



International
Scientific Conference



Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Normal subgroups of iterated wreath products of symmetric groups and alternating with symmetric groups

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In this research we continue our previous investigation of wreath product normal structure [1].

The lattice of normal subgroups and their properties for finite iterated wreath products $S_{n_1} \wr \dots \wr S_{n_m}$, $n, m \in \mathbb{N}$ are found. Special classes of normal subgroups and their orders and generators are found. Further, the monolith of these wreath products is investigated by us.

Let $k(\pi)$ be the number of cycles in decomposition of permutation π of degree n .

The number $n - k(\pi)$ is denoted by $dec(\pi)$, and is called a decrement [2] of permutation π .

As well known [2] the minimal number of transpositions in factorization of a permutation π on transpositions is happen to be equal to $dec(\pi)$. We set $dec(e) = 0$. Therefore the decrement of n -cycle is $n - 1$.

If $\pi_1, \pi_2 \in S_n$, then the following formula holds:

$$dec(\pi_1 \cdot \pi_2) = dec(\pi_1) + dec(\pi_2) - 2m, m \in \mathbb{N}, \quad (1)$$

where m is number of joint simplifying transpositions in π_1 and π_2 .

The trivial subgroup of S_n we denote by E .

Definition 1. The set of elements from $S_n \wr S_n$, $n \geq 5$ or $n = 3$ of the tableaux form: $[e]_1, [a_1, a_2, \dots, a_n]_2$, satisfying the following condition

$$\sum_{i=1}^n dec([a_i]_2) = 2k, k \in \mathbb{N}, \quad (2)$$

we will call set of type $\tilde{A}^{(2)}$ and denote this set by $E \wr \tilde{A}_n$. For brevity of notation this subgroup be also denoted by $\tilde{A}_n^{(2)}$. It follows directly from the definition that the set of these elements

supplemented by the operation of multiplication in the subdirect product, coincides with the group $E \times \underbrace{(S_n \boxtimes S_n \boxtimes S_n \dots \boxtimes S_n)}_n$, where subdirect product satisfies to condition (2).

We remind that the intersection of all non-trivial normal subgroups $Mon(G)$ of G is called the monolith of a group G .

Proposition 2. *Elements of first type form the subgroup $e \setminus A_n$. This subgroup is the **monolith** of $S_n \wr S_n$.*

Now we can recursively construct easiest and elegant subgroup $E \setminus \tilde{A}_n^{(2)}$ of $S_n \wr S_n \wr S_n$.

Definition 3. The subgroup $E \setminus \tilde{A}_n^{(2)}$ be denoted by $\tilde{A}_n^{(3)}$.

The order of $E \setminus \tilde{A}_n^{(2)}$ is $(n!)^{3n} : 2^3$. Furthermore we prove that $E \setminus \tilde{A}_n^{(2)} \triangleleft S_n \wr S_n \wr S_n$. Let the set of elements from $S_n \wr S_n \wr S_n, n \geq 3$ of the form:

$$[e]_1, [e, e, \dots, e]_2, [a_1, a_2, \dots, a_{n^2}]_3$$

satisfying the following condition

$$\sum_{i=1}^{n^2} dec([a_i]_3) = 2k, k \in \mathbb{N}, \quad (3)$$

be denoted by $\tilde{A}_{n^2}^{(3)}$.

Proposition 4. *The set of elements of type $\tilde{A}_{n^2}^{(3)}$ forms a subgroup in $S_n \wr S_n \wr S_n$. Moreover $\tilde{A}_{n^2}^{(3)} \triangleleft S_n \wr S_n \wr S_n$.*

Remark 5. We note that $\tilde{A}_n^{(3)} < \tilde{A}_{n^2}^{(3)}$. The order of $\tilde{A}_{n^2}^{(3)}$ is $(n!)^{n^2} : 2$. Furthermore $\tilde{A}_n^{(3)} \triangleleft S_n \wr S_n$.

Definition 6. A subgroup in $S_n \wr S_n$ is called \tilde{T}_n if it consists of:

- 1) elements of $E \setminus A_n$,
- 2) elements with the tableau [3] presentation $[e]_1, [\pi_1, \dots, \pi_n]_2$, that $\pi_i \in S_n \setminus A_n$.

One easy can validates a correctness of this definition, i.e. that the set of such elements form a subgroup and its normality. This subgroup has structure $\tilde{T}_n \simeq \underbrace{(A_n \times A_n \times \dots \times A_n)}_n \times C_2 \simeq \underbrace{S_n \boxplus S_n \dots \boxplus S_n}_n$, where the operation of a subdirect product \boxplus is subject to items 1) and 2).

Definition 7. A subgroup in $S_n \wr S_n \wr S_n$ is of the type $\tilde{T}_{n^2}^{(3)}$ if it consists of:

- 1) elements of the form $E \setminus E \setminus A_n$,
- 2) elements with the tableau [3] presentation $[e]_1, [e \dots, e]_2, [\pi_1 \dots, \pi_n, \pi_{n+1} \dots, \pi_{n^2}]_3$, wherein $\forall i = 1, \dots, n: \pi_i \in S_n \setminus A_n$.

We define recursively the subgroup $\tilde{T}_n^{(3)}$ having n different intervals of elements with the same parity permutations on X^2 .

Definition 8. The subgroup of $S_n \wr S_n \wr S_n$ having structure $E \wr \widetilde{T}_n^{(3)}$ is denoted by $\widetilde{T}_n^{(3)}$. The following isomorphism $\widetilde{T}_n^{(3)} \simeq \underbrace{S_n \boxplus S_n \dots \boxplus S_n}_n \times \underbrace{S_n \boxplus S_n \dots \boxplus S_n}_n \times \dots \times \underbrace{S_n \boxplus S_n \dots \boxplus S_n}_n$, where a tuple $S_n \boxplus S_n \dots \boxplus S_n$ repeats n times, holds. The operation of a subdirect product \boxplus is determined by Definition 6.

The operation \boxplus accords with the properties described in item 1 and 2 of Definition 6, also \boxplus is determined by automorphism in $\widetilde{T}_n \simeq \underbrace{(A_n \times A_n \times \dots \times A_n)}_n \rtimes C_2$ in this case.

Remark 9. Note that in $\widetilde{T}_n^{(3)}$ vertex permutation of tableau third part satisfy the condition: elements with the tableau presentation $[e]_1, [e \dots, e]_2, [\pi_1 \dots, \pi_n; \pi_{n+1} \dots, \pi_{n^2}]_3$, that either all $\pi_i \in S_n \setminus A_n$ or all $[\pi_i]_3 \in A_n$ for $1 < i \leq n, n+1 \leq i < 2n, \dots, n^2 - n < i \leq n^2$.

Here are the names of (almost all) predefined theorem-like environments.

Proposition 10. *The subgroup $E \wr A_n$ is the **monolith** of $S_n \wr S_n$.*

We call level of $AutX^*$ as active if it has at least one non-trivial permutation. Denote by $Aut_f X^*$ the group of all finite automorphism of spherically homogeneous rooted tree.

Proposition 11. *Let $H \triangleleft Aut_f X^*$ with depth k then H contains k -th level subgroup P having all even vertex permutations $p_{ki} \in A_n$ on X^k and trivial permutations in vertices of rest of levels. Furthermore P is normal in W provided k is last active level of $Aut_f X^*$.*

Theorem 12. *Proper normal subgroups in $S_n \wr S_m$ (action of group is left), where $n, m \geq 3$ with $n, m \neq 4$ are of the following types:*

- 1) the subgroups of the first level stabilizer $[1, 4]$ are

$$E \wr \widetilde{A}_m, \widetilde{T}_m, E \wr S_m, E \wr A_n,$$

- 2) the subgroups that act on both levels are $A_n \wr \widetilde{A}_m, S_n \wr \widetilde{A}_m, A_n \wr S_m,$

wherein the subgroup $S_n \wr \widetilde{A}_m \simeq S_n \ltimes \underbrace{(S_m \boxtimes S_m \boxtimes S_m \dots \boxtimes S_m)}_n$ endowed with the subdirect product [4] satisfying to condition (3), moreover $S_n \wr \widetilde{A}_m$ has two isomorphic copies, embedded into $S_n \wr S_m$ in different ways.

In total there are 8 proper normal subgroups in $S_n \wr S_m$.

Proposition 13. *All normal subgroups of $S_n \wr (S_m \times S_k)$ can be partitioned in 2 types:*

- 1) $E \wr (N_i \times N_j)$, where $N_i \triangleleft \prod_{k=1}^n S_m^{(k)}$ and $N_j \triangleleft \prod_{l=1}^n S_l^{(l)}$.
- 2) $\widetilde{A}_i \wr (N_i \times N_j)$, where $\widetilde{A}_i \triangleleft S_n$, N_i and N_j are subgroups from item 1) possessing an extension by \widetilde{A}_i in a correspondent groups $S_n \wr S_m$ and in $S_n \wr S_k$. The full list of them: $S_n \wr (S_m \times \widetilde{A}_k), S_n \wr (\widetilde{A}_m \times \widetilde{A}_k), S_n \wr (\widetilde{A}_m \times S_k)$, also $A_n \wr (S_m \times \widetilde{A}_k), A_n \wr (\widetilde{A}_m \times \widetilde{A}_k), A_n \wr (\widetilde{A}_m \times S_k)$.

We denote the set of normal subgroup of $S_n \wr S_n$ by $N(S_n \wr S_n)$. Subgroup with number i from $N(S_n \wr S_n)$ is denoted by $N_i(S_n \wr S_n)$.

Theorem 14. *The full list of normal subgroups of $S_n \wr S_n \wr S_n$ consists of 50 normal subgroups. These subgroups are the following:*

- 1) **Type** T_{023} contains: $E \wr \tilde{A}_n \wr H$, $\tilde{T}_n \wr H$, where $H \in \{\tilde{A}_n, \tilde{A}_{n^2}, S_n\}$. There are 6 subgroups.
- 2) **The second type of subgroups is subclass in** T_{023} with new base of wreath product subgroup \tilde{A}_{n^2} : $E \wr S_n \wr \tilde{A}_{n^2}$, $E \wr A_n \wr \tilde{A}_{n^2}$, $E \wr N_i(S_n \wr S_n)$. Therefore this class has 12 new subgroups. Thus, the total number of normal subgroups in **Type** T_{023} is 18.
- 3) **Type** T_{003} : $A_{00(n^2)}^{(3)} = E \wr E \wr \tilde{A}_{n^2}$, \tilde{T}_{n^2} , $\tilde{T}_n^{(3)}$. Hence, here are 3 new subgroups.
- 4) **Type** T_{123} : $N_i(S_n \wr S_n) \wr S_n$, $N_i(S_n \wr S_n) \wr \tilde{A}_n$ and $N_i(S_n \wr S_n) \wr \tilde{A}_{n^2}$. Thus, there are 29 new normal subgroups in T_{123} , taking into account repetition [5].

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Asymptotic behavior of the widths of classes of the generalized Poisson integrals

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Let L_p , $1 \leq p \leq \infty$, and C be the spaces of 2π -periodic functions with standard norms $\|\cdot\|_{L_p}$ and $\|\cdot\|_C$, respectively.

Denote by $C_{\beta, p}^\psi$, $1 \leq p \leq \infty$, the set of all 2π -periodic functions f , representable as convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(x-t) \Psi_{\bar{\beta}}(t) dt, \quad a_0 \in \mathbb{R}, \quad \varphi \in B_p^0 = \{g \in L_p : \|g\|_p \leq 1, g \perp 1\}, \quad (1)$$

with a fixed generated kernel $\Psi_{\bar{\beta}} \in L_{p'}$, $1/p + 1/p' = 1$, the Fourier series of which has the form

$$S[\Psi_{\bar{\beta}}](t) = \sum_{k=1}^{\infty} \psi(k) \cos\left(kt - \frac{\beta_k \pi}{2}\right), \quad \beta_k \in \mathbb{R}, \quad \psi(k) \geq 0.$$

A function f in the representation (1) is called $(\psi, \bar{\beta})$ -integral of the function φ and is denoted by $\mathcal{J}_{\bar{\beta}}^\psi \varphi$ ($f = \mathcal{J}_{\bar{\beta}}^\psi \varphi$). If $\psi(k) \neq 0$, $k \in \mathbb{N}$, then the function φ in the representation (1) is called

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