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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
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НТБ ОНАФТ

Finiteness of pretangent spaces at infinity

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Let (X, d) be an unbounded metric space, p be a point of X and $\tilde{r} = (r_n)_{n \in \mathbb{N}}$ be a scaling sequence of positive real numbers tending to infinity. Denote by $\tilde{X}_{\infty, \tilde{r}}$ the set of all sequences $\tilde{x} = (x_n)_{n \in \mathbb{N}} \subset X$ for each of which $\lim_{n \rightarrow \infty} d(x_n, p) = \infty$ and there is a limit $\tilde{d}_{\tilde{r}}(\tilde{x}) = \lim_{n \rightarrow \infty} \frac{d(x_n, p)}{r_n}$.

Define the equivalence relation \equiv on $\tilde{X}_{\infty, \tilde{r}}$ as

$$(\tilde{x} \equiv \tilde{y}) \Leftrightarrow \left(\lim_{n \rightarrow \infty} \frac{d(x_n, y_n)}{r_n} = 0 \right).$$

Let $\Omega_{\infty, \tilde{r}}^X$ be the set of equivalence classes generated by \equiv on $\tilde{X}_{\infty, \tilde{r}}$. We shall say that points $\alpha, \beta \in \Omega_{\infty, \tilde{r}}^X$ are *mutually stable* if for $\tilde{x} \in \alpha$ and $\tilde{y} \in \beta$ there is a finite limit

$$\rho(\alpha, \beta) := \lim_{n \rightarrow \infty} \frac{d(x_n, y_n)}{r_n}. \quad (1)$$

Let us consider the weighted graph $(G_{X, \tilde{r}}, \rho)$ with the set of vertices $V(G_{X, \tilde{r}}) = \Omega_{\infty, \tilde{r}}^X$, the edge set $E(G_{X, \tilde{r}})$ such that

$$(\{u, v\} \in E(G_{X, \tilde{r}})) \Leftrightarrow (u \text{ and } v \text{ are mutually stable and } u \neq v),$$

and the weight $\rho : E(G_{X, \tilde{r}}) \rightarrow \mathbb{R}^+$ defined by formula (1).

Definition 1. The *pretangent spaces* (to (X, d) at infinity w.r.t. \tilde{r}) are the maximal cliques of $(G_{X, \tilde{r}}, \rho)$ with metrics defined by (1).

Recall that a *clique* in a graph G is a set $A \subseteq V(G)$ such that every two distinct points of A are adjacent. A *maximal clique* in G is a clique C such that the inclusion

$$V(C) \subseteq V(A)$$

implies the equality $V(C) = V(A)$ for every clique A in G .

We study the conditions under which pretangent spaces are finite.

For natural $n \geq 2$ define the function $F_n : X^n \rightarrow \mathbb{R}$ by the rule

$$F_n(x_1, \dots, x_n) := \begin{cases} \frac{\min_{1 \leq k \leq n} d(x_k, p) \prod_{1 \leq k < l \leq n} d(x_k, x_l)}{\binom{n(n-1)}{2} + 1} & \text{if } (x_1, \dots, x_n) \neq (p, \dots, p), \\ 0 & \text{if } (x_1, \dots, x_n) = (p, \dots, p). \end{cases}$$

Theorem 2. Let (X, d) be an unbounded metric space, $p \in X$ and let $n \geq 2$. Then the inequality $|\Omega_{\infty, \tilde{r}}^X| \leq n$ holds for every pretangent space $\Omega_{\infty, \tilde{r}}^X \subseteq \Omega_{\infty, \tilde{r}}^X$ if and only if

$$\lim_{x_1, \dots, x_n \rightarrow \infty} F_n(x_1, \dots, x_n) = 0.$$

Note that for every unbounded metric space (X, d) there is a pretangent space $\Omega_{\infty, \tilde{r}}^X$ consisting at least two points.

Let $(n_k)_{k \in \mathbb{N}} \subset \mathbb{N}$ be an infinite and strictly increasing sequence. Denote by \tilde{r}' the subsequence $(r_{n_k})_{k \in \mathbb{N}}$ of the scaling sequence $\tilde{r} = (r_n)_{n \in \mathbb{N}}$ and, for every $\tilde{x} = (x_n)_{n \in \mathbb{N}} \in \tilde{\mathbf{X}}_{\infty, \tilde{r}}$, write $\tilde{x}' := (x_{n_k})_{k \in \mathbb{N}}$. It is clear that $\tilde{d}_{\tilde{r}'}(\tilde{x}') = \tilde{d}_{\tilde{r}}(\tilde{x})$ for every $\tilde{x} \in \tilde{\mathbf{X}}_{\infty, \tilde{r}}$. Moreover, if $\tilde{y} \in \tilde{\mathbf{X}}_{\infty, \tilde{r}}$ and there is $\lim_{n \rightarrow \infty} \frac{d(x_n, y_n)}{r_n}$, then

$$\lim_{k \rightarrow \infty} \frac{d(x_{n_k}, y_{n_k})}{r_{n_k}} = \lim_{n \rightarrow \infty} \frac{d(x_n, y_n)}{r_n}.$$

Consequently, the map $\pi' : \Omega_{\infty, \tilde{r}}^X \rightarrow \Omega_{\infty, \tilde{r}'}^X$ with $\pi'(\alpha) = \{(x_{n_k})_{k \in \mathbb{N}} : (x_n)_{n \in \mathbb{N}} \in \alpha\}$ is an embedding of $(G_{X, \tilde{r}}, \rho)$ in $(G_{X, \tilde{r}'}, \rho)$, i.e. if v and u are adjacent in $G_{X, \tilde{r}}$, then $\pi'(v)$ and $\pi'(u)$ are adjacent in $G_{X, \tilde{r}'}$ and $\rho(\{u, v\}) = \rho(\{\pi'(u), \pi'(v)\})$. Hence, $\pi'(C)$ is a clique in $G_{X, \tilde{r}'}$ if C is a clique in $G_{X, \tilde{r}}$.

Definition 3. A pretangent space $\Omega_{\infty, \tilde{r}}^X$ is *tangent* if $\pi'(\Omega_{\infty, \tilde{r}}^X)$ is a maximal clique in $G_{X, \tilde{r}'}$ for every infinite, strictly increasing sequence $(n_k)_{k \in \mathbb{N}} \subset \mathbb{N}$.

Theorem 2 gives us a condition under which all pretangent spaces $\Omega_{\infty, \tilde{r}}^X$ are finite. Let us consider now the problem of existence of finite tangent space.

Definition 4. Let $E \subseteq \mathbb{R}^+$. The porosity of E at infinity is the quantity

$$p(E, \infty) := \limsup_{h \rightarrow \infty} \frac{l(\infty, h, E)}{h}$$

where $l(\infty, h, E)$ is the length of the longest interval in the set $[0, h] \setminus E$. The set E is *strongly porous* at infinity if $p(E, \infty) = 1$.

The standard definition of the porosity at a point can be found in [3].

Let (X, d) be a metric space and let $p \in X$. Write $S_p(X) := \{d(x, p) : x \in X\}$.

Theorem 5. Let (X, d) be an unbounded metric space, $p \in X$. The following statements are equivalent:

- (i) The set $S_p(X)$ is strongly porous at infinity;
- (ii) There is a single-point, tangent space $\Omega_{\infty, \tilde{r}}^X$;
- (iii) There is a finite tangent space $\Omega_{\infty, \tilde{r}}^X$;
- (iv) There is a compact tangent space $\Omega_{\infty, \tilde{r}}^X$;
- (v) There is a bounded, separable tangent space $\Omega_{\infty, \tilde{r}}^X$.

Some results similar to Theorem 2 and Theorem 5 can be found in [1] and [2] respectively.

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