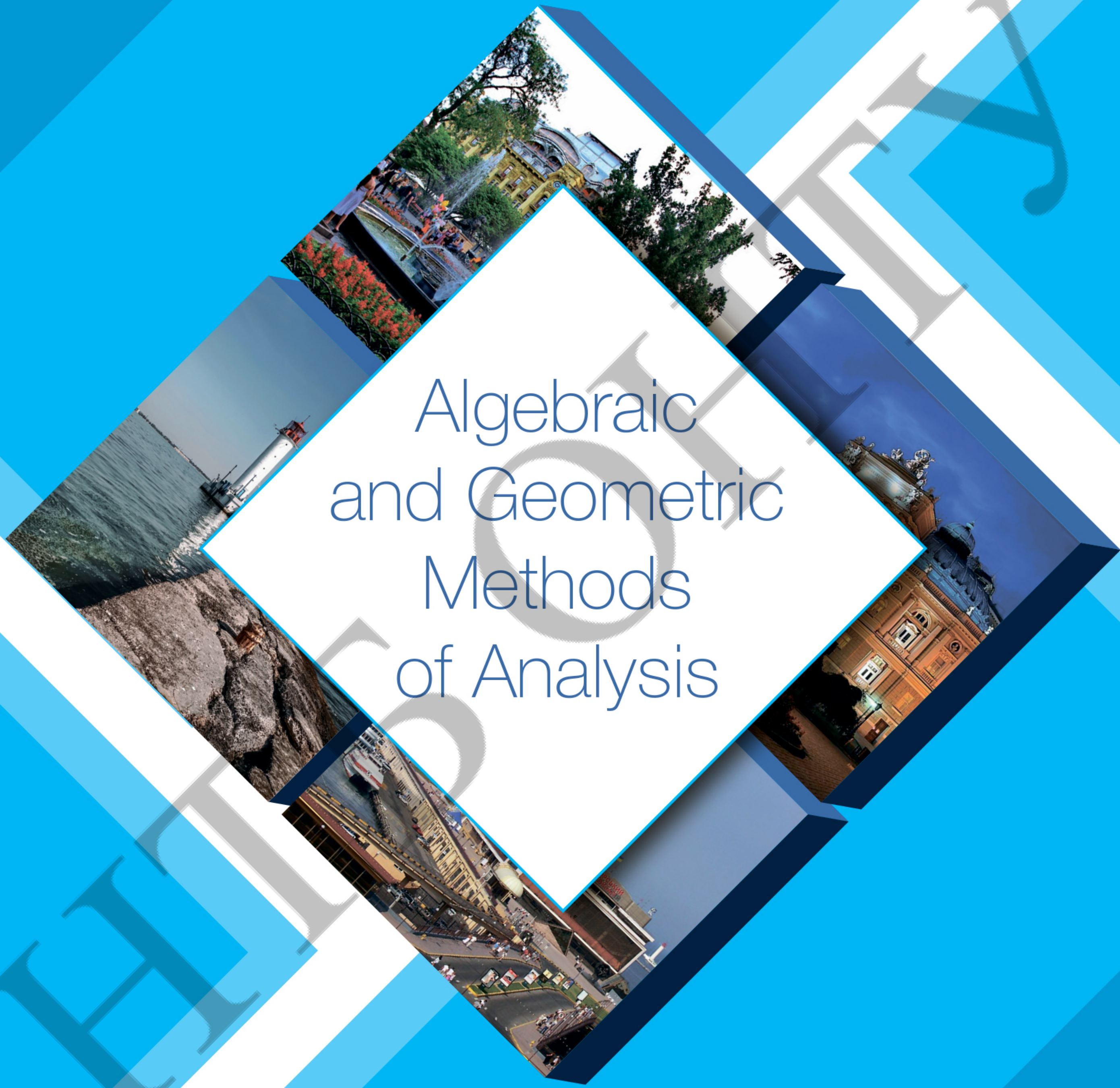
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Two problems in the theory of metric preserving functions

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The following is a particular case of J. Jachymski and F. Turoboś concept, see [1] for more details.

Definition 1. Let $\mathbf{A}$ be a class of metric spaces. Let us denote by $\mathbf{P_A}$ the set of all functions $f : [0,\infty) \to [0,\infty)$ such that the implication

$$((X,d) \in \mathbf{A}) \Rightarrow ((X, f \circ d) \in \mathbf{A})$$

is valid for every metric space (X,d).

We will use the following notations:

- $\mathbf{F}$, set of functions $f : [0,\infty) \to [0,\infty)$;
- $\mathbf{M}$, class of metric spaces;
- $\mathbf{U}$, class of ultrametric spaces;

Definition 2. A function $f \in \mathbf{F}$ is *metric preserving* (*ultrametric preserving*) iff $f \in \mathbf{P_M}$ ($f \in \mathbf{P_U}$).

Remark 3. The concept of metric preserving functions can be traced back to Wilson [2]. Similar problems were considered by Blumenthal in [3]. The theory of metric preserving functions was developed by Borsík, Doboš, Piotrowski, Vallin and other mathematicians. See also lectures by Doboš [4], and the introductory paper by Corazza [5]. The study of ultrametric preserving functions begun by P. Pongsriiam and I. Termwuttipong in 2014 [6].

Our main purpose is to give the answers on the following problems.

Problem 4. Let $\mathbf{A} \subseteq \mathbf{P_M}$. Find conditions under which the equation

$$\mathbf{P_X} = \mathbf{A} \tag{1}$$

has a solution $\mathbf{X} \subseteq \mathbf{M}$.

Problem 5. Let $\mathbf{A} \subseteq \mathbf{P_U}$. Find conditions under which equation (1) has a solution $\mathbf{X} \subseteq \mathbf{U}$.

Let us recall some basic concepts of semigroup theory, see, for example, John M. Howie [7].

A *semigroup* is a pair $(S, *)$ consisting of a nonempty set S and an associative operation $* : S \times S \to S$ which is called the *multiplication* on S. A semigroup $S = (S, *)$ is a *monoid* if there is $e \in S$ such that

$$e * s = s * e = s$$

for every $s \in S$.

Definition 6. Let $(S, *)$ be a semigroup and $\varnothing \neq T \subseteq S$. Then T is a *subsemigroup* of S if $a, b \in T \Rightarrow a * b \in T$. If $(S, *)$ is a monoid with the identity e, then T is a *submonoid* of S if T is a subsemigroup of S and $e \in T$.

Solutions to Problems 4 and 5 are given, respectively, in Theorems 7 and 8 below.

Theorem 7. *Let* $\mathbf{A}$ *be a nonempty subset of the set* $\mathbf{P_M}$ *of all metric preserving functions. Then the following statements are equivalent.*

(*i*) *The equality*

$$\mathbf{P_X} = \mathbf{A} \tag{2}$$

has a solution $\mathbf{X} \subseteq \mathbf{M}$.

(*ii*) $\mathbf{A}$ *is a submonoid of* $(\mathbf{F}, \circ)$.

(*iii*) $\mathbf{A}$ *is a submonoid of* $(\mathbf{P_M}, \circ)$.

The next theorem is an ultrametric analog of the previous theorem.

Theorem 8. *Let* $\mathbf{A}$ *be a nonempty subset of the set* $\mathbf{P_U}$ *of all ultrametric preserving functions. Then the following statements are equivalent.*

(*i*) *The equality* $\mathbf{P_X} = \mathbf{A}$ *has a solution* $\mathbf{X} \subseteq \mathbf{U}$.

(*ii*) $\mathbf{A}$ *is a submonoid of* $(\mathbf{F}, \circ)$.

(*iii*) $\mathbf{A}$ *is a submonoid of* $(\mathbf{P_U}, \circ)$.

Some properties of the monoids of $\mathbf{P_M}$ and $\mathbf{P_U}$ were described in [8] and [9].

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