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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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On controllability problems for the heat equation with variable coefficients controlled by the Dirichlet boundary condition on a half-axis

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Consider the heat equation

$$w_t = \frac{1}{\rho} (kw_x)_x + \gamma w, \quad x \in (0, +\infty), \quad t \in (0, T), \quad (1)$$

controlled by the Dirichlet boundary condition

$$w(0, \cdot) = u, \quad t \in (0, T), \quad (2)$$

under the initial condition

$$w(\cdot, 0) = w^0, \quad x \in (0, +\infty), \quad (3)$$

where $T = \text{const} > 0$; ρ, k, γ, w^0 are given functions; $u \in L^\infty(0, T)$ is a control. We assume $\rho, k \in C^1[0, +\infty)$, $\rho > 0$ and $k > 0$ on $[0, +\infty)$, $(\rho k) \in C^2[0, +\infty)$, $(\rho k)'(0) = 0$, and

$$\sigma(x) = \int_0^x \sqrt{\rho(\mu)/k(\mu)} d\mu \rightarrow +\infty \quad \text{as } x \rightarrow +\infty.$$

In addition, we assume

$$(P(k, \rho) - \gamma) \in L^\infty(0, +\infty) \cap C^1[0, +\infty) \quad \text{and} \quad \sigma \sqrt{\frac{\rho}{k}} (P(k, \rho) - \gamma) \in L^1(0, +\infty),$$

where $P(k, \rho) = \frac{1}{4} \sqrt{\frac{k}{\rho}} \left(\sqrt{\frac{k}{\rho}} \left(\frac{k'}{k} + \frac{\rho'}{\rho} \right) \right)' + \left(\frac{1}{4} \sqrt{\frac{k}{\rho}} \left(\frac{k'}{k} + \frac{\rho'}{\rho} \right) \right)^2$.

Control system (1)–(3) is considered in modified Sobolev spaces. Denote $\eta = (k\rho)^{1/4}$, $\theta = \left(\frac{k}{\rho}\right)^{1/4}$, $\mathcal{D}_{\eta\theta} = \theta^2 \left(\frac{d}{dx} + \frac{\eta'}{\eta} \right)$. Denote also $\mathbb{R}_+ = (0, +\infty)$. For $p = 1, 2$, denote

$$\tilde{\mathbb{H}}_+^0 = \left\{ \varphi \in L_{\text{loc}}^2(\mathbb{R}_+) \mid \left(\frac{\eta}{\theta} \varphi \right) \in L^2(\mathbb{R}_+) \right\}, \quad \tilde{\mathbb{H}}_+^p = \left\{ \varphi \in \tilde{\mathbb{H}}_+^{p-1} \mid \left(\frac{\eta}{\theta} \mathcal{D}_{\eta\theta}^p \varphi \right) \in L^2(\mathbb{R}_+) \text{ and } \varphi(0) = 0 \right\},$$

with the norm

$$\|\varphi\|_+^s = \sqrt{\sum_{m=0}^s \left(\left\| \frac{\eta}{\theta} \left(\mathcal{D}_{\eta\theta}^m \varphi \right) \right\|_{L^2(\mathbb{R}_+)} \right)^2}, \quad \varphi \in \tilde{\mathbb{H}}_+^s, \quad s = 0, 1, 2,$$

and the dual spaces $\tilde{\mathbb{H}}_+^{-s} = \left(\tilde{\mathbb{H}}_+^s \right)^*$ with the norms associated with the strong topology of these spaces, $s = 0, 1, 2$.

We suppose $\left(\frac{d}{dt} \right)^p w : [0, T] \rightarrow \tilde{\mathbb{H}}_+^{-2p}$, $p = 0, 1$, and $w^0 \in \tilde{\mathbb{H}}_+^0$ in system (1)–(3).

Let $T > 0$ and $w^0 \in \tilde{\mathbb{H}}_+^0$. By $\mathcal{R}_T(w^0)$, denote the set of all states $w^T \in \tilde{\mathbb{H}}_+^0$ for which there exists a control $u \in L^\infty(0, T)$ such that for the solution w to system (1)–(3) we have $w(\cdot, T) = w^T$.

Definition 1. A state $w^0 \in \tilde{\mathbb{H}}_+^0$ is said to be controllable to a state $w^T \in \tilde{\mathbb{H}}_+^0$ with respect to system (1)–(3) in a given time $T > 0$ if $w^T \in \mathcal{R}_T(w^0)$.

Definition 2. A state $w^0 \in \tilde{\mathbb{H}}_+^0$ is said to be null-controllable with respect to system (1)–(3) in a given time $T > 0$ if $0 \in \mathcal{R}_T(w^0)$.

Definition 3. A state $w^0 \in \tilde{\mathbb{H}}_+^0$ is said to be approximately controllable to a state $w^T \in \tilde{\mathbb{H}}_+^0$ with respect to system (1)–(3) in a given time $T > 0$ if $w^T \in \overline{\mathcal{R}_T(w^0)}$, where the closure is considered in the space $\tilde{\mathbb{H}}_+^0$.

Consider also control system with the simplest heat operator (the case $\rho = k = 1, \gamma = 0$)

$$z_t = z_{yy}, \quad y \in (0, +\infty), \quad t \in (0, T), \quad (4)$$

$$z(0, \cdot) = v, \quad t \in (0, T), \quad (5)$$

$$z(\cdot, 0) = z^0, \quad y \in (0, +\infty), \quad (6)$$

where $v \in L^\infty(0, T)$ is a control, $(\frac{d}{dt})^m z : [0, T] \rightarrow \tilde{H}_+^{-2m}$, $m = 0, 1$, $w^0 \in \tilde{H}_+^0 = L^2(\mathbb{R}_+)$. Here

$$\tilde{H}_+^s = \left\{ \varphi \in L^2(\mathbb{R}_+) \mid \left(\forall k = \overline{0, s} \varphi^{(k)} \in L^2(\mathbb{R}_+) \right) \text{ and } \left(\forall k = \overline{0, s-1} \varphi^{(k)}(0^+) = 0 \right) \right\}, \quad s = 0, 1, 2,$$

with the norm

$$\|\varphi\|_+^s = \sqrt{\sum_{k=0}^s \binom{s}{k} \left(\|\varphi^{(k)}\|_{L^2(\mathbb{R}_+)} \right)^2}, \quad \varphi \in \tilde{H}_+^s, \quad s = 0, 1, 2,$$

and $\tilde{H}_+^{-s} = (\tilde{H}_+^s)^*$ with the norms associated with the strong topology of these spaces, $s = 0, 1, 2$.

Controllability problems for system (4)–(6) were investigated in [1].

To study controllability problems for system (1)–(3), we use the transformation operator $\tilde{\mathbb{T}} : \tilde{H}_+^{-2} \rightarrow \tilde{\mathbb{H}}_+^{-2}$. It was introduced and studied in [2]. In particular, it has been proved therein that $\tilde{\mathbb{T}}$ is a continuous one-to-one mapping between the spaces $\tilde{H}_+^s$ and $\tilde{\mathbb{H}}_+^s$, $s = -2, -1, 0$.

In the present talk, we prove that the transformation operator $\tilde{\mathbb{T}}$ is one-to-one mapping between the sets of the solutions to system (4)–(6) and to system (1)–(3). The application of the operator $\tilde{\mathbb{T}}$ allows us to conclude that the control system (1)–(3) replicates the controllability properties of the control system (4)–(6) and vice versa. A relation between controls u and v is also found. Thus, we obtain the following main results.

Theorem 4. *If a state $w^0 \in \tilde{\mathbb{H}}_+^0$ is null-controllable with respect to system (1)–(3) in a time $T > 0$, then $w^0 = 0$.*

Theorem 5. *Each state $w^0 \in \tilde{\mathbb{H}}_+^0$ is approximately controllable to any target state $w^T \in \tilde{\mathbb{H}}_+^0$ with respect to system (1)–(3) in a given time $T > 0$.*

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