

International scientific conference
**«Algebraic and geometric
methods of analysis»**

Book of abstracts



May 30 - June 4, 2018,
Odesa,
Ukraine

<https://www.imath.kiev.ua/~topology/conf/agma2018>

Hyperbolic quasiperiodic motion of charged particle on 2-sphere

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Let \mathbb{E}^3 be 3-D Euclidean space endowed with a scalar product $\langle \cdot, \cdot \rangle$ and cross-product $\cdot \times \cdot$, and let $\iota : \mathbb{S}^2 \hookrightarrow \mathbb{E}^3$ stands for the inclusion map of 2-D sphere \mathbb{S}^2 into \mathbb{E}^3 : $\iota(\mathbb{S}^2) := \{ \mathbf{x} \in \mathbb{E}^3 : \|\mathbf{x}\|^2 := \langle \mathbf{x}, \mathbf{x} \rangle = 1 \}$.

We consider a partial case of Newton equation

$$\nabla_{\dot{x}} \dot{x} = f(t\omega, x) + P(t\omega, x)\dot{x} \quad (1)$$

that governs the motion of quasiperiodically excited particle on \mathbb{S}^2 . Here ∇ stands for the Levi-Civita connection of naturally induced Riemannian metric on \mathbb{S}^2 , $\{f(\varphi, \cdot)\}_{\varphi \in \mathbb{T}^k}$ is a smooth family of vector fields on \mathbb{S}^2 parametrized by points of the standard k -dimensional torus $\mathbb{T}^k := \mathbb{R}^k/2\pi\mathbb{Z}^k$, $\{P(\varphi, \cdot)\}_{\varphi \in \mathbb{T}^k}$ is a smooth family of $(1, 1)$ -tensor fields, and $\omega \in \mathbb{R}^k$ is the basic frequency vector with rationally independent components. For Eq. (1), there naturally arise the problem of quasiperiodic response, i.e. the existence problem for ω -quasiperiodic solution $t \mapsto x(t) := u(t\omega)$ associated with a continuous mapping $u(\cdot): \mathbb{T}^k \mapsto \mathbb{S}^2$. Such a solution is said to be hyperbolic if the corresponding system in variations

$$\begin{aligned} \nabla_{\dot{x}(t)} \eta &= \zeta \\ \nabla_{\dot{x}(t)} \zeta &= [\nabla f(t\omega, x)\eta - R(\eta, \dot{x})\dot{x} + \nabla P(t\omega, x)(\eta, \dot{x}) + P(t\omega, x)\zeta]_{x=x(t)}, \end{aligned}$$

where R is the Riemann curvature tensor, is exponentially dichotomic.

We consider the case where the charged particle of unit mass is constrained to move on $\iota(\mathbb{S}^2) := \{ \mathbf{x} \in \mathbb{E}^3 : \|\mathbf{x}\|^2 = 1 \}$ by the applied force Φ represented in the form

$$\Phi(t\omega, \mathbf{x}, \dot{\mathbf{x}}) = -\frac{\mathbf{x} - \mathbf{a}}{\|\mathbf{x} - \mathbf{a}\|^3} + \mathbf{E}(t\omega) + \dot{\mathbf{x}} \times \mathbf{B}(t\omega).$$

Here $\mathbf{a} \in \mathbb{E}^3$ is a constant vector with norm $a := \|\mathbf{a}\|$; $\mathbf{E}(\cdot) : \mathbb{T}^k \mapsto \mathbb{E}^3$ and $\mathbf{B}(\cdot) : \mathbb{T}^k \mapsto \mathbb{E}^3$ are smooth mappings. The force Φ can be naturally interpreted as the superposition of the Coulomb force caused by a charge placed at point \mathbf{a} and the Lorentz force caused by the electric field \mathbf{E} and the magnetic field \mathbf{B} . Let ι_* stands for the derivative of the inclusion map. In the case under consideration, the forces affecting the motion of the constrained particle are

$$\begin{aligned} \iota_* f(t\omega, x) &= \mathbf{f}(t\omega, \mathbf{x}) = -\frac{\mathbf{x} + a\mathbf{k}}{\|\mathbf{x} + a\mathbf{k}\|^3} + \mathbf{E}(t\omega) + \left\langle \frac{\mathbf{x} + a\mathbf{k}}{\|\mathbf{x} + a\mathbf{k}\|^3} - \mathbf{E}(t\omega), \mathbf{x} \right\rangle \mathbf{x}, \\ \iota_* P(t\omega, x)\dot{x} &= \dot{\mathbf{x}} \times \mathbf{B}(t\omega) - \langle \dot{\mathbf{x}} \times \mathbf{B}(t\omega), \mathbf{x} \rangle \mathbf{x}. \end{aligned}$$

where $\mathbf{x} := \iota(x)$, $\dot{\mathbf{x}} := \iota_* \dot{x}$, $\mathbf{k} := -\mathbf{a}/a$. First consider the case where the influence of magnetic field can be neglected.

Theorem 1. *Let $\mathbf{B}(\varphi) \equiv 0$. If there holds the inequality*

$$\frac{a}{(1+a)^3} - \langle \mathbf{E}(\varphi), \mathbf{k} \rangle > 0 \quad \forall \varphi \in \mathbb{T}^k \quad (1)$$

and there exists a point $\varphi_0 \in \mathbb{T}^k$ such that $\mathbf{E}(\varphi_0) \not\parallel \mathbf{k}$, then the system of charged particle on \mathbb{S}^2 has a unique ω -quasiperiodic solution $t \mapsto x(t)$ such that $0 < \langle \mathbf{x}(t), \mathbf{k} \rangle \leq 1$ for all $t \in \mathbb{R}$ where $\mathbf{x}(t) := \iota \circ x(t)$. This solution is hyperbolic.

This theorem is obtained by applying results of [1]. We essentially use the so-called U -monotonicity property of the system $\nabla_{\dot{x}}\dot{x} = f$ in the hemisphere $S^+ := \{x \in \mathbb{S}^2 : 0 < \langle \iota(x), \mathbf{k} \rangle \leq 1\}$.

Namely, to ensure such a property we have constructed a function $U(\cdot) \in C^\infty(S^+ \mapsto \mathbb{R})$ satisfying the conditions

$$\lambda_f(\varphi, x) + \frac{\langle \nabla U(x), f(\varphi, x) \rangle}{2} > 0 \quad , \quad \mu_U(x) \geq 2 \quad \forall (\varphi, x) \in \mathbb{T}^k \times S^+$$

where

$$\lambda_f(\varphi, x) := \min_{\eta \in T_x \mathbb{S}^2} \left\{ \frac{\langle \nabla f(\varphi, x)\eta, \eta \rangle}{\|\eta\|^2} \right\},$$

$$\mu_U(x) := \min_{\eta \in T_x \mathbb{S}^2} \left\{ \frac{\langle \nabla_\eta \nabla U(x), \eta \rangle}{\|\eta\|^2} - \frac{\langle \nabla U(x), \eta \rangle^2}{2\|\eta\|^2} \right\}.$$

When $\mathbf{B}(\varphi) \neq 0$, we restrict ourselves to the case where $\langle \mathbf{E}(\varphi), \mathbf{k} \rangle = 0$ and $\mathbf{B}(\varphi) \perp \mathbf{E}(\varphi)$. We show how to establish sufficient condition for the existence of hyperbolic ω -quasiperiodic solution in the domain $\{x \in \mathbb{S}^2 : 0.5 < \langle \iota(x), \mathbf{k} \rangle \leq 1\}$. Set

$$E := \max_{\varphi \in \mathbb{T}^k} \|\mathbf{E}(\varphi)\|, \quad B := \max_{\varphi \in \mathbb{T}^k} \|\mathbf{B}(\varphi)\|$$

Define $z_+ = z_+(B, E)$ and $z_* = z_*(B, E)$, respectively, as the greatest roots of the equations

$$J(z) := z^2 - \frac{B^2}{4}z - \sqrt{3}(E+1) = 0,$$

$$I(z) := \frac{z^3}{3} - \frac{B^2}{8}z^2 - \sqrt{3}(E+1)z = I(z_+) + \sqrt{3}(E+1)z_+.$$

It turns out that the sought sufficient condition take the form

$$\frac{4a}{(1+a)^3} > \max \left\{ 9B^2z_*, \frac{4E}{\sqrt{3}} + B^2 \right\}.$$

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