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Online Conference



**Algebraic
and Geometric
Methods of Analysis**

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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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Some applications of transversality for infinite dimensional manifolds

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We present some transversality results for a category of Fréchet manifolds, the so-called MC^k -Fréchet manifolds. In this context, we apply the obtained transversality results to construct the degree of nonlinear Fredholm mappings by virtue of which we prove a rank theorem, an invariance of domain theorem and a Borsuk-Ulam type theorem.

We refer to [1, 2] for the basic definitions and result regarding MC^k -Fréchet manifolds. We assume that E, F are Fréchet spaces and $\mathcal{U} \subseteq E$ is an open subset, also that M, N are MC^k -Fréchet manifolds.

Theorem 1 (Transversality Theorem). *Let $\varphi : M \rightarrow N$ be an MC^k -mapping, $k \geq 1$, $S \subset N$ an MC^k -submanifold and $\varphi \pitchfork S$. Then, $\varphi^{-1}(S)$ is either empty of MC^k -submanifold of M with*

$$(T_x\varphi)^{-1}(T_yS) = T_x(\varphi^{-1}(S)), \quad x \in \varphi^{-1}(S), \quad y = \varphi(x).$$

If S has finite co-dimension in N , then $\text{codim}(\varphi^{-1}(S)) = \text{codim}S$. Moreover, if $\dim S = m < \infty$ and φ is an MC^k -Lipschitz-Fredholm mapping of index l , then $\dim \varphi^{-1}(S) = l + m$.

Theorem 2 (The Parametric Transversality Theorem). *Let A be a manifold of dimension n , $S \subset N$ a submanifold of finite co-dimension m . Let $\varphi : M \times A \rightarrow N$ be an MC^k -mapping, $k \geq \{1, n - m\}$. If φ is transversal to S , $\varphi \pitchfork S$, then the set of all points $x \in M$ such that the mappings*

$$\varphi_x : A \rightarrow N, \quad (\varphi_x(\cdot) := \varphi(x, \cdot))$$

are transversal to S , is residual in M .

Theorem 3 (Rank theorem for MC^k -mappings). *Let $\varphi : \mathcal{U} \subseteq E \rightarrow F$ be an MC^k -mapping, $k \geq 1$. Suppose $u_0 \in \mathcal{U}$ and $D\varphi(u_0)$ has closed split image \mathbf{F}_1 with closed complement \mathbf{F}_2 and split kernel \mathbf{E}_2 with closed complement \mathbf{E}_1 . Also, assume $D\varphi(\mathcal{U})(E)$ is closed in F and $D\varphi(u)|_{\mathbf{E}_1} : \mathbf{E}_1 \rightarrow D\varphi(u)(E)$ is an MC^k -isomorphism for each $u \in \mathcal{U}$. Then, there exist open sets $\mathcal{U}_1 \subseteq \mathbf{F}_1 \oplus \mathbf{E}_2$, $\mathcal{U}_2 \subseteq E$, $\mathcal{V}_1 \subseteq F$, and $\mathcal{V}_2 \subseteq F$ and there are MC^k -diffeomorphisms $\phi : \mathcal{V}_1 \rightarrow \mathcal{V}_2$ and $\psi : \mathcal{U}_1 \rightarrow \mathcal{U}_2$ such that*

$$(\phi \circ \varphi \circ \psi)(f, e) = (f, 0), \quad \forall (f, e) \in \mathcal{U}_1.$$

Theorem 4 (Invariance of domain for Lipschitz-Fredholm mappings). *Let $\varphi : M \rightarrow N$ be an MC^k -Lipschitz-Fredholm mapping of index zero, $k > 1$. If φ is locally injective, then φ is open.*

Definition 5. Let $\varphi : M \rightarrow N$ be a non-constant closed Lipschitz-Fredholm mapping with index $l \geq 0$ of class MC^k such that $k > l + 1$. We associate to φ a degree, denoted by $\text{deg } \varphi$, defined as the non-oriented cobordism class of $\varphi^{-1}(q)$ for some regular value q . If $l = 0$, then $\text{deg } \varphi \in \mathbb{Z}_2$ is the number modulo 2 of preimage of a regular value.

Theorem 6 (Borsuk-Ulam Theorem). *Let $\varphi : \bar{U} \rightarrow F$ be a non-constant closed Lipschitz-Fredholm mapping of class MC^2 with index zero, where $U \subseteq F$ is a centrally symmetric and bounded. If φ is odd and for $u \in \bar{U}$ we have $u \notin \varphi(\partial\bar{U})$. Then $\text{deg}(\varphi, 0_F) \equiv 1 \pmod{2}$.*

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