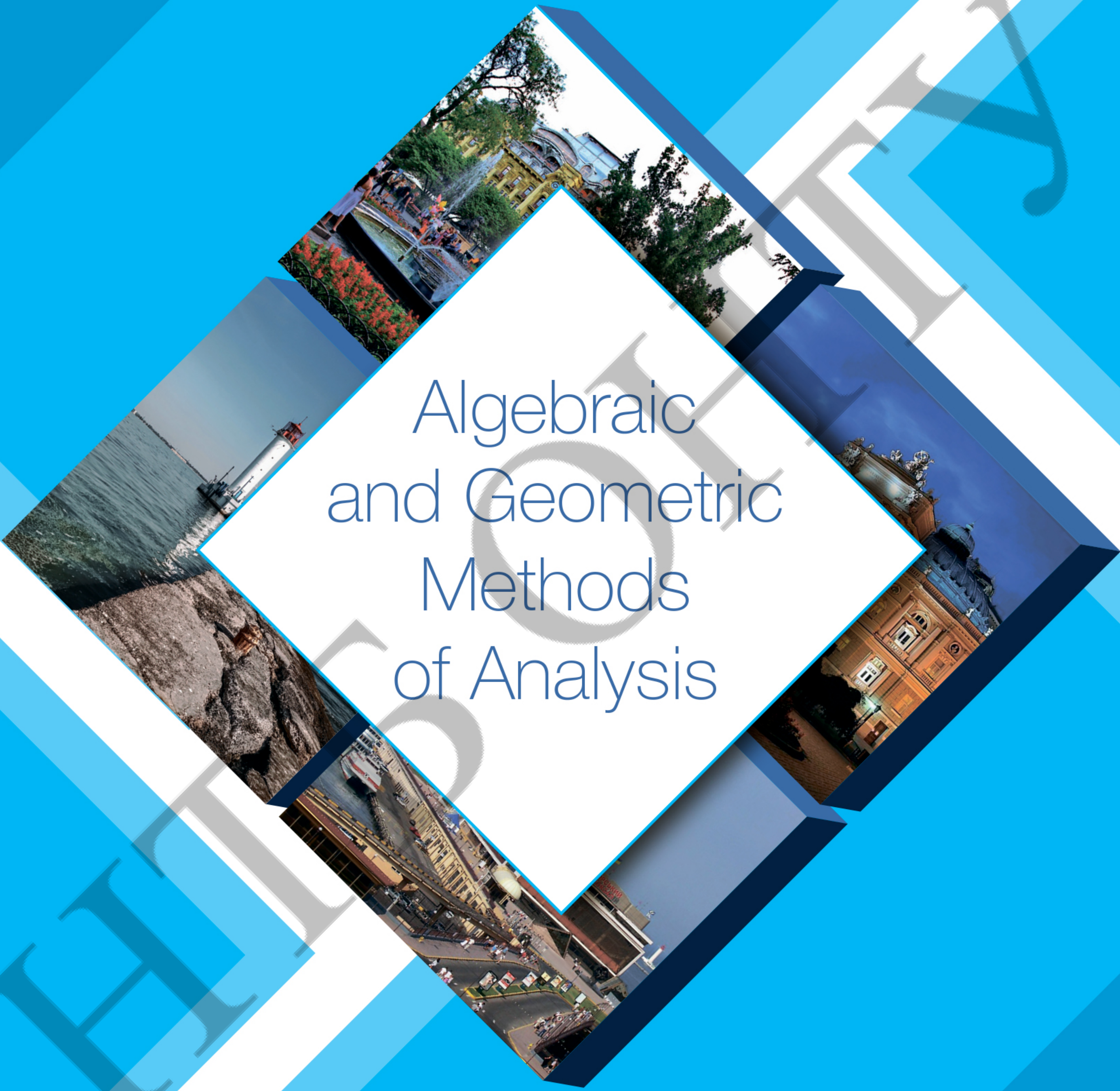
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Open billiards, chaos and limit theorems

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Abstract: Chaos is one of the important subjects in the theory of dynamical systems. In 1958, Kolmogorov made a discovery regarding the statistical properties exhibited by certain chaotic dynamical systems.

I will talk about the relationship between chaotic billiard systems and their statistical properties. More precisely, I will show that

(1) Poisson limit theorems can characterize chaotic behaviors of billiard systems
(2) The convergence rates of Poisson limit theorems and Zeta-Functions have certain connections.

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On the inverse Poletsky inequality with a cotangent dilatation

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The following definitions are from [1]. A path γ in $\mathbb{R}^n$ is a continuous mapping $\gamma : \Delta \to \mathbb{R}^n$ where Δ is an interval in $\mathbb{R}$. Its locus $\gamma(\Delta)$ is denoted by $|\gamma|$. Given a family Γ of paths γ in $\mathbb{R}^n$, a Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, abbr. $\rho \in \operatorname{adm} \Gamma$, if

$$\int\limits_{\gamma} \rho(x)|dx| \geqslant 1$$

for each (locally rectifiable) $\gamma \in \Gamma$. Given $p \geqslant 1$, the *p-modulus* of Γ is defined by the relation

$$M_p(\Gamma) := \inf_{\rho \in \operatorname{adm} \Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x) dm(x) \tag{1}$$

interpreted as $+\infty$ if $\operatorname{adm}\Gamma=\varnothing$.

We will need the following definitions related to paths, their lengths and mappings defined on them, see [2, section 8]. If $\gamma:\Delta\to\mathbb{R}^n$ is a locally rectifiable path, then there is the unique nondecreasing length function l_γ of Δ onto a length interval $\Delta_\gamma\subset\mathbb{R}$ with a prescribed normalization $l_\gamma(t_0)=0\in\Delta_\gamma$, $t_0\in\Delta$, such that $l_\gamma(t)$ is equal to the length of the subpath $\gamma|_{[t_0,t]}$ of γ if $t>t_0$, $t\in\Delta$, and $l_\gamma(t)$ is equal to minus length of $\gamma|_{[t,t_0]}$ if $t<t_0$, $t\in\Delta$. Let $g:|\gamma|\to\mathbb{R}^n$ be a continuous mapping, and suppose that the path $\widetilde{\gamma}=g\circ\gamma$ is also locally rectifiable. Then there is a unique non-decreasing function $L_{\gamma,g}:\Delta_\gamma\to\Delta_{\widetilde{\gamma}}$ such that $L_{\gamma,g}\left(l_\gamma(t)\right)=l_{\widetilde{\gamma}}(t)$ for all $t\in\Delta$. A path γ in D is called here a (whole) *lifting* of a path $\widetilde{\gamma}$ in $\mathbb{R}^n$ under $f:D\to\mathbb{R}^n$ if $\widetilde{\gamma}=f\circ\gamma$.

Further, we use the notation I for the segment $[a,b]$. Given a closed rectifiable path $\gamma:I\to\mathbb{R}^n$, we define a length function $l_\gamma(t)$ by the rule $l_\gamma(t)=S\left(\gamma,[a,t]\right)$, where $S(\gamma,[a,t])$ is the length of the path $\gamma|_{[a,t]}$. Let $\alpha:[a,b]\to\mathbb{R}^n$ be a rectifiable path in $\mathbb{R}^n$, $n\geqslant 2$, and $l(\alpha)$ be its length. A *normal representation* α^0 of α is defined as a path $\alpha^0:[0,l(\alpha)]\to\mathbb{R}^n$ which can be got from α by change of parameter such that $\alpha(t)=\alpha^0\left(S\left(\alpha,[a,t]\right)\right)$ for every $t\in[0,l(\alpha)]$. Such a normal representation always exists and is unique (see [1, Theorem 2.4]).

The following definition may be found in [1, 2.5, item 2, section I]. Let $\alpha:[a,b]\to\mathbb{R}^n$ be a closed rectifiable path in $\mathbb{R}^n$, $n\geqslant 2$. A mapping $f:|\alpha|\to\mathbb{R}^n$ is said to be *absolutely continuous on* α, if the function $f\circ\alpha^0$ is absolutely continuous on $[0,l(\alpha)]$, where $l(\alpha)$ denotes the length of α, and α^0 is its normal representation.

In the following, we say that some property P holds for *p-almost all paths in the domain* D if this property may be violated only for some family Γ_0 of paths in D such that $M_p(\Gamma_0)=0$, where $M_p(\Gamma_0)$ denotes the p-module of the family of paths Γ_0 defined in (1). We will say that the mapping $f:D\to\mathbb{R}^n$ has the *ACP-property with respect to p-modulus,* write $f\in ACP_p$, if the length function $L_{\gamma,f}$ is absolutely continuous on all closed intervals Δ_γ for p-almost all paths γ in D.

Let X and Y be two spaces with measures μ and μ', respectively. We say that a mapping $f:X\to Y$ has *N-property of Luzin*, if from the condition $\mu(E)=0$ it follows that $\mu'(f(E))=0$. Similarly, we say that a mapping $f:X\to Y$ has *N^{-1}-Luzin property,* if from the condition $\mu'(E)=0$ it follows that $\mu(f^{-1}(E))=0$.

Let $x\in D$ be a differentiability point of f. We set

$$l(f'(x))=\min_{h\in\mathbb{R}^n\setminus\{0\}}\frac{|f'(x)h|}{|h|},\quad \|f'(x)\|=\max_{h\in\mathbb{R}^n\setminus\{0\}}\frac{|f'(x)h|}{|h|},\quad J(x,f)=\det f'(x)\,.$$

Given sets E and F and a given domain D in $\overline{\mathbb{R}^n}=\mathbb{R}^n\cup\{\infty\}$, we denote by $\Gamma(E,F,D)$ the family of all paths $\gamma:[0,1]\to\overline{\mathbb{R}^n}$ joining E and F in D, that is, $\gamma(0)\in E$, $\gamma(1)\in F$ and $\gamma(t)\in D$ for all $t\in(0,1)$. Everywhere below, unless otherwise stated, the boundary and the closure of a set are understood in the sense of the extended Euclidean space $\overline{\mathbb{R}^n}$. Let $x_0\in\overline{D}$, $x_0\neq\infty$,

$$B(x_0,r)=\{x\in\mathbb{R}^n:|x-x_0|<r\}\,,\quad \mathbb{B}^n=B(0,1)\,,$$

$$S(x_0,r)=\{x\,\in\,\mathbb{R}^n:|x-x_0|=r\}\,,S_i=S(x_0,r_i)\,,\quad i=1,2\,,$$

$$A=A(x_0,r_1,r_2)=\{x\,\in\,\mathbb{R}^n:r_1<|x-x_0|<r_2\}\,.$$

Let $f:D\to\mathbb{R}^n$, $n\geqslant 2$, and let $Q:\mathbb{R}^n\to[0,\infty]$ be a Lebesgue measurable function such that $Q(y)\equiv 0$ for $y\in\mathbb{R}^n\setminus f(D)$. Let $A=A(y_0,r_1,r_2)$ and $\Gamma_f(y_0,r_1,r_2)$ denotes the family of all paths $\gamma:[a,b]\to D$ such that $f(\gamma)\in\Gamma(S(y_0,r_1),S(y_0,r_2),A(y_0,r_1,r_2))$, i.e., $f(\gamma(a))\in S(y_0,r_1)$,

$f(\gamma(b)) \in S(y_0, r_2)$, and $f(\gamma(t)) \in A(y_0, r_1, r_2)$ for any $a < t < b$. We say that f *satisfies the inverse Poletsky inequality at $y_0 \in f(D)$ with respect to p-modulus,* if the relation

$$M_p(\Gamma_f(y_0, r_1, r_2)) \leqslant \int\limits_A Q(y) \cdot \eta^p(|y - y_0|)\, dm(y) \qquad (2)$$

holds for any $0 < r_1 < r_2 < r_0 := \sup\limits_{y \in f(D)} |y - y_0|$ and any Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r)\, dr \geqslant 1$. A mapping $f : D \to \mathbb{R}^n$ is called *weakly light,* if, for any $y \in \mathbb{R}^n$, each connected component $\{f^{-1}(y)\}$ does not contain a non-degenerate path (see, e.g., Remark 8.3 in [2]).

Theorem 1. *Let $p > 1$, and let $f : D \to \mathbb{R}^n$ be a weakly light mapping which is differentiable a.e. and has Luzin N- and N^{-1}-properties with respect to the Lebesgue measure in $\mathbb{R}^n$, besides that, $f \in ACP_p(D)$. Let $y_0 \in \overline{f(D)} \setminus \{\infty\}$. Set*

$$K_{CT,p,y_0}(y, f) = \sum\limits_{x \in f^{-1}(y)} \frac{\left(\sup\limits_{|h|=1} \left|\left(f'(x)h, \frac{f(x)-y_0}{|f(x)-y_0|}\right)\right|\right)^p}{|J(x, f)|}\,. \qquad (3)$$

Then f satisfies the inverse Poletsky inequality 2 at y_0 for $Q_*(y) := K_{CT,p,y_0}(y, f)$.

The result mentioned above is published in [3].

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Hasse norm theorem for 3-manifolds

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Abstract:Following the analogies between knots and primes, 3-manifolds and number rings in arithmetic topology, we show a topological analogue of the Hasse norm principle for finite cyclic coverings of 3-manifolds, which was originally stated for finite cyclic extensions of number fields.

Theorem 1. *Let M be an integral homology 3-sphere endowed with a very admissible link $\mathcal{L}$. Let $f : N \to M$ be a finite cyclic covering branched over a finite sublink L_0 of $\mathcal{L}$. Then,*

$$P_{M,\mathcal{L}} \cap f_*(I_{N,f^{-1}(\mathcal{L})})) = f_*(P_{N,f^{-1}(\mathcal{L})}).$$