

Ministry of Education and Science of Ukraine  
Black Sea Universities Network

# ODESA NATIONAL UNIVERSITY OF TECHNOLOGY

International Competition of  
Student Scientific Works

# BLACK SEA SCIENCE 2022 PROCEEDINGS



ODESA, ONUT 2022

Ministry of Education and Science of Ukraine

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International Competition of Student Scientific Works

# **BLACK SEA SCIENCE 2022**

**Proceedings**

Odesa, ONUT 2022

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## INTRODUCTION

International Competition of Student Scientific Works “Black Sea Science” has been held annually since 2018 at the initiative of Odesa National University of Technology (formerly Odesa National Academy of Food Technologies) with the support of the Ministry of Education and Science of Ukraine. It has been supported by Black Sea Universities Network (the Association of 110 higher education institutions from 12 countries of the Black Sea Region) since 2019, and by Iseki-FOOD Association (European Integrating Food Science and Engineering Knowledge into the Food Chain Association) since 2020.

The goal of the competition is to expand international relations and attract students to research activities. It is held in the following fields:

- Food science and technologies
- Economics and administration
- Information technologies, automation and robotics
- Power engineering and energy efficiency
- Ecology and environmental protection

The jury includes both Ukrainian and foreign scientists. In the 4 years that the competition has been held, the jury included scientists from universities of 24 countries: Angola, Azerbaijan, Benin, Bulgaria, China, Czech Republic, France, Georgia, Germany, Greece, Israel, Italy, Kazakhstan, Latvia, Lithuania, Moldova, Pakistan, Poland, Romania, Serbia, Slovakia, Switzerland, Turkey, USA.

At the same time, every year the geography has expanded and the number of foreign jury members has increased: from 46 jury members representing 25 universities from 12 countries in 2018, to 73 jury members of the 46 universities from 19 countries in 2022.

More than a thousand student research papers have been submitted to the competition from both Ukrainian and foreign institutions from 25 countries: China, Poland, Mexico, USA, France, Greece, Germany, Canada, Costa Rica, Brazil, India, Pakistan, Israel, Macedonia, Lithuania, Latvia, Slovakia, Romania, Kyrgyzstan, Kazakhstan, Bulgaria, Moldova, Georgia, Turkey, Serbia.

The interest of foreign students in the competition grew every year. In 2018, the students representing 15 institutions from 7 countries have submitted 33 works. In 2021 the number of submitted works increased to 73, authored by the students of 40 institutions from 18 countries.

The competition is held in two stages. In the first stage, student research papers are reviewed by members of the jury who are experts in the relevant fields. In the second stage of the competition, the winners of the first stage have the opportunity to present their work to a wide audience in person or online.

All participants of the competition and their scientific supervisors are awarded appropriate certificates, and the scientific works of the winners are included in the electronic proceedings of the competition. Every year the competition receives a large number of positive responses from Ukrainian and foreign colleagues with the desire to participate in the coming years.

# **3. INFORMATION TECHNOLOGIES, AUTOMATION AND ROBOTICS**

## IMPROVING THE LEVEL OF DETAILING IN THE FORMATION OF REALISTIC THREE-DIMENSIONAL SCENES

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**Abstract.** *The issue of increasing the level of detail in the formation of realistic three-dimensional scenes is considered. An expression is obtained to determine the vector at any point on the surface bounded by a triangle; splitting the output triangle into components with the same area, to achieve a balanced load of shader processors; The obtained relations for detailing the surfaces of three-dimensional objects by Serpinsky triangulation; For the first time, it is proposed to perform triangulations of three-dimensional objects depending on the maximum values of colour intensities on the edges of the triangle, which allows the use of adaptive triangulation.*

**Keywords:** *rendering, triangulation, vectors of normals, polygonal network, detailing of surfaces, normalization of vectors, barycentric coordinates*

### I. INTRODUCTION

The graphic form [1-13] of the representation is the most informative, so realistic images are useful in almost all engineers and scientific tasks for the visual reproduction of real objects.

At the present stage of the development of computer graphics, the development of new methods and means to increase the realism of the formation of three-dimensional images is a priority [1, 2], as traditional approaches do not always provide the required quality of image formation.

The aim of the work – is to improve the realism of the formation of three-dimensional images by increasing the detailing of the surfaces of the scene.

Scientific innovation.

1. An expression is obtained to determine the vector at an arbitrary point of the surface bounded by a triangle by the values of the vectors at its vertices and the barycentric coordinates of the current point, which allows to detail of the surface to increase realism.
2. The proposed division of the output triangle into components with the same area allows achieving a balanced load of shader processors.
3. The obtained relations for detailing the surfaces of three-dimensional objects by Serpinsky triangulation, which makes it possible to increase the realism of the formation of three-dimensional images.
4. For the first time, it has been proposed to perform triangulations of three-dimensional objects depending on the maximum values of colour intensity on the edges of a triangle, which makes it possible to use adaptive triangulation.

The practical significance of the work is to develop on the basis of theoretical research algorithms and triangulation programs and integrate them into the professional graphics engine idx3d.

8 scientific works have been published on the topic of research, including one article in a professional publication.

## II. LITERATURE ANALYSIS

To simplify the task of constructing three-dimensional graphic objects, its surface is approximated by piecewise-linear sections [1-3], using a network of spatial triangles in the predominant event.

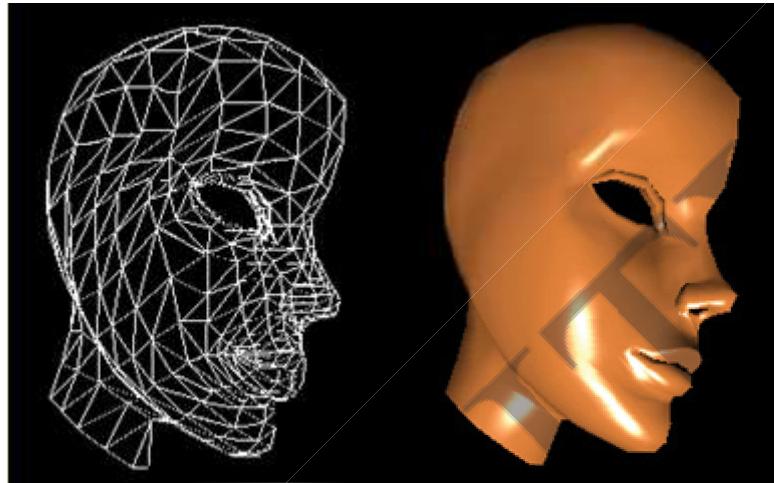


Figure 1 - Example of image formation by triangulation

The most common division of images into triangles is explained by the following reasons [1-3]: a) a triangle is the simplest polygon, the vertices of which uniquely define the face; b) any surface can be divided into triangles; c) the computational complexity of algorithms for dividing areas into triangles is significantly less than when using other polygons; d) the implementation of rendering procedures is the easiest for the area bounded by a triangle; e) for a triangle it is easy to determine its three nearest neighbours that have common faces with it.

Any surface can be represented, with the required accuracy, by a network of triangles [1-3], the accuracy of the approximation is determined by the number of triangles and the method of their selection. At the same time, it is important to reach a compromise between the quality of the final visualization and the load on graphics.

The number of triangles in a network is largely determined by the complexity of the object. For example, building a model of the human head requires about 60 thousand triangles.

Triangulation is an important procedure when using three-dimensional modelling to diagnose diseases in medical practice [7, 11].

Of course, the use of a triangle as the simplest polygon leads to the fact that the surface is insufficiently smoothed. One possible approach to improving the quality of image formation is to condense the network of triangles until an acceptable by some criteria local surface smoothness is achieved.

Triangulation of surfaces for the subsequent painting is carried out.

When painting a 3D object, normalized vectors [1, 2] to the object surface, light source and observer, as well as auxiliary vectors depending on the choice of lighting model are determined. Vector normalization requires three-division operations, three multiplication operations, two addition operations, and a square root finding operation,

which are quite time-consuming. In this regard, the urgent issue is to simplify the normalization procedure in order to implement it in hardware.

NVIDIA scientists have obtained a formula for the approximate normalization of vectors using a single step of Newton-Rafson iteration:  $\vec{N}_n \approx \vec{N} \cdot (3 - \vec{N} \cdot \vec{N}) / 3$  [1] Although the given formula is quite simple, large errors in determining the orthogonal components of the vector limit its use for painting problems.

R. Lyon [1, 2], using the Taylor series decomposition of the expression,  $1/\sqrt{\vec{N} \cdot \vec{N}}$ , obtained a formula for the approximate normalization of the normal vector

$$\vec{N}_n \approx \vec{N} \left( 1 - \frac{1}{2}((\vec{N} \cdot \vec{N}) - 1) + \frac{3}{8}((\vec{N} \cdot \vec{N}) - 1)^2 \right), \quad (1)$$

in which the division operation is replaced by an offset, which simplifies hardware implementation. Unfortunately, the formula has a large approximation error, which limits its application.

The interpolation of unit vectors between the initial and final vectors can be performed by the formula [1]

$$\vec{N}(w) = \vec{N}_a \frac{\sin((1-w)\psi)}{\sin\psi} + \vec{N}_b \frac{\sin(w\psi)}{\sin\psi}, \quad (2)$$

where  $w \in [0, 1]$ , while  $\psi$  – is the angle between the vectors  $\vec{N}_a$  and  $\vec{N}_b$ . The calculation of vectors involves determining the sine, the resource-intensive function of the arccosine to find the unknowns  $w$  and  $\psi$ , as well as performing the division operation.

The given short analysis of painting problems has shown that it is necessary not only to perform triangulation, but also to find vectors to the vertices of the obtained triangles.

## 2. Development of methods for improving the level of detail in the formation of realistic three-dimensional scenes

### 2.1 Balanced triangulation of the triangle

When developing methods to increase the level of detail in the formation of realistic three-dimensional scenes, it is important to achieve a balanced load of shader processors. This can be achieved by dividing the original triangle into constituent triangles with the same area.

Let's prove that in a triangle with vertices  $P_1(x_1, y_1)$ ,  $P_2(x_2, y_2)$ ,  $P_3(x_3, y_3)$  there is a point with coordinates

$$x = \frac{x_1 + x_2 + x_3}{3}, \quad y = \frac{y_1 + y_2 + y_3}{3},$$

which is connected to the vertices  $P_1, P_2, P_3$  divides  $\Delta P_1 P_2 P_3$  into triangles of equal areas.

Let  $P_1P_2P_3$  – be a given triangle, the coordinates of the point  $O$  satisfy the following conditions:

$$x = \frac{x_1 + x_2 + x_3}{3}, \quad y = \frac{y_1 + y_2 + y_3}{3}. \quad (3)$$

Areas  $\Delta P_1P_2O$ ,  $\Delta P_2P_3O$ ,  $\Delta P_3P_1O$  (Fig. 2) are respectively equal to:

$$S_1 = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \frac{x_1 + x_2 + x_3}{3} & \frac{y_1 + y_2 + y_3}{3} & 1 \end{vmatrix} \quad S_2 = \frac{1}{2} \begin{vmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ \frac{x_1 + x_2 + x_3}{3} & \frac{y_1 + y_2 + y_3}{3} & 1 \end{vmatrix}$$

$$S_3 = \frac{1}{2} \begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ \frac{x_1 + x_2 + x_3}{3} & \frac{y_1 + y_2 + y_3}{3} & 1 \end{vmatrix}$$

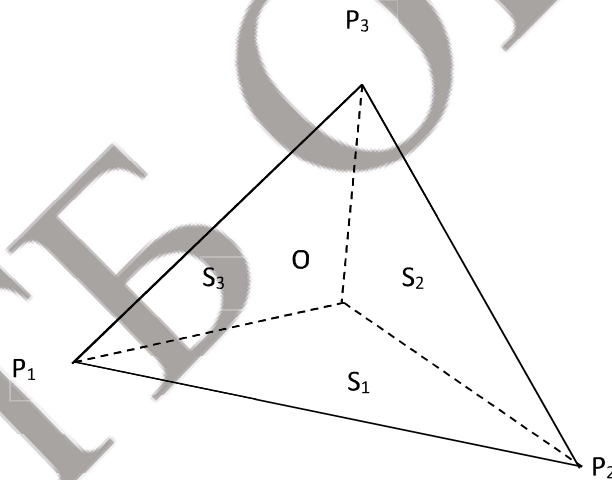


Figure 2 - Triangulation of a triangle into comparable triangles of equal areas

Let's find the values of  $S_1$ ,  $S_2$ ,  $S_3$ .

$$S_1 = \frac{1}{2} \left( x_1 y_2 + y_1 \frac{x_1 + x_2 + x_3}{3} + x_2 \frac{y_1 + y_2 + y_3}{3} - y_2 \frac{x_1 + x_2 + x_3}{3} - x_2 \cdot y_1 - x_1 \frac{y_1 + y_2 + y_3}{3} \right),$$

$$S_1 = (x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 \cdot y_1 - x_3 y_2) / 6 ;$$

$$S_2 = \frac{1}{2} \left( x_2 y_3 + y_2 \frac{x_1 + x_2 + x_3}{3} + x_3 \frac{y_1 + y_2 + y_3}{3} - y_3 \frac{x_1 + x_2 + x_3}{3} - x_3 \cdot y_2 - x_2 \frac{y_1 + y_2 + y_3}{3} \right),$$

$$S_2 = (x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 \cdot y_1 - x_3 y_2) / 6 ;$$

$$S_3 = \frac{1}{2} \left( x_3 y_1 + y_3 \frac{x_1 + x_2 + x_3}{3} + x_1 \frac{y_1 + y_2 + y_3}{3} - y_1 \frac{x_1 + x_2 + x_3}{3} - x_1 \cdot y_3 - x_3 \frac{y_1 + y_2 + y_3}{3} \right),$$

$$S_3 = (x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 \cdot y_1 - x_3 y_2) / 6.$$

Comparing the expressions for S1, S2, S3, conclude that S1 = S2 = S3. Therefore, the condition is correct.

As the vectors include three components, to each of which it can apply the proved property, the vector of the normal at the point will be equal to

$$\vec{N}_o = \frac{N_{p1} + N_{p2} + N_{p3}}{3} \quad (4)$$

The resulting formulas can be obtained for balanced [1] triangulation in computer graphics problems.

### 2.2. Definition of vectors for Serpinsky triangulation

In the Serpinsky triangulation of the 1st order, the triangle is divided into 4 components by connecting the midpoints of its sides (Fig. 3). The result is four triangles that are equal in area. The constituent triangles are painted in parallel.

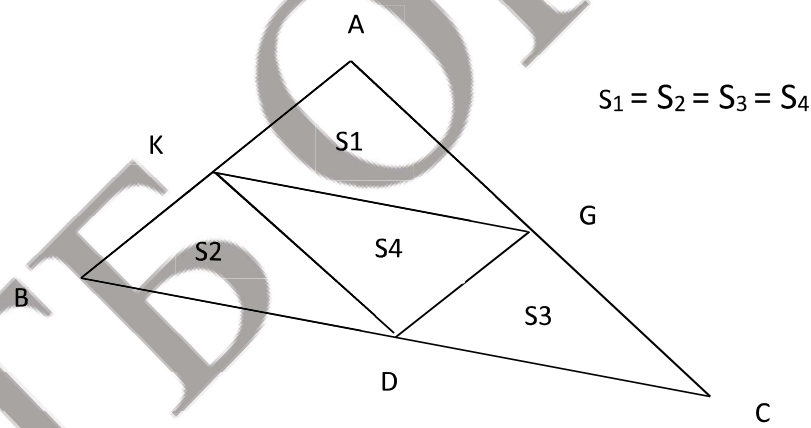


Figure 3 - Dividing a triangle into components using Serpinsky triangulation

When implementing Guro painting, only one dye is used, which paints one of the triangles, and the results of its work are transferred to all other triangles with a certain offset in the coordinates and the corresponding color intensity transformations of the points. Due to the parallelization of the computational process, the output triangle will be painted 4 times faster.

When painting according to Fong, it is necessary to find the normals in the midpoints of the edges of the triangle.

Let`s prove that the increase in color intensity along the corresponding edges of the four constituent triangles is a constant value.

Let's find the vector  $\vec{N}_{(1/2)}$ , which forms with the vectors  $\vec{N}_a$  and  $\vec{N}_b$  the angle  $\psi/2$ . According to formula (2) it can be written

$$\vec{N}_{(1/2)} = \vec{N}_a \frac{\sin(\frac{1}{2} \cdot \psi)}{\sin \psi} + \vec{N}_b \frac{\sin(\frac{1}{2} \cdot \psi)}{\sin \psi} = \frac{\vec{N}_a + \vec{N}_b}{2 \cos \frac{\psi}{2}} = \frac{\vec{N}_a + \vec{N}_b}{\sqrt{2} \cdot (1 + \cos \psi)}.$$

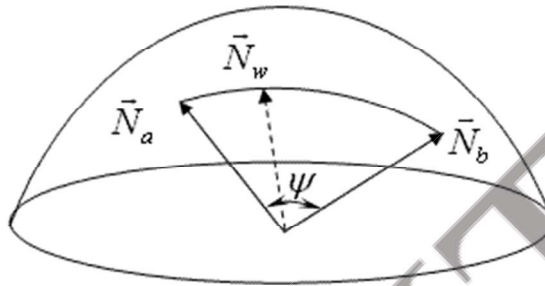


Figure 4 - Outputs  $\vec{N}_a$ ,  $\vec{N}_b$  and flow  $\vec{N}_w$  vectors of normals

Let's find the vector of the normal  $\vec{N}_{(1/2^2)}$ , which forms the same angles between the vectors  $\vec{N}_a$ ,  $\vec{N}_{1/2}$ .

$$\vec{N}_{(1/2^2)} = \frac{\vec{N}_a + \vec{N}_{1/2}}{\sqrt{2(1 + \cos \frac{\psi}{2})}} = \frac{\vec{N}_a + \vec{N}_{1/2}}{\sqrt{2(1 + \sqrt{\frac{1 + \cos \psi}{2}})}} = \frac{\vec{N}_a + \vec{N}_{1/2}}{\sqrt{2 + \sqrt{2(1 + \cos \psi)}}}.$$

After the first iteration, the angle between the vectors of the normals changes from zero to  $90^\circ$ , therefore  $\sqrt{2} \leq z_{2^n} \leq 2$ . The expression  $\frac{1}{\sqrt{2 + z_{2^n}}}$  was approximated by a number of Chebyshev. When using a first-degree polynomial

$$\frac{1}{\sqrt{2 + z_{2^n}}} \approx -0,07 \cdot z_{2^n} + 0,64.$$

The maximum absolute approximation error does not exceed 0,0005, and the relative 0,12%. When using the second-degree polynomial

$$\frac{1}{\sqrt{2 + z_{2^n}}} \approx 0,014 \cdot z_{2^n}^2 - 0,119 \cdot z_{2^n} + 0,681.$$

the maximum absolute error of the approximation does not exceed  $2 \cdot 10^{-5}$ , and the relative 0,004 %. The first formula should be used for low-resolution screens for which the triangles that make up the surface of a three-dimensional object are small enough.

The analysis showed that when using the last approximation formula, the time

of calculation of the vector  $\vec{N}_{(1/2^n)}$  is reduced by 2,5 times compared to the classical implementation. Figure 5 shows an example of object formation using different triangulation network densities.

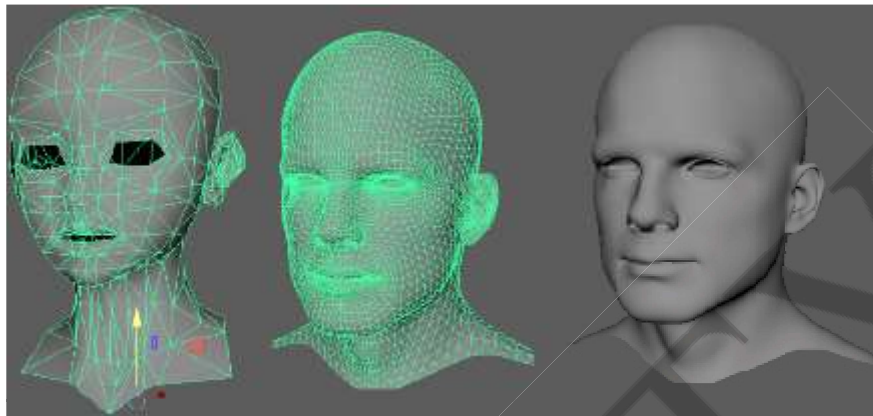


Figure 5 - Example of image formation using additional triangulation

The resulting formulas can be obtained for balanced [1] triangulation in computer graphics problems.

**2.3. The method of triangulation of three-dimensional objects depending on the maximum values of color intensity on the edges of the triangle**

Find the highest intensity of the specular component of color on the edges of the triangle (Fig. 6).

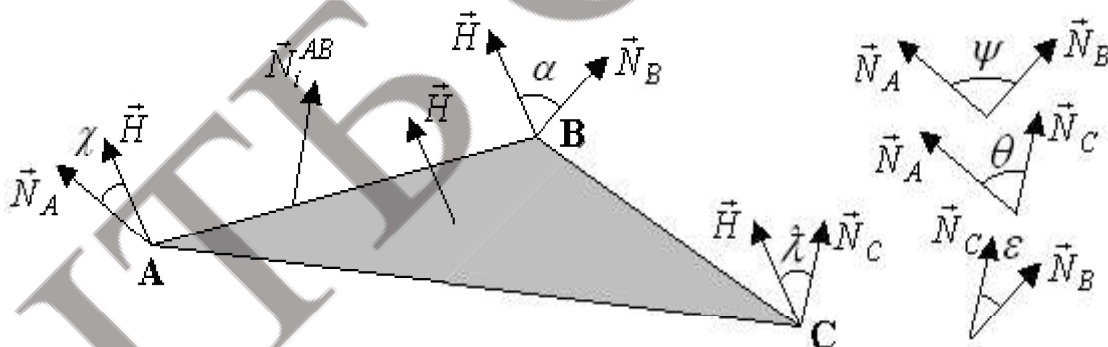


Figure 6 - Vectors of the normal of the ABC triangle

The vectors of normals to the points of the edge AB can be found from the parametric equation  $\vec{N}_i^{AB} = \vec{N}_A + t_1 \cdot (\vec{N}_B - \vec{N}_A)$ . Let's perform normalization of the vector  $\vec{N}_i^{AB}$ .

$$\frac{\vec{N}_i^{AB}}{|\vec{N}_i^{AB}|} = \frac{\vec{N}_A + t_1 \cdot (\vec{N}_B - \vec{N}_A)}{\sqrt{(\vec{N}_A)^2 + 2 \cdot t_1 \cdot \vec{N}_A \cdot (\vec{N}_B - \vec{N}_A) + t_1^2 \cdot (\vec{N}_B - \vec{N}_A)^2}}$$

As  $\vec{N}_A, \vec{N}_B$  - are single vectors, then  $\vec{N}_A^2 = \vec{N}_B^2 = 1$ . Given the last equation, as well as the fact that  $\vec{N}_A \cdot \vec{N}_B = \cos \psi$ , find

$$\frac{\vec{N}_i^{AB}}{|\vec{N}_i^{AB}|} = \frac{\vec{N}_A + t_1 \cdot (\vec{N}_B - \vec{N}_A)}{\sqrt{2 \cdot t_1^2 \cdot (1 - \cos \psi) - 2 \cdot t_1 \cdot (1 - \cos \psi) + 1}}.$$

Let's find the positions on the edges of the triangle ABC, where the glare is most intense. This can be judged by the value of the cosine of the angle between the vector  $\vec{H}$  and the unit vectors of normals to the points of the edge.

For example, for the edge AB

$$\frac{\vec{N}_i^{AB} \cdot \vec{H}}{|\vec{N}_i^{AB}|} = \frac{(\vec{N}_A + t_1 \cdot (\vec{N}_B - \vec{N}_A)) \cdot \vec{H}}{\sqrt{2t_1^2(1 - \cos \psi) - 2t_1(1 - \cos \psi) + 1}} = \frac{\cos \chi + t_1(\cos \alpha - \cos \chi)}{\sqrt{2t_1^2(1 - \cos \psi) - 2t_1(1 - \cos \psi) + 1}},$$

where  $\chi, \alpha$ - respectively, the angles between the vector  $\vec{H}$  and the vectors  $\vec{N}_A, \vec{N}_B$ . In the above expression, all quantities are scalar.

Let's find  $t_1$ , at which the scalar component of the color on the edge AB has the maximum value. To do this, find the derivative of the previous expression and equate it to zero.

$$\left( \frac{\vec{N}_i^{AB} \cdot \vec{H}}{|\vec{N}_i^{AB}|} \right)' = \frac{t_1(1 - \cos \psi) \cdot (\cos \alpha - \cos \chi) - \cos \chi \cdot \cos \psi + \cos \alpha}{(2t_1^2(1 - \cos \psi) - 2t_1(1 - \cos \psi) + 1)^{3/2}} = 0.$$

The last equation has a root

$$t_1 = \frac{\cos \chi \cdot \cos \psi - \cos \alpha}{(\cos \psi - 1)(\cos \alpha + \cos \chi)}. \quad (5)$$

Similarly, for the edges AC i BC the values of the parameters  $t_2, t_3$ , at which the maximum value of the speculative component of color is achieved, respectively, are

$$t_2 = \frac{\cos \chi \cdot \cos \theta - \cos \lambda}{(\cos \theta - 1)(\cos \lambda + \cos \chi)}, \quad t_3 = \frac{\cos \alpha \cdot \cos \varepsilon - \cos \lambda}{(\cos \varepsilon - 1)(\cos \lambda + \cos \alpha)}. \quad (6)$$

By value  $t$  it is easy to find the coordinates  $x, y$  on the edges of the triangle, where the speculative component of color is maximum. For example, for the edge AB

$$x = [x_A + t \cdot (x_B - x_A)], \quad y = [y_A + t \cdot (y_B - y_A)].$$

In determining the diffuse component of color instead of the vector  $\vec{H}$  using the vector  $\vec{L}$ .

The output triangle can be divided into several, depending on the maximum values of color intensity at points  $t_1, t_2, t_3$ . If all three values of color intensity are greater than the threshold, then the triangle is divided into four components (Fig. 7, a). In the case when only in one of the points  $t_1, t_2, t_3$  the value of color intensity is reached, more than the threshold, the triangle is divided into two components (Fig. 7, b). If the maximum values of the speculative component of color on the edges of the triangle are greater than the threshold value at only two points, the triangle can be divided into three components as shown in Fig. 7, s, d. In the latter case, at the point  $D$ , which bisecting the segment  $AC$ , it is easy to find the value of color intensity as the

average value of color intensity at points  $A$  and  $C$ .

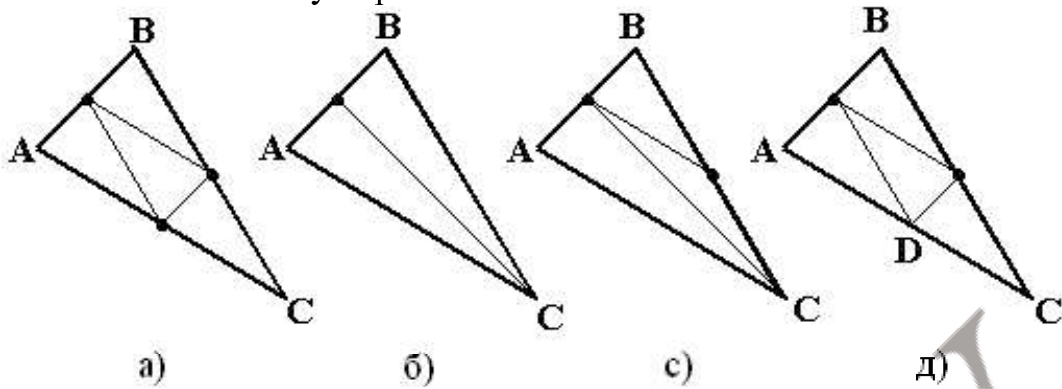


Figure 7 - Dividing the triangle into components

The proposed approach allows to increase the realism of the Guro method compared to the classical implementation by more detailed reproduction of glare on the surface of three-dimensional graphics. Figure 8 shows an example of the formation of a three-dimensional object using the Fong method.

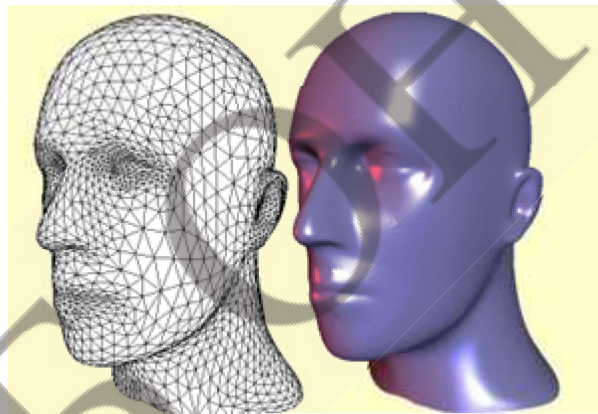


Figure 8 - Forming a three-dimensional object using the Fong method

#### 2.4. Determination of a vector at an arbitrary point on a surface bounded by a triangle

Consider the division of the original triangle into components with an arbitrary interior point [8].

When a triangle is divided into components, it is necessary to determine not only the addresses of the components of the triangles, but also the vectors of the normals. Consider the method of determining the vectors of normals at any point of the triangle, which will make it possible to divide the triangle into its components.

It is convenient to use the barycentric coordinate system, while working with triangles [2]. Figure 9 shows a triangle  $A_1A_2A_3$ , at which point  $C$  – is an arbitrary point located inside a given triangle. If the point  $C$  is connected to the vertices of the triangle  $A_1A_2A_3$ , then it will divide this triangle into triangles  $A_1CA_2$ ,  $A_3CA_2$  and  $A_1CA_3$ , whose area will be  $S_1$ ,  $S_2$  and  $S_3$  accordingly.

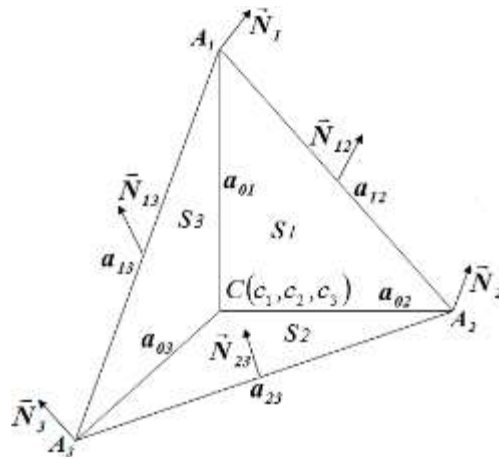


Figure 9 - Dividing the output triangle into 3 components

The barycentric coordinates of the point  $C$  are defined as the ratio of the areas of the constituent triangles [2, 8], into which the point  $C$  of the triangle divides  $A_1, A_2, A_3$ , to its total area.

$$c_1 = \frac{S_1}{\sum_{i=1}^3 S_i}, \quad c_2 = \frac{S_2}{\sum_{i=1}^3 S_i}, \quad c_3 = \frac{S_3}{\sum_{i=1}^3 S_i}, \quad (7)$$

$$c_1 + c_2 + c_3 = 1. \quad (8)$$

For any point lying on one side of the triangle, the corresponding barycentric coordinate will be equal to 0, which simplifies the calculations. For example, for a point that lies on the side  $A_1A_2$ , the values of the area  $S_3 = 0$  and, accordingly, the barycentric coordinate will be equal to  $c_3 = 0$ , while  $c_1 = 1 - c_2$ .

If the point  $C$  is considered as a pixel of the graphic image, then with the help of barycentric coordinates you can find other parameters of the point, necessary for its visualization, namely the value of light intensity, texture coordinates or normal vector at this point.

Consider how the vector of normal along one of the sides of a triangle will change  $A_1, A_2, A_3$ , namely along the side  $A_1A_2$ , provided that the vectors of normals  $\vec{N}_1$  and  $\vec{N}_2$  are known. Find the vector  $\vec{N}_{12}$  at the midpoint of the side  $A_1A_2$ , by the formula:

$$\vec{N}_{12} = \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{(\vec{N}_1 + \vec{N}_2)}} = \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{N_1^2 + N_2^2 + 2\vec{N}_1 \cdot \vec{N}_2}} \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \cos\psi_{12})}}, \quad (9)$$

where  $\psi_{12}$  – is the angle between the vectors of normals  $\vec{N}_1$  and  $\vec{N}_2$ . Note that the vectors  $\vec{N}_1, \vec{N}_2, \vec{N}_3$  are normalized. The scalar product of two vectors is the product of the modules of these vectors by the cosine of the angle between them. Considering that the vectors  $\vec{N}_1$  i  $\vec{N}_2$  are normalized, their modules are equal to one, therefore

$$\vec{N}_1 \cdot \vec{N}_2 = |\vec{N}_1| \cdot |\vec{N}_2| \cdot \cos\psi_{12} = 1 \cdot 1 \cdot \cos\psi_{12} = \cos\psi_{12}. \quad (10)$$

Considering equation (10), replace in formula (9)  $\cos\psi_{12}$  by  $\vec{N}_1 \cdot \vec{N}_2$ .

The change in the vector of the normal along the side can be described by the quadratic Bézier curve [1, 2]:

$$\vec{N}(t) = (1-t)^2 T_1 + 2t(1-t) T_2 + t^2 T_3, \quad (11)$$

where  $t$  – is a parameter that varies along the side  $A_1 A_2$  and lies in the range  $[0;1]$ ,  $T_1$ ,  $T_2$  and  $T_3$  – reference points..

If  $t = 0$ , then  $\vec{N}(0) = \vec{N}_1 = T_1$ . When  $t = 1$ ,  $\vec{N}(1) = \vec{N}_2 = T_3$ . If  $t = 0,5$ , then  $N(0.5) = \vec{N}_{12} = 0,25 T_1 + 2 \cdot 0,25 \cdot T_2 + 0,25 T_3 = 0,25 \vec{N}_1 + 0,5 T_2 + 0,25 \vec{N}_2$ .

From the last equation find

$$T_2 = \frac{\vec{N}_{12} - 0,25 \vec{N}_1 - 0,25 \vec{N}_2}{0,5} = 2 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} - 0,5 \vec{N}_1 - 0,5 \vec{N}_2.$$

$$\text{Then } 2T_2 = 4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} - \vec{N}_1 - \vec{N}_2.$$

Considering that in this case  $c_3 = 0$ , then  $c_1 = 1-t$ ,  $c_2 = t$ . The equation in the barycentric coordinates will look like this

$$\vec{N}(c_1, c_2) = \vec{N}_1 \cdot c_1^2 + \vec{N}_2 \cdot c_2^2 + \left( 4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} - \vec{N}_1 - \vec{N}_2 \right) \cdot c_1 c_2. \quad (12)$$

Similarly, you can display a change in the normal along the sides  $A_2 A_3$  i  $A_3 A_1$

$$\vec{N}(c_2, c_3) = \vec{N}_2 \cdot c_2^2 + \vec{N}_3 \cdot c_3^2 + \left( 4 \frac{\vec{N}_2 + \vec{N}_3}{\sqrt{2(1 + \vec{N}_2 \cdot \vec{N}_3)}} - \vec{N}_2 - \vec{N}_3 \right) \cdot c_2 c_3, \quad (13)$$

$$\vec{N}(c_3, c_1) = \vec{N}_3 \cdot c_3^2 + \vec{N}_1 \cdot c_1^2 + \left( 4 \frac{\vec{N}_3 + \vec{N}_1}{\sqrt{2(1 + \vec{N}_3 \cdot \vec{N}_1)}} - \vec{N}_3 - \vec{N}_1 \right) \cdot c_3 c_1. \quad (14)$$

The function of changing the normal vector on the surface of a triangle  $A_1 A_2 A_3$  can be represented by the sum of equations (12)–(14), subtracting the product of the normal vector by the square of the corresponding barycentric coordinate at the vertices of the triangle:

$$\begin{aligned} \vec{N}(c_1, c_2, c_3) &= \vec{N}_1 c_1^2 + \vec{N}_2 c_2^2 + \left( 4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} - \vec{N}_1 - \vec{N}_2 \right) c_1 c_2 + \vec{N}_2 c_2^2 + \\ &+ \vec{N}_3 c_3^2 + \left( 4 \frac{\vec{N}_2 + \vec{N}_3}{\sqrt{2(1 + \vec{N}_2 \cdot \vec{N}_3)}} - \vec{N}_2 - \vec{N}_3 \right) c_2 c_3 + \vec{N}_3 c_3^2 + \vec{N}_1 c_1^2 + \\ &+ \left( 4 \frac{\vec{N}_3 + \vec{N}_1}{\sqrt{2(1 + \vec{N}_3 \cdot \vec{N}_1)}} - \vec{N}_3 - \vec{N}_1 \right) c_3 c_1 - \vec{N}_1 c_1^2 - \vec{N}_2 c_2^2 - \vec{N}_3 c_3^2. \end{aligned}$$

Simplifying, we obtain the following formula

$$\begin{aligned} \vec{N}(c_1, c_2, c_3) = & \vec{N}_1 c_1^2 + \vec{N}_2 c_2^2 + \vec{N}_3 c_3^2 + \left( 4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} - \vec{N}_1 - \vec{N}_2 \right) c_1 c_2 + \\ & + \left( 4 \frac{\vec{N}_2 + \vec{N}_3}{\sqrt{2(1 + \vec{N}_2 \cdot \vec{N}_3)}} - \vec{N}_2 - \vec{N}_3 \right) c_2 c_3 + \left( 4 \frac{\vec{N}_3 + \vec{N}_1}{\sqrt{2(1 + \vec{N}_3 \cdot \vec{N}_1)}} - \vec{N}_3 - \vec{N}_1 \right) c_3 c_1. \end{aligned}$$

Having opened brackets and regrouped members, we will receive

$$\begin{aligned} \vec{N}(c_1, c_2, c_3) = & \vec{N}_1 c_1 (c_1 - c_2 - c_3) + \vec{N}_2 c_2 (c_2 - c_1 - c_3) + \vec{N}_3 c_3 (c_3 - c_2 - c_1) + \\ & + 4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} \cdot c_1 c_2 + 4 \frac{\vec{N}_2 + \vec{N}_3}{\sqrt{2(1 + \vec{N}_2 \cdot \vec{N}_3)}} \cdot c_2 c_3 + 4 \frac{\vec{N}_3 + \vec{N}_1}{\sqrt{2(1 + \vec{N}_3 \cdot \vec{N}_1)}} \cdot c_3 c_1. \end{aligned}$$

The last formula can be simplified by performing the transformation in parentheses. It is known that if add and subtract 1 to the expression, its value will not change. Taking into account this property and equation (14) find

$$\begin{aligned} c_1 - c_2 - c_3 &= c_1 - 1 + 1 - c_2 - c_3 = c_1 - 1 + c_1 = 2c_1 - 1, \\ c_2 - c_1 - c_3 &= c_2 - 1 + 1 - c_1 - c_3 = c_2 - 1 + c_2 = 2c_2 - 1, \\ c_3 - c_2 - c_1 &= c_3 - 1 + 1 - c_2 - c_1 = c_3 - 1 + c_3 = 2c_3 - 1. \end{aligned} \quad (15)$$

Considering (15) the formula for determination  $\vec{N}(c_1, c_2, c_3)$  will look like

$$\begin{aligned} \vec{N}(c_1, c_2, c_3) = & \vec{N}_1 c_1 (2c_1 - 1) + \vec{N}_2 c_2 (2c_2 - 1) + 4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}} c_1 c_2 + \\ & + \vec{N}_3 c_3 (2c_3 - 1) + 4 \frac{\vec{N}_2 + \vec{N}_3}{\sqrt{2(1 + \vec{N}_2 \cdot \vec{N}_3)}} c_2 c_3 + 4 \frac{\vec{N}_3 + \vec{N}_1}{\sqrt{2(1 + \vec{N}_3 \cdot \vec{N}_1)}} c_3 c_1. \end{aligned} \quad (16)$$

Expressions  $4 \frac{\vec{N}_1 + \vec{N}_2}{\sqrt{2(1 + \vec{N}_1 \cdot \vec{N}_2)}}$ ,  $4 \frac{\vec{N}_2 + \vec{N}_3}{\sqrt{2(1 + \vec{N}_2 \cdot \vec{N}_3)}}$  and  $4 \frac{\vec{N}_3 + \vec{N}_1}{\sqrt{2(1 + \vec{N}_3 \cdot \vec{N}_1)}}$  are

constant for any point of the surface bounded by a triangle, so there is no need to calculate them for each point. Denote these expressions by  $\vec{M}_{12}$ ,  $\vec{M}_{23}$  and  $\vec{M}_{31}$ .

Equation (16) will look like

$$\begin{aligned} \vec{N}(c_1, c_2, c_3) = & \vec{N}_1 c_1 (2c_1 - 1) + \vec{N}_2 c_2 (2c_2 - 1) + \vec{N}_3 c_3 (2c_3 - 1) + \\ & + \vec{M}_{12} c_1 c_2 + \vec{M}_{23} c_2 c_3 + \vec{M}_{31} c_3 c_1. \end{aligned} \quad (17)$$

The obtained expression allows to find the vector at any point of the surface bounded by a triangle, according to the values of the vectors in its vertices and the barycentric coordinates of the current point.

### 3. Software implementation

A computer program based on an open professional conveyor for modeling and testing methods has been developed.

To develop a software product, the Java programming language was chosen, the implementation of which is adapted to different platforms (the most famous of which are Win32, Unix/x86, Unix/Alpha, Solaris/Sparc, MacOS). A large number of libraries have been developed for the Java language, which greatly simplifies the development of software products. The advantage of the Java language is that the software module with an interface based on Swing or AWT can be easily adapted for use in the form of a Java applet, which allows you to use the software module directly from an Internet browser.

The software module is based on the idx3d library, which has basic functions for working with 3D models and three-dimensional geometry. The object-oriented model of the software module is shown in Fig. 11.

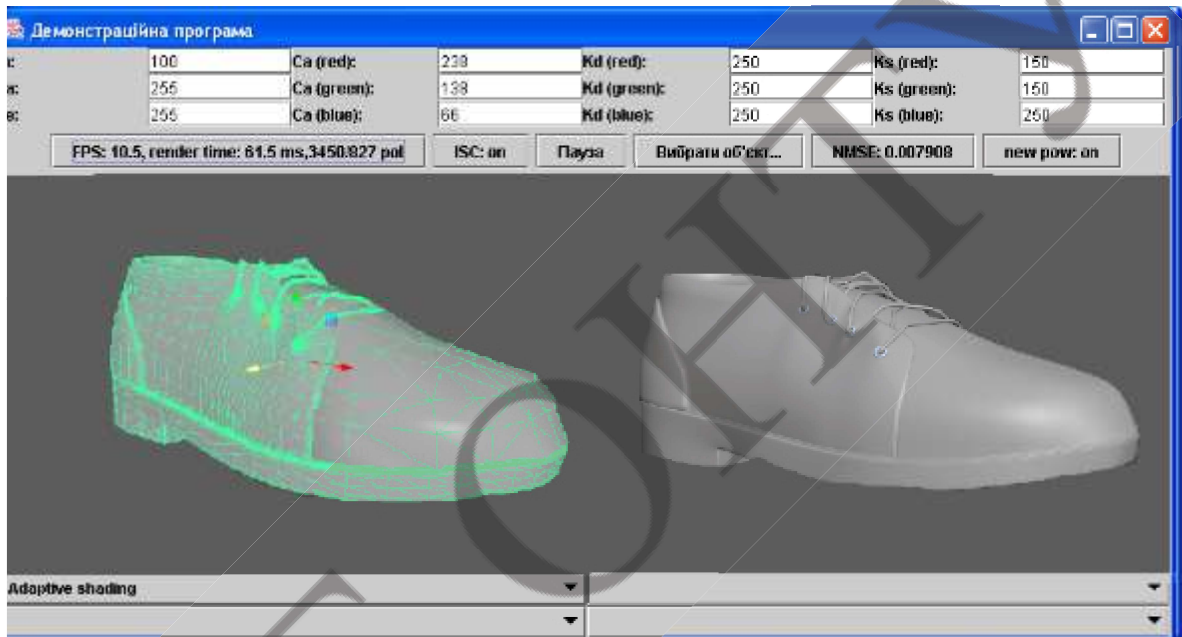


Figure 10 - Example of image formation using additional triangulation

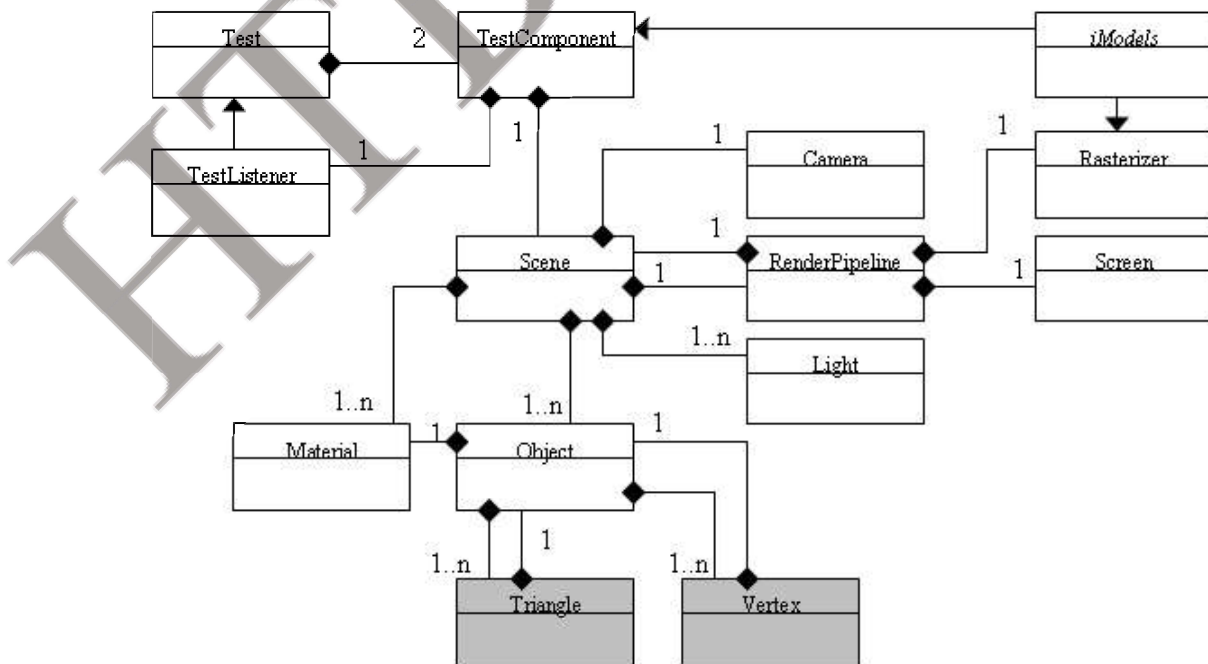


Figure 11 - Oriented model of the software module



Figure 12 - Example of triangulation

The simulation performed using the developed program confirmed the validity of the obtained theoretical positions.

**Scientific novelty of the work.**

1. An expression is obtained to determine the vector at an arbitrary point of the surface bounded by a triangle by the values of the vectors at its vertices and

the barycentric coordinates of the current point, which allows to detail the surface to increase realism.

2. The division of the output triangle into components with the same area is proposed, which allows to achieve a balanced loading of shader processors.
3. The obtained relations for detailing the surfaces of three-dimensional objects by Serpinsky triangulation makes it possible to increase the realism of image formation.
4. For the first time, it is proposed to perform triangulations of three-dimensional objects depending on the maximum values of color intensity on the edges of the triangle, which allows the use of adaptive triangulation.

The practical significance of the work is to develop on the basis of theoretical studies of algorithms and programs for triangulation and integrate them into a professional graphics engine `idx3d`.

8 scientific works have been published on the topic of research, including one article in a professional publication.

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