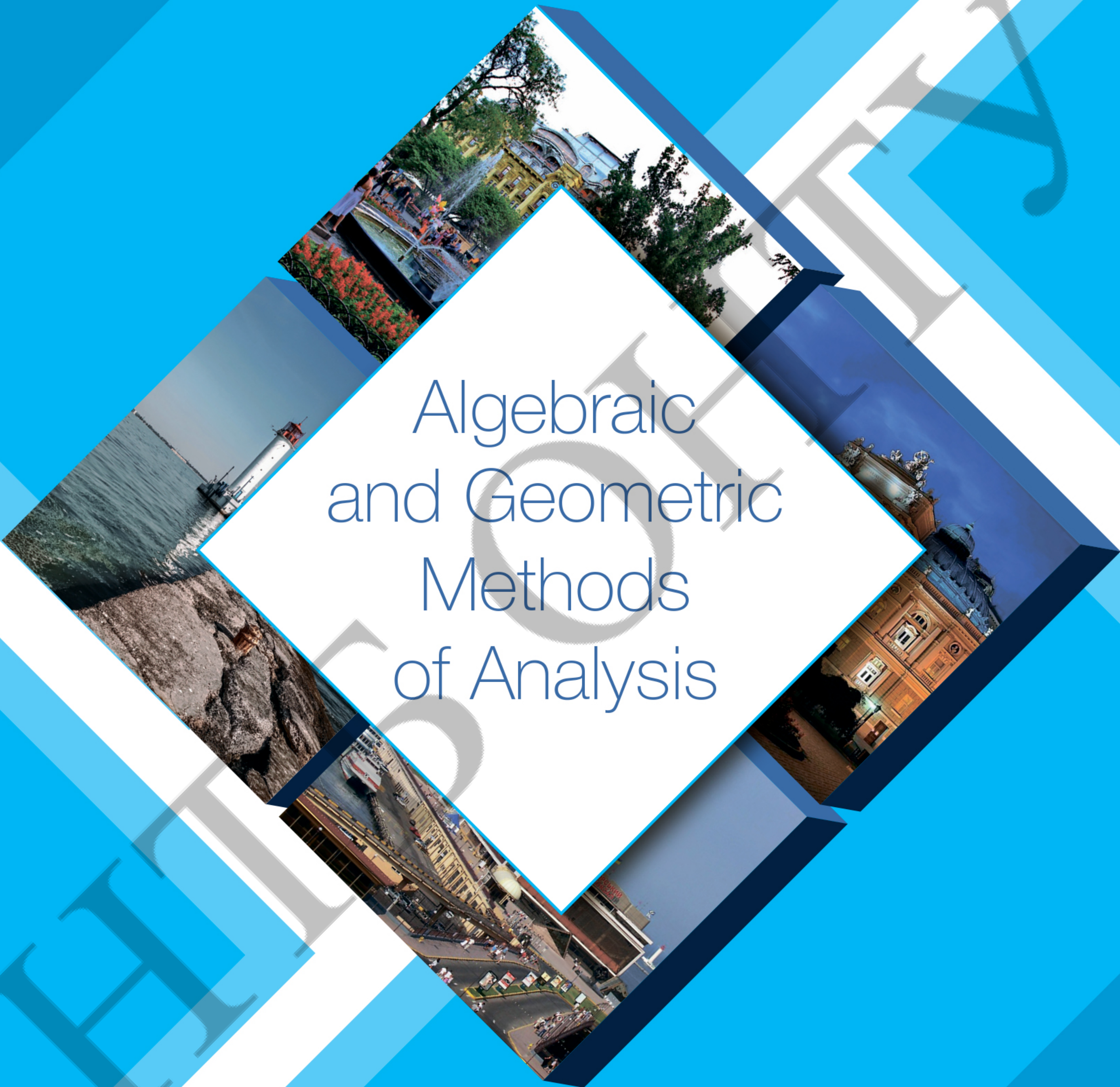
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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On some vanishing theorems of global character about geodesic mappings of complete Riemannian spaces

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Generalization of Bocher technique (see for example, [1]) allows to broad to the complete Riemannian spaces a lot of theorems of geodesic unique definiteness on the whole proved previousely only for the compact ones (see for example, [2]). It seems to be interesting to indicate some of them.

Theorem 1. *Complete Ricci semi-symmetric Riemannian C^r-spaces V^n $(n > 2,\ r > 4)$ with positively definite metric form, Einstein tensor of which doesn't equal to zero, don't admit non-trivial (different from the affine) geodesic mappings on the whole.*

Theorem 2. *Complete Riemannian C^r-spaces V^n $(n > 2,\ r > 4)$ with positively definite metric form and non-negative scalar curvature $(R \geq 0)$ don't admit non-trivial (different from the affine) geodesic mappings on the whole.*

Theorem 3. *Complete Ricci semi-symmetric Riemannian C^r-spaces V^n $(n > 2,\ r > 4)$ with positively definite Ricci form, Einstein tensor of which doesn't equal to zero, scalar curvature of which preserves its sign ($R \geq 0$ or $R \leq 0$ everywhere in V^n) don't admit non-trivial (different from the affine) geodesic mappings on the whole.*

Theorem 4. *Complete Ricci semi-symmetric Riemannian C^r-spaces V^n $(n > 2,\ r > 4)$ with positively definite Einstein form don't admit non-trivial (different from the affine) geodesic mappings on the whole.*

Examples of Riemannian spaces of the considered types are known.

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Subwreath product as structure of normal subgroups of permutational wreath products

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In this research we continue our previous investigation of wreath product normal structure [1, 2] Normal subgroups and there structures for finite and infinite iterated wreath products $S_{n_1} \wr \ldots \wr S_{n_m}$, $n, m \in \mathbb{N}$ and $A_n \wr S_n$ are founded.

Let $k(\pi)$ be the number of cycles in decomposition of permutation π of degree n.

The number $n-k(\pi)$ is denoted by $dec(\pi)$, and is called a decrement [6] of permutation π. As well known [6] the minimal number of transpositions in factorization of a permutation π on transpositions is happen to be equal to $dec(\pi)$. We set $dec(e)=0$. If $\pi_1,\pi_2 \in S_n$, then the following formula holds:

$$dec(\pi_1 \cdot \pi_2) = dec(\pi_1) + dec(\pi_2) - 2m, m \in \mathbf{N}, \tag{1}$$

Definition 1. The permutational *subwreath product* $G \wr\wr H$ is the semi-direct product $G \ltimes \tilde{H}^X$, where G acts on the subdirect product [4] $\tilde{H}^X$ by the respective permutations of the subdirect factors. Provided the specification of $\tilde{H}^X$ is established separately.

Definition 2. The set of elements from $S_n \wr S_n, n \geqslant 3$ which presented by the tableaux of form: $[e]_0,\ [a_1,a_2,\ldots,a_n]_1$, satisfying the following condition

$$\sum_{i=1}^{n} dec([a_i]_1) = 2k, k \in \mathbb{N}, \tag{2}$$

be called set of type $\widetilde{A}_n^{(1)}$. Note that condition (2) uniquely identifies subdirect product.

The set $\widetilde{A}_n^{(1)}$ is normal subgroup having **normal rank** 2 [7] in $S_n \wr S_n$ and be denoted by $E \wr\wr \widetilde{A}_n$. We spread this definition on 3-multiple wreath product by recursive way.

Definition 3. *The subgroup* $E \wr \widetilde{A}_n^{(1)}$ *be denoted by* $\widetilde{A}_n^{(2)}$.

Furthermore we prove that $E \wr \widetilde{A}_n^{(2)} \triangleleft S_n \wr S_n \wr S_n$. The order of $E \wr \widetilde{A}_n^{(2)}$ is $(n!)^{3n} : 2^3$. The subgroup $\widetilde{A}_n^{(1)}$ has **normal rank** 2 in $S_n \wr S_n$.

Definition 4. The set of elements from $S_n \wr S_n \wr S_n, n \geqslant 3$ presented by the tables [3] form: $[e]_1,\ [e,e,\ldots,e]_2, [a_1,a_2,\ldots,a_n]_2$, satisfying the following condition

$$\sum_{i=1}^{n} dec([a_i]_2) = 2k, k \in \mathbb{N}, \tag{3}$$

be denoted by $\widetilde{A}_{n^2}^{(2)}$. Note that condition (3) uniquely identifies subdirect product in $\prod_{i=1}^{n^2} S_n$ as base of subwreath product, the similar subdirect product describing commutator of wreath product was investigated by us in [8] in research of pronormality it appears in [9].

Proposition 5. *The subgroup* $\widetilde{A}_n^{(1)} \triangleleft S_n \wr S_n$ *as well as* $\widetilde{A}_n^{(2)} \triangleleft S_n \wr S_n \wr S_n$. *Furthermore* $\widetilde{A}_n^{(2)} \triangleleft \widetilde{A}_{n^2}^{(2)}$.

Definition 6. *A subgroup in* $S_n \wr S_n$ *is called* $\widetilde{T_n}$ *if it consists of:*

(1) elements of $E \wr A_n$ *,*
(2) elements with the tableau [3] presentation $[e]_1,\ [\pi_1,\ldots,\pi_n]_2$*, that* $\pi_i \in S_n \setminus A_n$.

One easy can validates a correctness of this definition, i.e. that the set of such elements form a subgroup and its normality. This subgroup has structure

$$\tilde{T}_n \simeq (\underbrace{A_n \times A_n \times \cdots \times A_n}_{n}) \rtimes C_2 \simeq \underbrace{S_n \boxplus S_n \ldots \boxplus S_n}_{n},$$

where the operation $\boxplus$ *of a subdirect product is subject of item 1) and 2)*

Remark 7. The order of $\widetilde{T_n}$ is $\frac{(n!)^n}{2^{n-1}}$.

Definition 8. The unique minimal normal subgroup is called the monolith.

Theorem 9. *The monolith of* $S_n \wr S_m$ *is* $e \wr A_m$.

Theorem 10. *Proper normal subgroups in* $S_n \wr S_m$*, where* $n, m \geq 3$ *with* $n, m \neq 4$ *are of the following types:*

(1) subgroups that act only on the second level are

$$E \wr\wr \widetilde{A_m},\ \widetilde{T_m},\ E \wr S_m, E \wr A_m,$$

(2) subgroups that act on both levels are $A_n \wr\wr \widetilde{A_m}$, $S_n \wr\wr \widetilde{A_m}$, $A_n \wr S_m$,

wherein the subgroup $S_n \wr\wr \widetilde{A_m} \simeq S_n \leftthreetimes (\underbrace{S_m \boxtimes S_m \boxtimes S_m \ldots \boxtimes S_m}_{n})$ *endowed with the subdirect product satisfying to condition* (2).

Theorem 11. *The full list of normal subgroups of* $W = S_n \wr S_n \wr S_n$ *consists of 50 normal subgroups. These subgroups are the following:*

1. ***Type*** T_{023} *contains:* $E \wr\wr \tilde{A}_n \wr\wr H$, $\widetilde{T_n} \wr H$*, where* $H \in \{\tilde{A}_n, \tilde{A}_{n^2}, S_n\}$. *There are 6 subgroups.*
2. ***The second type of subgroups is subclass in*** T_{023} *with new base of wreath product subgroup* $\tilde{A}_{n^2}$*:* $E \wr S_n \wr\wr \tilde{A}_{n^2}$, $E \wr N_i(S_n \wr S_n)$. *Therefore this class has 12 new subgroups. Thus, the total number of normal subgroups in* ***Type*** T_{023} *is 18.*
3. ***Type*** T_{003}*:* $A^{(3)}_{00(n^2)}$, $\widetilde{T_{n^2}}$, $\widetilde{T_n}^{(3)}$.
4. ***Type*** T_{123}*:* $N_i(S_n \wr S_n) \wr S_n$, $N_i(S_n \wr S_n) \wr \tilde{A}_n$ *and* $N_i(S_n \wr S_n) \wr \tilde{A}_{n^2}$. *Thus, there are 29 new normal subgroups in* T_{123}*, taking into account a repetition.*

Remark 12. Note that $E \wr \widetilde{A}_n^{(1)} \simeq E \wr (E \wr\wr \widetilde{A}_n)$ contains in the family $E \wr N_i(S_n \wr S_n)$.

We denote by $Aut_f X^*$ the group of all finite automorphism of spherically homogeneous rooted tree.

Theorem 13. *Let* $H \lhd Aut_f X^*$ *having depth* k*, then* H *contains* k*-th level subgroup* P *having all even vertex permutations* $p_{ki} \in A_n$ *on* X^k *and trivial permutations in vertices of rest of levels.*

Furthermore P *is normal in* $Aut_f X^*$ *provided* k *is last active level of* $Aut_f X^*$.

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