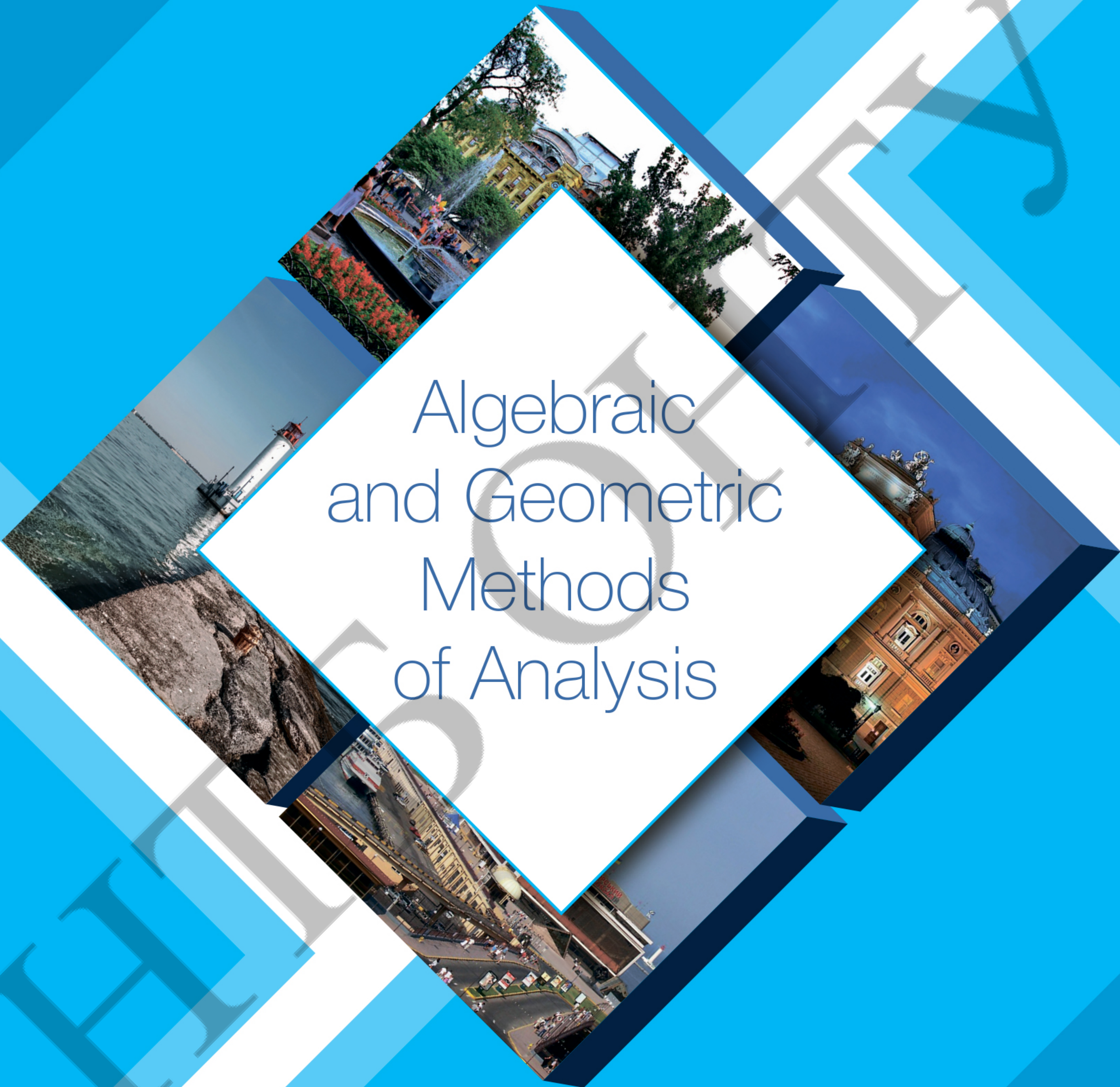
International Scientific Conference

Algebraic and Geometric Methods of Analysis



27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Disjoint dynamical properties of wedge operators

Stefan Ivković

(Mathematical Institute of the Serbian Academy of Sciences and Arts, p.p. 367, Kneza Mihaila 36, 11000 Beograd, Serbia)

E-mail: stefan.iv10@outlook.com

Let H be a separable Hilbert space and $B_0(H)$ be the C^*-algebra of compact operators on H. Given an invertible bounded operator W and a unitary operator $\mathcal{U}$ on H, we let $T_{\mathcal{U},\mathcal{W}}$ be the operator on $B_0(H)$ given by $T_{\mathcal{U},W}(F) = WF\mathcal{U}$ for all $F \in B_0(H)$. Such operators are called wedge operators. In this talk, we characterize disjoint hypercyclic finite sequences of wedge operators. We provide also sufficient conditions for a finite sequence of the dual wedge operators to be disjoint topologically transitive. Finally, we give concrete examples and applications. The talk will be based on [1].

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On semi-symmetric (α, β, γ)-inverse quasigroup

Richard Ilemobade

(Obafemi Awolowo University, Ile Ife, Nigeria)

E-mail: richardilemobade@gmail.com

Temitope Gbolahan Jaiyeola

(University of Lagos, Akoka, Nigeria and Obafemi Awolowo University, Ile Ife, Nigeria)

E-mail: tgjaiyeola@unilag.edu.ng, tjayeola@oauife.edu.ng

Quasigroups and loops are generalizations of groups (see [2, 9, 10]).

Definition 1. Let $(Q, \cdot)$ be a system of non-empty set Q and a binary operation $(\cdot)$. $(Q, \cdot)$ will be called a quasigroup if for $a, b \in Q$, the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions $(x, y) \in Q \times Q$.

Definition 2. A quasigroup $(Q, \cdot)$, in which there is a unique element $\mu \in Q$, such that $x \cdot \mu = x = \mu \cdot x \ \ \forall\, x \in Q$, is called a loop. The element μ is called the identity element in Q.

In associative algebraic systems, the notion of an inverse element or property holds significance only when the system possesses a neutral element, as seen in groups, for instance. Nevertheless, in quasigroups, the inverse property can be meaningfully established even when there is no neutral element present.

Definition 3. A quasigroup $(Q,\cdot)$ will be said to have the inverse property if there are permutations on Q: $J_\lambda : x \longrightarrow x^\lambda$ and $J_\rho : x \longrightarrow x^\rho$ such that $x^\lambda(xy) = y$ and $(yx)x^\rho = y$ for $x, y \in Q$.

Certain varieties of quasigroups or loops lack the inverse property, yet exhibit characteristics that can be viewed as variations of the inverse property.

Definition 4. A quasigroup $(Q,\cdot)$ has the cross-inverse-property (and is called a CIP quasigroup) if there exists a permutation $J : \ Q \rightarrow Q; \ x \longmapsto xJ$ such that either of the following holds: $(x \cdot y) \cdot xJ = y$ or $xJ \cdot (y \cdot x) = y$ for all $x, \ y \in Q$. If $(Q,\cdot)$ is a loop with the neutral element μ, then $J = J_\lambda$ or $J = J_\rho$ and we have a CIP loop.

This class of quasigroup and loop, and their generalizations have been studied and found to be applicable to cryptography (see [7, 3]). Among such generalizations is the m-inverse quasigroup and loop (see [4]) .

Definition 5. If there is a permutation J of elements of a quasigroup $(Q,\cdot)$ such that $\forall\ x, y \in Q$ $(x \cdot y)J^m \cdot xJ^{m+1} = yJ^m$, where m is an integer, then $(Q,\cdot)$ is called an m-inverse quasigroup. In the special case $(Q,\cdot)$ is a loop with neutral element μ and $x \cdot xJ = \mu$ for all $x \in Q$, then we have an m-inverse loop.

Another of such is the (r, s, t)-inverse quasigroup (see [1, 5, 6]) which (α, β, γ)-inverse quasigroup generalizes.

Definition 6. If there is a permutation J of elements of a quasigroup $(Q,\cdot)$ such that $\forall\ x, y \in Q$ $(x \cdot y)J^r \cdot xJ^s = yJ^t$, where r, s and t are integers, then $(Q,\cdot)$ is called an (r, s, t)-inverse quasigroup. If in addition, $(Q,\cdot)$ is a loop and the permutation J is such that $x \cdot xJ = \mu$, where μ is the neutral element in Q, then $(Q,\cdot)$ is an (r, s, t)-inverse loop.

A quasigroup $(Q,\cdot)$ will be called an (α, β, γ)-inverse quasigroup, if there exist fixed permutations α, β and γ of Q, such that $(x \cdot y)\alpha \cdot x\beta = y\gamma \ \ \forall\ (x, y) \in Q \times Q$.

Conjecture 7. *A quasigroup can have more than one triple of bijections (α, β, γ), for which the (α, β, γ)-inverse property holds.*

In this work, examples were given to illustrate that a quasigroup can have more than one (α, β, γ)-inverse property.

Definition 8. Let $(Q,\cdot)$ be a quasigroup. Define the set Δ_Q as follows:

$$\Delta_Q := \{\omega = \langle \alpha, \beta, \gamma \rangle \colon (x \cdot y)\alpha \cdot x\beta = y\gamma, \ x, y \in Q\}$$

where α, β, and γ are permutations of Q.

Definition 9. A quasigroup $(Q,\cdot)$ is said to be semi-symmetric if it satisfies the identity $(x \cdot y) \cdot x = y$ for all $x, y \in Q$.

For non-empty set Δ_Q of quasigroup $(Q,\cdot)$, it was shown that if the semi-symmetry law holds in $(Q,\cdot)$, it induces a binary operation on Δ_Q for which Δ_Q is a group.

Theorem 10. *Let $(Q,\cdot)$ be an (α, β, γ)-inverse quasigroup. If $(Q,\cdot)$ is semi-symmetric, then there exists a binary operation $\otimes$ on Δ_Q, such that $(\Delta_Q, \otimes)$ is a group.*

Conjecture 11. *There is relationship between Δ_Q and the autotopism group $ATP(Q)$, for a quasigroup $(Q,\cdot)$.*

Interestingly, this relation is actually an isomorphism between Δ_Q and the autotopism group of $(Q,\cdot)$.

Theorem 12. *For an (α,β,γ)-inverse quasigroup $(Q,\cdot)$ that is semi-symmetric, $(\Delta_Q,\otimes)$ and $ATP(Q)$ are isomorphic i.e $\Delta_Q \cong ATP(Q)$.*

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About Rolewicz theorem on inversion of continuous bijection between F-spaces

Olena Karlova

(Yurii Fedkovych Chernivtsi National University, Chernivtsi, Ukraine)

E-mail: o.karlova@chnu.edu.ua

Well-known result of Stefan Banach states that if X and Y are F-spaces and $f : X \to Y$ is a bijective additive continuous mapping, then the inverse mapping $f^{-1} : Y \to X$ is continuous. In general case the inverse mapping can be everywhere discontinuous.

In article [1] Stefan Rolewicz presented sufficient conditions on spaces X and Y under which the inverse mapping to a continuous bijection belongs to the first Baire class.

Theorem 1 (Rolewicz, 1958)**.** *Let X, Y be F-spaces and let X be separable locally compact. Then for every continuous bijection $f : X \to Y$ the inverse mapping $f^{-1} : Y \to X$ is Baire 1.*

The aim of this talk is a discussion of possible generalizations of the above mentioned result of Rolewicz on spaces X which are not linear. In order to do this we introduce a notion of weak Rolewicz space and prove the auxiliary fact about uniform limit of Baire 1 functions which is of self contained interest and extends corresponding results from [2].