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Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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On the properties smooth manifolds defined by intersections

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The report is devoted to study properties and construction an examples of the (3, 4)-dim smooth manifolds contained the surfaces of constant curvature.

At the first will be considered the Dyck-surface (W.Dyck,1888) defined by the algebraic equation

$$(z_1^2 + z_2^2) (z_1^2 + z_2^2 + z_3^2) - z_3 (4 z_1^2 + 2 z_2^2) = 0, \quad (1)$$

where z_1, z_2, z_3 are the complex coordinates: $z_1 = x + Ia$, $z_2 = y + Ib$, $z_3 = z + Ic$, $I^2 = -1$. The complex surface (1) is generalization of real projective surface and belongs to the class of the one-side surfaces having an important applications in various branch of modern algebraic topology (J.Milnor,1968).

Proposition 1. *Joint consideration both equations (1) and the equation of the 5D-sphere $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$ with real coordinates (x, y, z, a, b, c) , in general, lead us to the some 4D-space, where (t) -is auxiliary parameter:*

$${}^4G(a, b, c, z, t) = bc^2 - b + b^3 + bz^2 + 2ct - ztc - bt^2 + ba^2 = 0, \quad (2)$$

containing 3D-subspace with the equation

$${}^3H(a, b, c, z) = 4a^2b^2 - 4b^2 + 4b^4 + 4c^2 + 4b^2c^2 - 4zc^2 + 4b^2z^2 + z^2c^2 = 0. \quad (3)$$

With the equation (3) at the condition $b = b(a)$ can be associated an invariant second order ODE of the form $b'' - A_1b'^3 - 3A_2b'^2 - 3A_3b' - A_4 = 0$, $A_i = A_i(a, b)$ having the General integral

$${}^3F(a, b, C_1, C_2) = -C_1 - b^4 - a^2b^2 + 2b^2C_2 = 0.$$

having an algebraic curve of genus $g_{a,b} = 1$, and which is placed on the 2D-surface, equipped by the metrics of const positive curvature $K = 1$

$$\phi^2 ds^2 = \psi_1(x, y)dx^2 + 2\psi_2(x, y)dx dy + \psi_3(x, y)dy^2, \quad \phi(x, y) = \psi_1(x, y)\psi_3(x, y) - (\psi_2(x, y))^2. \quad (4)$$

The components ψ_i of the metrics are determined from the system (M.R.Liouville,1897)

$$\psi_{1x} + 2A_3\psi_1 - 2A_4\psi_2 = 0, \quad \psi_{3y} + 2A_1\psi_2 - 2A_2\psi_3 = 0,$$

$$\psi_{1y} + 2\psi_{2x} - 2A_3\psi_2 + 4A_2\psi_1 - 2A_4\psi_3 = 0, \quad \psi_{3x} + 2\psi_{2y} + 2A_2\psi_2 - 4A_3\psi_3 + 2A_1\psi_1 = 0. \quad (5)$$

In the second part of report we consider some examples of the Brieskorn type manifolds which are the intersection of the five-dimensional sphere with the singular manifold ($l = 2, m = 3, n = 5$)

$$|z_1|^2 + |z_2|^2 + |z_3|^2 = 1, \quad z_1^2 + z_2^3 + z_3^5 = 0. \quad (6)$$

Proposition 2. *From the equations of the system (6)*

$$x^2 + a^2 + y^2 + b^2 + z^2 + c^2 - 1 = 0, \quad 2xa + 3y^2b - b^3 + 5z^4c - 10z^2c^3 + c^5 = 0,$$

$$x^2 - a^2 + y^3 - 3yb^2 + z^5 - 10z^3c^2 + 5zc^4 = 0, \quad (7)$$

on the six real coordinates $z_1 = x + Ia$, $z_2 = y + Ib$, $z_3 = z + Ic$, in the case the relation $z = z(y)$ holds, the linearizable the second order ODE

$$\frac{d^2}{dy^2}z(y) = \frac{\left(\frac{d}{dy}z(y)\right) \left(-y\frac{d}{dy}z(y) + z(y)\right)}{yz(y)}$$

can be derived. It has general integral

$$F(z, y, C_1, C_2) = z - \sqrt{C_1 y^2 + 2 C_2} = 0,$$

containing the 2D-surface of constant curvature with the components of the metrics defined from (1),

$$\psi_2(x, y) = \frac{(C_3 x^2 + C_4) y^{5/3}}{\sqrt[3]{x}} + \frac{C_1 x^2 + C_2}{\sqrt[3]{x} \sqrt[3]{y}}, \quad \psi_1(x, y) = -\frac{x^{2/3} (-C_6 + C_3 y^4 + 2 C_1 y^2)}{y^{4/3}},$$

$$\psi_3(x, y) = -\frac{y^{2/3} (-C_5 + x^4 C_3 + 2 x^2 C_4)}{x^{4/3}}$$

with the parameters C_i .

By analogy are considered the case of tetrahedral space which corresponds to the intersection of the five-dimensional sphere with singular manifold ($l = 2, m = 3, n = 4$)

$$|z_1|^2 + |z_2|^2 + |z_3|^2 = 1, \quad z_1^2 + z_2^3 + z_3^4 = 0, \quad (8)$$

for which corresponding ODE has the form

$$\frac{d^2}{dy^2} z(y) = 1/2 \frac{\left(\frac{d}{dy} z(y)\right) \left(3 \left(\frac{d}{dy} z(y)\right)^2 y^2 - 6 \left(\frac{d}{dy} z(y)\right) y z(y) + 2 (z(y))^2\right)}{y (z(y))^2}$$

and the octahedral space defined by the condition

$$|z_1|^2 + |z_2|^2 + |z_3|^2 = 1, \quad z_1^2 + z_2^3 + z_3^3 = 0, \quad (9)$$

with the ODE: $\frac{d^2}{dy^2} z(y) = -\frac{\left(\frac{d}{dy} z(y)\right)^2}{z(y)}$, and corresponding metrics (4) with the components $\psi_i(x, y)$

$$\psi_3(x, y) = -2 y^{2/3} C_3 x^2 - 4 y^{2/3} C_4 x + C_5 y^{2/3}, \quad \psi_2(x, y) = \frac{y^2 C_3 x + y^2 C_4 + C_1 x + C_2}{\sqrt[3]{y}},$$

$$\psi_1(x, y) = \frac{-1/2 C_3 y^4 - C_1 y^2 + C_6}{y^{4/3}}.$$

To studying a more detail properties of considered spaces can be used the 4D-Riemann extensions

$$ds^2 = 2 (zA_3 - tA_4) dx^2 + 4 (zA_2 - tA_3) dx dy + 2 (zA_1 - tA_2) dy^2 + 2 dy dz + 2 dy dt$$

of 2D-metrics and with help of the Liouville-Tresse-Cartan invariants to investigated topological properties of the Brieskorn manifolds.

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