topology of the basin of attraction of surface endomorphisms.</i>	100
Voloshyna V. <i>About some properties of functions determined as transformations from W^n to W^m-representation</i>	101
Vyhivska L. <i>On the problem of product of inner radii symmetric non-overlapping domains</i>	103
Yildirim S., Ayyildiz N. <i>A Study on Rectifying Curves in Semi-Euclidean Spaces</i>	104
Арсеньева О. Е., Кириченко В. Ф., Суровцева Е. В. <i>Эрмитова геометрия почти контактного метрического многообразия</i>	105