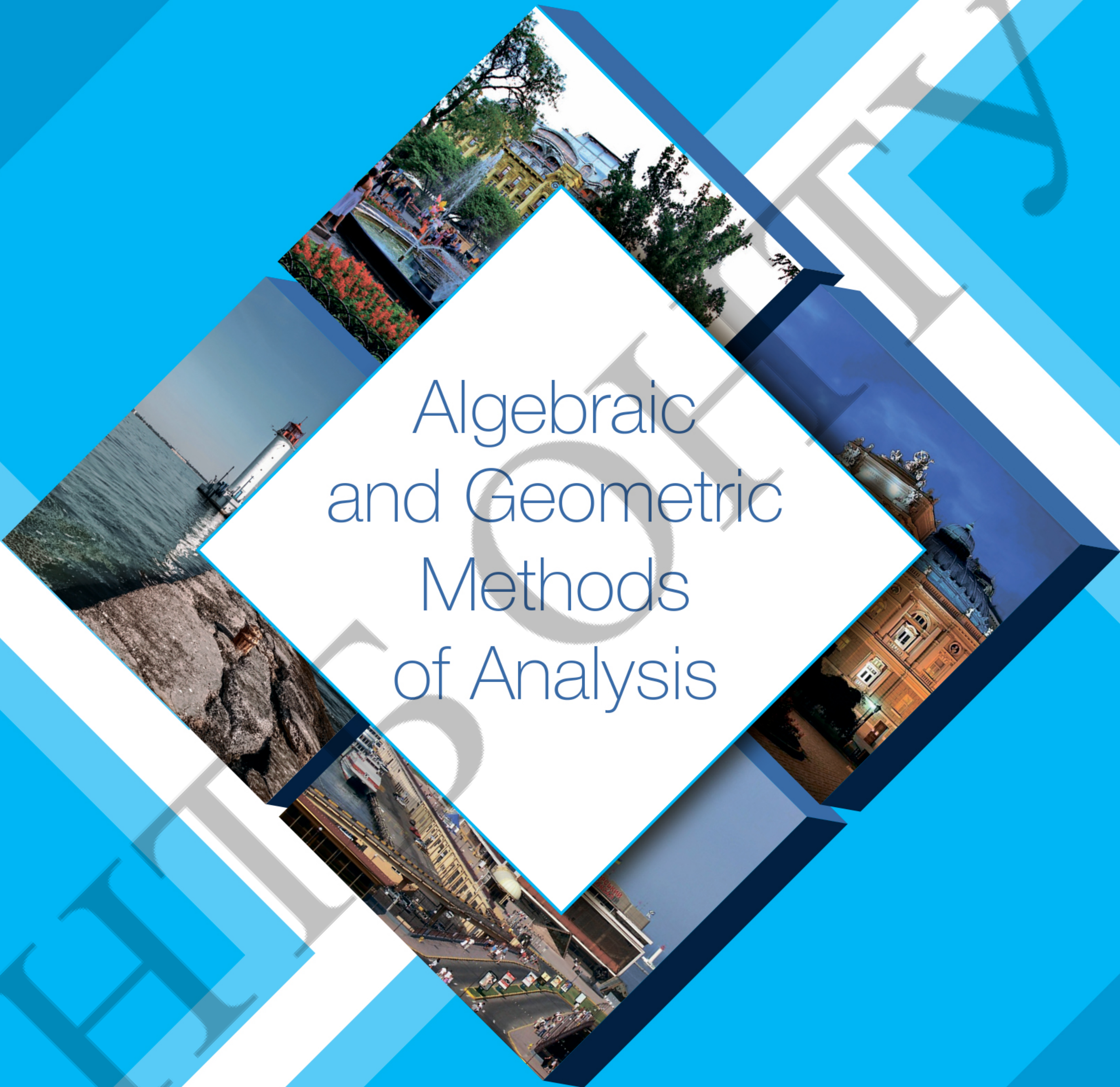
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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N-foci balls in hyperbolic geometry

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Let's suppose that $\mathbb{H}^2 = \{(x, y) \mid y > 0\}$ is an upper half-plane with the Riemannian metric $\dfrac{dx^2 + dy^2}{y^2}$. It is called a hyperbolic plane and has a constant negative Gaussian curvature -1. Besides, $\mathbb{H}^2$ is a Hadamard space, which is a complete Riemannian manifold of nonpositive sectional curvature.

Between two any points $x, y \in \mathbb{H}^2$ there is a unique geodesic $\sigma_{x,y}$. So we can define a notion of a *geodesically convex* (or just *convex*) set in hyperbolic plane — it is a set that for two arbitrary points x and y of its $\sigma_{x,y}$ belongs to this set. Particularly, the mapping

$$\rho : \mathbb{H}^2 \times \mathbb{H}^2 \to \mathbb{R}, \rho(x, y) = \ell(\sigma_{x,y}), x, y \in \mathbb{H}^2,$$

where ℓ denotes a length of curve in $\mathbb{H}^2$, satisfies all the axioms of metric space.

We also can define a notion of convex function in $\mathbb{H}^2$.

Definition 1. We will call a parametrization $\gamma : [0, 1] \to \mathbb{H}^2$ of the geodesics between points a and b in $\mathbb{H}^2$, $\gamma(0) = a$, $\gamma(1) = b$, *standard*, if for all $\alpha \in (0; 1)$ the equality

$$\rho(a, \gamma(\alpha)) = \alpha\ell$$

holds. Here ℓ denotes a length of the appropriate geodesics.

Definition 2. A function $f : \mathbb{H}^2 \to \mathbb{R}$ is called *convex* in a convex set $A \subset \mathbb{H}^2$, if for arbitrary points $x_1, x_2 \in A$ and a standard parametrization $\gamma : [0, 1] \to \mathbb{H}^2$ of the geodesics between them, $\gamma(0) = x_2$, $\gamma(1) = x_1$, next inequality holds:

$$\forall\alpha \in [0, 1] : f(\gamma(\alpha)) \leq \alpha f(x_1) + (1 - \alpha)f(x_2). \tag{1}$$

Definition 3. Let's fix in $\mathbb{H}^2$ any mutually distinct points $x_1, \ldots, x_N$, where $N \in \mathbb{N}$, and such positive numbers $w_1, \ldots, w_N, a$ that $\sum\limits_{k=1}^{N} w_k = 1$. *Open weighted N-foci ball*, or *weighted N-foci ball*, is a set

$$A = \{x \in \mathbb{H}^2 \,|\, w_1\, \rho(x, x_1) + \cdots + w_N\, \rho(x, x_N) < a\}, \tag{2}$$

where $x_1, \ldots, x_N$ are called *foci of the weighted N-foci ball*, a is called *a radius of the weighted N-foci ball*, $w_1, \ldots, w_N$ are called *weights of the foci* $x_1, \ldots, x_N$.

We can define closed weighted N-foci balls the same way, having replaced the symbol "$<$" by the symbol "$\leq$" in the formula (2).

Let's fix any point $x_0 \in \mathbb{H}^2$ and define the distance function for it:

$$f : \mathbb{H}^2 \to \mathbb{R},\ f(x) = \rho(x, x_0),\ x \in \mathbb{H}^2.$$

Theorem 4. *The distance function f is convex in the hyperbolic plane $\mathbb{H}^2$.*

It is known, that such a function is convex in any Hadamard space [2]. In this work we got a direct proof of convexity of f for the case of the hyperbolic plane.

From the convexity of f we obtain another result.

Theorem 5. *All open and closed weighted N-foci balls are geodesically convex sets in the hyperbolic plane $\mathbb{H}^2$.*

We also proved geodesical convexity of 1-foci ball, which is a hyperbolic ball, with geometrical methods.

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A retraction from the space of pseudometrics to the space of ultrapseudometrics

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Definition 1. A **pseudometric** on a set X is a function $d : X \times X \to \mathbb{R}$ that satisfies the following properties for all $x, y, z \in X$:

(1) **Non-negativity**: $d(x, y) \geq 0$.
(2) **Identity of indiscernibles**: $d(x, x) = 0$. (However, it is not required that $d(x, y) = 0$ implies $x = y$, which differentiates a pseudometric from a metric.)
(3) **Symmetry**: $d(x, y) = d(y, x)$.
(4) **Triangle inequality**: $d(x, z) \leq d(x, y) + d(y, z)$.

An **ultrapseudometric** is a type of distance function defined on a set that generalizes the notion of a metric, incorporating properties specific to ultrametrics and pseudometrics. Formally: