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Representing trees of finite ultrametric spaces and weak similarities

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An *ultrametric* on a set X is a function $d: X \times X \rightarrow \mathbb{R}^+$, $\mathbb{R}^+ = [0, \infty)$, such that for all $x, y, z \in X$:

- (i) $d(x, y) = d(y, x)$,
- (ii) $(d(x, y) = 0) \Leftrightarrow (x = y)$,
- (iii) $d(x, y) \leq \max\{d(x, z), d(z, y)\}$.

The pair (X, d) is called an *ultrametric space*. If condition (iii) is omitted, then (X, d) is a *semimetric space*, see [1]. The *spectrum* of a semimetric space (X, d) is the set

$$\text{Sp}(X) = \{d(x, y) : x, y \in X\}.$$

Recall that a *graph* is a pair (V, E) consisting of a nonempty set V and a (probably empty) set E elements of which are unordered pairs of different points from V . For a graph $G = (V, E)$, the sets $V = V(G)$ and $E = E(G)$ are called *the set of vertices* and *the set of edges*, respectively. A connected graph without cycles is called a *tree*. A tree T may have a distinguished vertex called the *root*; in this case T is called a *rooted tree*. With every finite ultrametric space (X, d) it is possible to associate a labeled rooted tree T_X , which is called a *representing tree* of the space X , see, for example, [2, P. 109].

Definition 1. Let T_1 and T_2 be rooted trees with the roots v_1 and v_2 respectively. A bijective function $\Psi: V(T_1) \rightarrow V(T_2)$ is an isomorphism of T_1 and T_2 if

$$(\{x, y\} \in E(T_1)) \Leftrightarrow (\{\Psi(x), \Psi(y)\} \in E(T_2))$$

for all $x, y \in V(T_1)$ and $\Psi(v_1) = v_2$. If there exists an isomorphism of rooted trees T_1 and T_2 , then we will write $T_1 \simeq T_2$.

Definition 2. Let (X, d) and (Y, ρ) be semimetric spaces. A bijective mapping $\Phi: X \rightarrow Y$ is a *weak similarity* if there exists a strictly increasing bijection $f: \text{Sp}(X) \rightarrow \text{Sp}(Y)$ such that the equality

$$f(d(x, y)) = \rho(\Phi(x), \Phi(y))$$

holds for all $x, y \in X$. If $\Phi: X \rightarrow Y$ is a weak similarity, then we write $X \stackrel{w}{=} Y$ and say that X and Y are *weakly similar*.

The notion of weak similarity of semimetric spaces was introduced in [3] in a slightly different form, where also some properties of these mappings were studied.

Denote by $\tilde{\mathfrak{R}}$ the class of finite ultrametric spaces X for which T_X has exactly one inner node at each level except the last level. The rooted tree T_X without the labels we will denote by \bar{T}_X .

The next theorem gives a description of finite ultrametric spaces for which the isomorphism of representing trees implies the weak similarity of the spaces.

Theorem 3 ([2]). *Let X be a finite ultrametric space. Then the following statements are equivalent.*

- (i) *The implication $(\bar{T}_X \simeq \bar{T}_Y) \Rightarrow (X \stackrel{w}{=} Y)$ holds for every finite ultrametric space Y .*
- (ii) $X \in \tilde{\mathfrak{R}}$.

Denote by \mathfrak{D} the class of all finite ultrametric spaces X such that the different internal nodes of T_X have the different labels. It is clear that $\tilde{\mathfrak{R}}$ is a subclass of \mathfrak{D} . A question arises whether there exist finite ultrametric spaces $X, Y \in \mathfrak{D}$ which do not belong to the class $\tilde{\mathfrak{R}}$ and for which the isomorphism of \bar{T}_X and \bar{T}_Y implies $X \stackrel{w}{=} Y$.

Let us define a rooted tree T with n levels by the following two conditions:

(A) There is only one inner node at the level k of T whenever $k < n - 1$.

(B) If u and v are different inner nodes at the level $n - 1$ then the numbers of offsprings of u and v are equal.

Denote by \mathfrak{T} the class of all finite ultrametric spaces X for which T_X satisfies conditions (A) and (B).

Theorem 4 ([2]). *Let $X \in \mathfrak{D}$ be a finite ultrametric space. Then the following statements are equivalent.*

- (i) *The implication $(\overline{T}_X \simeq \overline{T}_Y) \Rightarrow (X \stackrel{w}{=} Y)$ holds for every finite ultrametric space $Y \in \mathfrak{D}$.*
- (ii) *$X \in \mathfrak{T}$.*

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