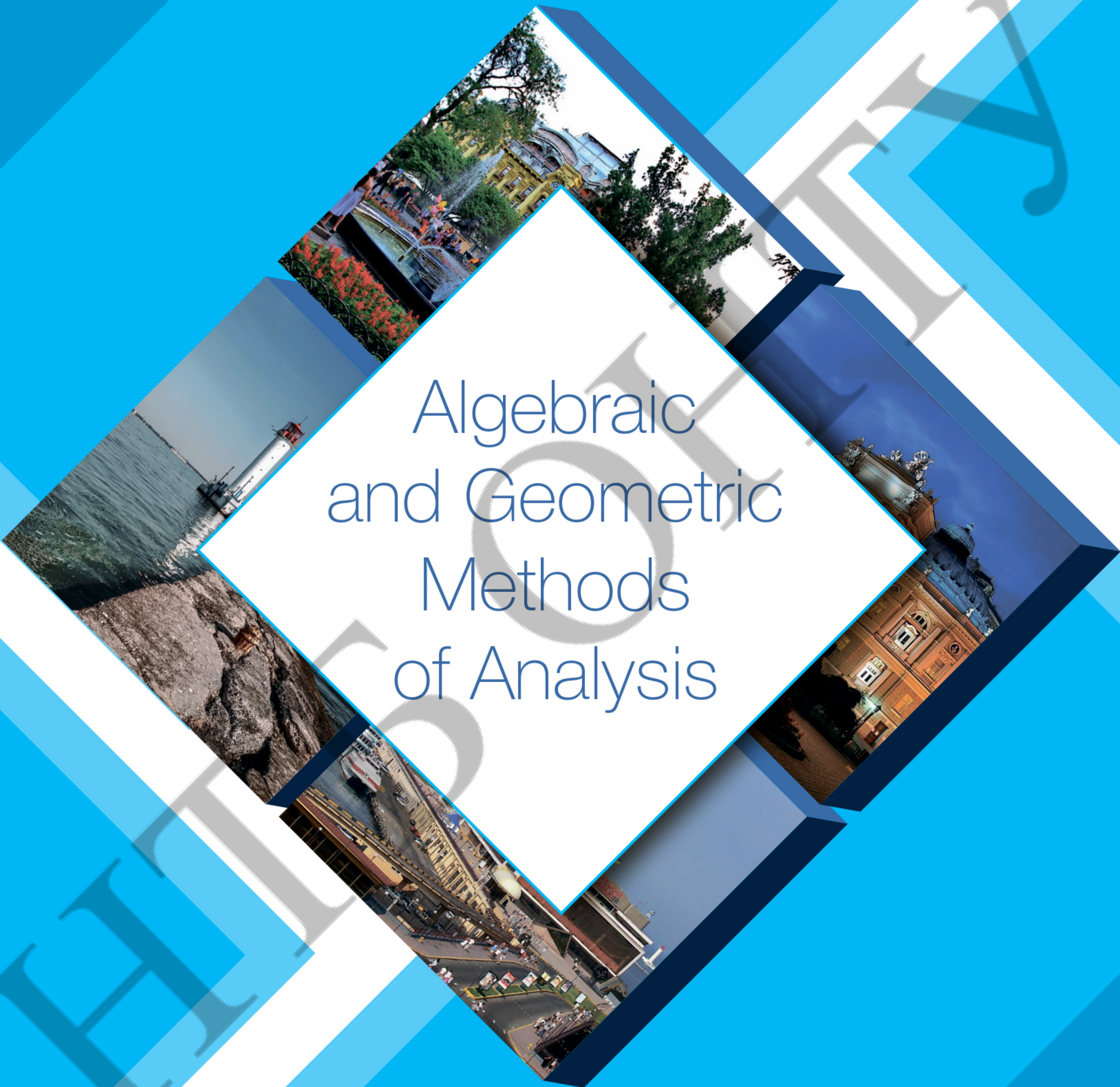
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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[2] A. S. Serdyuk and I. V. Sokolenko. Approximation by Fourier sums in the classes of Weyl–Nagy differentiable functions with high exponents of smoothness. *Ukr. Math. J.*, 74, No 5, 783–800, 2022.

On the asymptotic behavior of solutions to nonlinear Beltrami equation

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Let $\mathbb{C}$ be the complex plane. In the complex notation $f = u + iv$ and $z = x + iy$, the *Beltrami equation* in a domain $G \subset \mathbb{C}$ has the form

$$f_{\overline{z}} = \mu(z) f_z, \tag{1}$$

where $\mu\colon G \to \mathbb{C}$ is a measurable function and

$$f_{\overline{z}} = \frac{1}{2}(f_x + if_y) \qquad \text{and} \qquad f_z = \frac{1}{2}(f_x - if_y)$$

are formal derivatives of f in $\overline{z}$ and z, while f_x and f_y are partial derivatives of f in the variables x and y, respectively.

We consider the following equation written in the polar coordinates (r, θ)

$$f_\theta = \sigma(re^{i\theta})\, |f_r|^m\, f_r. \tag{2}$$

We rewrite the equation (2) in the Cartesian form:

$$f_{\overline{z}} = \frac{z}{\overline{z}} \, \frac{1 + i\sigma(z)\, |z|^{-m-1} |z f_z + \overline{z} f_{\overline{z}}|^m}{1 - i\sigma(z)\, |z|^{-m-1} |z f_z + \overline{z} f_{\overline{z}}|^m} \, f_z. \tag{3}$$

Assuming $m = 0$, the equation (3) also becomes the standard linear Beltrami equation (1) with

$$\mu(z) = \frac{z}{\overline{z}} \, \frac{1 + i\sigma(z)/|z|}{1 - i\sigma(z)/|z|} \ .$$

Choosing $m = 0$ and $\sigma = i|z|$ in (3), we arrive at the classical Cauchy-Riemann system. Later on we assume that $m > 0$.

A mapping $f\colon G \to \mathbb{C}$ is called *regular at a point* $z_0 \in G$, if f has the total differential at this point and its Jacobian $J_f = |f_z|^2 - |f_{\overline{z}}|^2$ does not vanish. A homeomorphism f of Sobolev class $W^{1,1}_{\mathrm{loc}}$ is called *regular*, if $J_f > 0$ a.e. By a *regular solution of the equation* (3) we call a regular homeomorphism $f\colon G \to \mathbb{C}$, which satisfies (3) a.e. in G.

Later on, we use the following notations:

$$B_r = \{z \in \mathbb{C} : |z| < r\}, \quad \mathbb{B} = \{z \in \mathbb{C} : |z| < 1\}$$

and

$$\gamma_r = \{z \in \mathbb{C} : |z| = r\}.$$

Theorem 1. *Let $f:\mathbb{B}\to\mathbb{C}$ be a regular homeomorphic solution of the equation (3) which belongs to Sobolev class $W^{1,2}_{\rm loc}$, and normalized by $f(0)=0$. Assume that $C>0$ and the coefficient $\sigma:\mathbb{B}\to\mathbb{C}$ satisfies the following condition*

$$\int\limits_{\gamma_r}\frac{|\sigma(z)|^{m+2}}{\left(\operatorname{Im}\sigma(z)\right)^{m+1}}\,|dz|\leqslant C\,r^2$$

for a.a. $r\in(0,1)$. Then

$$\limsup_{z\to0}\frac{|f(z)|}{|z|}\geqslant\left(\frac{2\pi}{C}\right)^{\frac{1}{m}}.$$

Corollary 2. *Let $f:\mathbb{B}\to\mathbb{C}$ be a regular homeomorphic solution of the equation (3) which belongs to Sobolev class $W^{1,2}_{\rm loc}$, and normalized by $f(0)=0$ and $K>0$. Assume that the coefficient $\sigma:\mathbb{B}\to\mathbb{C}$ satisfies the following condition*

$$\frac{|\sigma(z)|^{m+2}}{\left(\operatorname{Im}\sigma(z)\right)^{m+1}}\leqslant K\,|z|$$

for a.a. $z\in\mathbb{B}$. Then

$$\limsup_{z\to0}\frac{|f(z)|}{|z|}\geqslant K^{-\frac{1}{m}}.$$

Theorem 3. *Let $f:\mathbb{B}\to\mathbb{C}$ be a regular homeomorphic solution of the equation (3) which belongs to Sobolev class $W^{1,2}_{\rm loc}$, and normalized by $f(0)=0$. Suppose that*

$$\sigma_0=\liminf_{\varepsilon\to0}\frac{1}{\pi\varepsilon^2}\int\limits_{B_\varepsilon}\frac{|\sigma(z)|^{m+2}}{|z|\left(\operatorname{Im}\sigma(z)\right)^{m+1}}\,dxdy.$$

1) If $\sigma_0\in(0,\infty)$, then

$$\limsup_{z\to0}\frac{|f(z)|}{|z|}\geqslant c_m\,\sigma_0^{-\frac{1}{m}},$$

where c_m is a positive constant depending on the parameter m.

2) If $\sigma_0=0$, then

$$\limsup_{z\to0}\frac{|f(z)|}{|z|}=\infty.$$

References

[1] I. Petkov, R. Salimov, M. Stefanchuk. Nonlinear Beltrami equation: lower estimates of Schwarz Lemma's type. *Canadian Mathematical Bulletin*, 2023. DOI: https://doi.org/10.4153/S0008439523000942