International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Obviously, the covariant derivative in (3) is compatible with the Riemannian metric g. In this case the metric is referred to as a hyper-Kähler one.

In a hyper-Kähler manifold, let us consider a curve $x(t)$ satisfying differential equations:

$$\frac{d^2x^h}{dt^2}+\Gamma^h_{ij}\frac{dx^i}{dt}\frac{dx^i}{dt}=\alpha(t)\frac{dx^h}{dt}+\beta(t)F^h_i\frac{dx^i}{dt}+\gamma(t)G^h_i\frac{dx^i}{dt}+\delta(t)H^h_i\frac{dx^i}{dt},$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\delta(t)$ are certain functions of the parameter t, the symbol Γ^h_{ij} denotes connection compatible with the Riemannian metric g. We call such a curve a *hyper-holomorphically planar curve (HHP-curve).* The HHP-curves are a generalization of holomorphically planar curves [1].

Suppose two hyper-Kähler manifolds (M^n, g, F, G, H) and $(\overline{M}^n, \overline{g}, F, G, H)$ are given and the defined triple of the affinors F, G, H is the same in both manifolds.

A mapping $\pi : (M^n, g, F, G, H) \to (\overline{M}^n, \overline{g}, F, G, H)$ is an *hyper-holomorphically projective mapping (HHP-mapping)* if any HHP-curve of (M^n, g, F, G, H) is mapped under π onto an HHP-curve in $(\overline{M}^n, \overline{g}, F, G, H)$.

Theorem 2. *If two hyper-Kähler manifolds (M^n, g, F, G, H) and $(\overline{M}^n, \overline{g}, F, G, H)$ are in hyper-holomorphically projective correspondence, then their Levi-Civita connections related to each other as*

$$\overline{\Gamma}^h_{ij}=\Gamma^h_{ij}+\psi_{(i}\delta^h_{j)}-\psi_\alpha F^\alpha_{(i}F^h_{j)}-\psi_\alpha G^\alpha_{(i}G^h_{j)}-\psi_\alpha H^\alpha_{(i}H^h_{j)},$$

where ψ_i is some gradient vector.

Theorem 3. *Let a hyper-Kähler manifold (M^n, g, F, G, H) admit HHP-mappings. Then the object*

$$\overline{T}^h_{ij}=\Gamma^h_{ij}-\frac{1}{n+4}\left(\Gamma^\alpha_{\alpha(i}\delta^h_{j)}-\Gamma^\alpha_{\alpha\beta}F^\beta_{(i}F^h_{j)}-\Gamma^\alpha_{\alpha\beta}G^\beta_{(i}G^h_{j)}-\Gamma^\alpha_{\alpha\beta}H^\beta_{(i}H^h_{j)}\right)$$

is invariant under any HHP-mapping.

References

[1] Yano K. *Differential geometry on complex and almost complex spaces /K. Yano* –New York: Pergamon Press Book –New York: 326p. 1965.

[2] Calabi E. Métriques kählériennes et fibrés holomorphes. *Ann. Éc. Norm. Sup.* 1979, No. 12, p. 269-294.

[3] Hitchin N. Hyper-Kähler manifolds. *Séminaire Bourbaki, Astérisque* 1992, vol. 206, No. 748, 3, p. 137-166.

On a problem of Fejes Toth

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Let P be any convex n-gon in the plane with sides $A_j, j = 1, \ldots, n$ of lengths a_j. Denote by b_j the length of the longest chord parallel to the side A_j. Fejes Tóth conjectured that $\sum_{j=1}^{n} \frac{a_j}{b_j} \geq 3$, with equality only for a snub triangle obtained by cutting off three congruent triangles from the corners of a triangle. This question appears as B7 in the *Unsolved Problems* *in Geometry* by H. T. Croft, K. J. Falconer and R. K. Guy. We will present F. Nazarov's proof of Tóth's inequality and discuss its higher-dimensional analogues.

Gottlieb groups of some Moore spaces

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In this work, we present some computations of Gottlieb groups of Moore spaces $M(A,n)$ for some classes of finitely generated abelian groups A.

Given $m \geq 1$, recall that the m-th *Gottlieb group* $G_m(X)$ of a space X has been defined in [4, 5] as the subgroup of the homotopy group $\pi_m(X)$ consisting of all elements which can be represented by a map $f\colon \mathbb{S}^m \to X$ such that $f \vee \iota_X \colon \mathbb{S}^m \vee X \to X$ extends (up to homotopy) to a map $F\colon \mathbb{S}^m \times X \to X$. Notice that $\alpha \in G_m(\Sigma X)$ if and only if the generalized Whitehead product $[\alpha, \iota_{\Sigma X}] = 0$ (see [1, Proposition 5.1]).

First, we recall from [5, Theorems 5.2 and 5.4]:

Theorem 1. *Let A be a finitely generated abelian group and $n \geqslant 3$. Then,*

$$G_n(M(A,n)) = \begin{cases} 0, & \text{if } n \text{ is even,} \\ 0, & \text{if } n \text{ is odd and } \mathrm{rk}(A) \neq 1, \\ 2\mathbb{Z} \subseteq \mathbb{Z} = \pi_n(\mathbb{S}^n), & \text{if } n \neq 1,3,7 \text{ is odd and } A = \mathbb{Z}, \\ \mathbb{Z} = \pi_n(\mathbb{S}^n), & \text{if } n = 1,3,7 \text{ and } A = \mathbb{Z} \end{cases}$$

We point out that the result above has been stated also in [2] for $n \geqslant 3$. In addition, [2, Corollary 4.4] claims that if n is odd, then $G_n(M(\mathbb{Z} \oplus T, n))$ is infinite cyclic, where T is a finite abelian group.

As stated in [2, Remark 4.5], it would be interesting to compute other Gottlieb groups for some Moore spaces, such as $G_{n+1}(M(A,n))$. We will do this for a finitely generated abelian group A which its torsion subgroup has order 2 (mod 4). We notice that on [3, Chapter 3] there are some results on $G_{n+1}(M(A,n))$ only for A having torsion subgroup with odd order.

Our main result is:

Theorem 2. *Let A be a finite abelian group with order $|A| \equiv 2 \pmod 4$. Then $G_{n+1}(M(\mathbb{Z} \oplus A, n)) = 0$, for $n \geq 3$, and $G_{n+2}(M(\mathbb{Z} \oplus A, n)) = 0$, for $n \geq 4$.*

Furthermore, investigations of $G_{n+k}(M(\mathbb{Z} \oplus A, n))$ for $k = 3,4,5$ and A as above, is planned as well.

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