



International
Scientific Conference



Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Obviously, the covariant derivative in (3) is compatible with the Riemannian metric g . In this case the metric is referred to as a hyper-Kähler one.

In a hyper-Kähler manifold, let us consider a curve $x(t)$ satisfying differential equations:

$$\frac{d^2x^h}{dt^2} + \Gamma_{ij}^h \frac{dx^i}{dt} \frac{dx^j}{dt} = \alpha(t) \frac{dx^h}{dt} + \beta(t) F_i^h \frac{dx^i}{dt} + \gamma(t) G_i^h \frac{dx^i}{dt} + \delta(t) H_i^h \frac{dx^i}{dt},$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\delta(t)$ are certain functions of the parameter t , the symbol Γ_{ij}^h denotes connection compatible with the Riemannian metric g . We call such a curve a *hyper-holomorphically planar curve (HHP-curve)*. The HHP-curves are a generalization of holomorphically planar curves [1].

Suppose two hyper-Kähler manifolds (M^n, g, F, G, H) and $(\bar{M}^n, \bar{g}, F, G, H)$ are given and the defined triple of the affinors F, G, H is the same in both manifolds.

A mapping $\pi : (M^n, g, F, G, H) \rightarrow (\bar{M}^n, \bar{g}, F, G, H)$ is an *hyper-holomorphically projective mapping (HHP-mapping)* if any HHP-curve of (M^n, g, F, G, H) is mapped under π onto an HHP-curve in $(\bar{M}^n, \bar{g}, F, G, H)$.

Theorem 2. *If two hyper-Kähler manifolds (M^n, g, F, G, H) and $(\bar{M}^n, \bar{g}, F, G, H)$ are in hyper-holomorphically projective correspondence, then their Levi-Civita connections related to each other as*

$$\bar{\Gamma}_{ij}^h = \Gamma_{ij}^h + \psi_{(i} \delta_{j)}^h - \psi_\alpha F_{(i}^\alpha F_{j)}^h - \psi_\alpha G_{(i}^\alpha G_{j)}^h - \psi_\alpha H_{(i}^\alpha H_{j)}^h,$$

where ψ_i is some gradient vector.

Theorem 3. *Let a hyper-Kähler manifold (M^n, g, F, G, H) admit HHP-mappings. Then the object*

$$\bar{T}_{ij}^h = \Gamma_{ij}^h - \frac{1}{n+4} (\Gamma_{\alpha(i}^\alpha \delta_{j)}^h - \Gamma_{\alpha\beta}^\alpha F_{(i}^\beta F_{j)}^h - \Gamma_{\alpha\beta}^\alpha G_{(i}^\beta G_{j)}^h - \Gamma_{\alpha\beta}^\alpha H_{(i}^\beta H_{j)}^h)$$

is invariant under any HHP-mapping.

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On a problem of Fejes Toth

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Let P be any convex n -gon in the plane with sides $A_j, j = 1, \dots, n$ of lengths a_j . Denote by b_j the length of the longest chord parallel to the side A_j . Fejes Tóth conjectured that $\sum_{j=1}^n \frac{a_j}{b_j} \geq 3$, with equality only for a snub triangle obtained by cutting off three congruent triangles from the corners of a triangle. This question appears as B7 in the *Unsolved Problems*

in *Geometry* by H. T. Croft, K. J. Falconer and R. K. Guy. We will present F. Nazarov's proof of Tóth's inequality and discuss its higher-dimensional analogues.

Gottlieb groups of some Moore spaces

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In this work, we present some computations of Gottlieb groups of Moore spaces $M(A, n)$ for some classes of finitely generated abelian groups A .

Given $m \geq 1$, recall that the m -th *Gottlieb group* $G_m(X)$ of a space X has been defined in [4, 5] as the subgroup of the homotopy group $\pi_m(X)$ consisting of all elements which can be represented by a map $f: \mathbb{S}^m \rightarrow X$ such that $f \vee \iota_X: \mathbb{S}^m \vee X \rightarrow X$ extends (up to homotopy) to a map $F: \mathbb{S}^m \times X \rightarrow X$. Notice that $\alpha \in G_m(\Sigma X)$ if and only if the generalized Whitehead product $[\alpha, \iota_{\Sigma X}] = 0$ (see [1, Proposition 5.1]).

First, we recall from [5, Theorems 5.2 and 5.4]:

Theorem 1. *Let A be a finitely generated abelian group and $n \geq 3$. Then,*

$$G_n(M(A, n)) = \begin{cases} 0, & \text{if } n \text{ is even,} \\ 0, & \text{if } n \text{ is odd and } \text{rk}(A) \neq 1, \\ 2\mathbb{Z} \subseteq \mathbb{Z} = \pi_n(\mathbb{S}^n), & \text{if } n \neq 1, 3, 7 \text{ is odd and } A = \mathbb{Z}, \\ \mathbb{Z} = \pi_n(\mathbb{S}^n), & \text{if } n = 1, 3, 7 \text{ and } A = \mathbb{Z} \end{cases}$$

We point out that the result above has been stated also in [2] for $n \geq 3$. In addition, [2, Corollary 4.4] claims that if n is odd, then $G_n(M(\mathbb{Z} \oplus T, n))$ is infinite cyclic, where T is a finite abelian group.

As stated in [2, Remark 4.5], it would be interesting to compute other Gottlieb groups for some Moore spaces, such as $G_{n+1}(M(A, n))$. We will do this for a finitely generated abelian group A which its torsion subgroup has order 2 (mod 4). We notice that on [3, Chapter 3] there are some results on $G_{n+1}(M(A, n))$ only for A having torsion subgroup with odd order.

Our main result is:

Theorem 2. *Let A be a finite abelian group with order $|A| \equiv 2 \pmod{4}$. Then $G_{n+1}(M(\mathbb{Z} \oplus A, n)) = 0$, for $n \geq 3$, and $G_{n+2}(M(\mathbb{Z} \oplus A, n)) = 0$, for $n \geq 4$.*

Furthermore, investigations of $G_{n+k}(M(\mathbb{Z} \oplus A, n))$ for $k = 3, 4, 5$ and A as above, is planned as well.

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