International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Bolotov D.** (*Kharkiv, Ukraine*)
- **Bondarenko V.** (*Kyiv, Ukraine*)
- **Boychuk O.** (*Kyiv, Ukraine*)
- **Boyko V.** (*Kyiv, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Dorogovtsev A.** (*Kyiv, Ukraine*)
- **Drozd Yu.** (*Kyiv, Ukraine*)
- **Gerasymenko V.** (*Kyiv, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Kiosak V.** (*Odesa, Ukraine*)
- **Kochubei A.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Mikhailets V.** (*Kyiv, Ukraine*)
- **Ostrovskyi V.** (*Kyiv, Ukraine*)
- **Petravchuk A.** (*Kyiv, Ukraine*)
- **Plaksa S.** (*Kyiv, Ukraine*)
- **Portenko M.** (*Kyiv, Ukraine*)
- **Pratsiovytyi M.** (*Kyiv, Ukraine*)
- **Savchenko O.** (*Kherson, Ukraine*)
- **Romanyuk A.** (*Kyiv, Ukraine*)
- **Timokha O.** (*Kyiv, Ukraine*)

Organizing commitee

- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Osadchuk Ye.** (*Odesa, Ukraine*)
- **Sergeeva O.** (*Odesa, Ukraine*)

Lie structures of the Sheffer group over a Hilbert space

Eugene Lytvynov
(Swansea University, Bay Campus, Swansea, SA1 8EN, UK)
E-mail: `e.lytvynov@swansea.ac.uk`

Umbral calculus (also called calculus of finite differences) is essentially the theory of Sheffer polynomial sequences, which are characterised by the exponential form of their generating function. The class of Sheffer sequences includes the binomial sequences and Appell sequences. After a long period when one-dimensional umbral calculus was used for purely formal calculations, the theory became rigorous in the 1970s due to the seminal works of G.-C. Rota, S. Roman and their co-authors. Their theory is nowadays called the modern umbral calculus, see e.g. the monographs [4, 8]. Umbral calculus found applications in combinatorics, theory of special functions, approximation theory, probability and statistics, topology and physics, see e.g. the survey paper [2] for a long list of references. A central object of studies of umbral calculus is the umbral composition, which equips the set of all Sheffer sequences with a group structure. This group is isomorphic to the Riordan group of infinite lower triangular matrices [6, 10]. Recently, Cheon et al. [3] (see also Bacher [1]) introduced a Lie group structure on the Riordan group and found the corresponding Lie algebra.

A lot of research has been done to extend the classical umbral calculus to the multivariate case, see Section 4 in [2] for a list of references. However, this research had a significant drawback of being basis-dependent. The paper [5] developed foundations of infinite-dimensional, basis-independent umbral calculus.

In this talk, we will discuss Lie structures of the group of Sheffer polynomials over a Hilbert space. Let

$$\mathcal{H}_+ \subset \mathcal{H}_0 \subset \mathcal{H}_0$$

be standard triple of real separable Hilbert spaces, i.e., the Hilbert space $\mathcal{H}_+$ is densely and continuously embedded into $\mathcal{H}_0$ and $\mathcal{H}_-$ is the dual of $\mathcal{H}_+$, while the dual paring between elements of $\mathcal{H}_-$ and $\mathcal{H}_+$ is determined by the inner product in $\mathcal{H}_0$. Then, for each n, we also get a standard triple

$$\mathcal{H}_+^{\odot n} \subset \mathcal{H}_0^{\odot n} \subset \mathcal{H}_0^{\odot n}.$$

Here $\odot$ denotes the symmetric tensor product. For $F^{(n)} \in \mathcal{H}_-^{\odot n}$ and $f^{(n)} \in \mathcal{H}_+^{\odot n}$, we denote by $\langle F^{(n)}, f^{(n)} \rangle$ the dual pairing between $F^{(n)}$ and $f^{(n)}$. (For a real Hilbert space $\mathcal{H}$, we define $\mathcal{H}^{\odot 0} := \mathbb{R}$.)

A (continuous) polynomial on $\mathcal{H}_-$ is a function $p : \mathcal{H}_- \to \mathbb{R}$ of the form

$$p(\omega) = \sum_{i=0}^{n} \langle \omega^{\odot i}, f^{(i)} \rangle, \quad \omega \in \mathcal{H}_-,\ f^{(i)} \in \mathcal{H}_+^{\odot i}, \quad i = 0, 1, \dots, n,\ n \in \mathbb{N}_0. \tag{1}$$

We denote by $\mathcal{P}(\mathcal{H}_-)$ the vector space of all polynomials on $\mathcal{H}_-$. By identifying the polynomial $p(\omega)$ in (1) with the sequence $(f^{(i)})$, we endow $\mathcal{P}(\mathcal{H}_-)$ with the topology of the topological direct sum of the Hilbert spaces $\mathcal{H}_+^{\odot i}$, $i \in \mathbb{N}_0$.

A monic polynomial sequence on $\mathcal{H}_-$ is a continuous linear map $P \in \mathcal{L}(\mathcal{P}(\mathcal{H}_-))$ that satisfies

$$(P\langle \cdot^{\odot n}, f^{(n)} \rangle)(\omega) = \sum_{i=0}^{n} \langle \omega^{\odot i}, p_{in} f^{(n)} \rangle, \tag{2}$$

where $p_{in} \in \mathcal{L}(\mathcal{H}_+^{\odot n}, \mathcal{H}_+^{\odot i})$ and $p_{nn} = \mathbf{1}$. Denote by $p_{in}^* \in \mathcal{L}(\mathcal{H}_+^{\odot i}, \mathcal{H}_+^{\odot n})$ the adjoint (dual) operator of p_{in}. Then

$$(P\langle \cdot^{\odot n}, f^{(n)}\rangle)(\omega) = \langle p^{(n)}(\omega), f^{(n)}\rangle,$$

where $p^{(n)}(\omega) \in \mathcal{H}_-^{\odot n}$ is given by $p^{(n)}(\omega) := \sum_{i=0}^n p_{in}^* \omega^{\odot i}$. Thus, $p^{(n)} : \mathcal{H}_- \to \mathcal{H}_-^{\odot n}$, and we can identify the linear operator P from (2) with the sequence $(p^{(n)})_{n=0}^\infty$.

A monic polynomial sequence $(p^{(n)})_{n=0}^\infty$ is called a Sheffer sequence (on $\mathcal{H}_-$) if it has the exponential generating function of the form

$$\sum_{n=0}^\infty \frac{1}{n!}\langle p^{(n)}(\omega), \xi^{\odot n}\rangle = \exp\left[\langle \omega, B(\xi)\rangle\right] A(\xi), \quad \omega \in \mathcal{H}_-,\ \xi \in \mathcal{H}_+, \tag{3}$$

where $B(\xi) = \xi + \sum_{k=2}^\infty b_k \xi^{\odot k}$, $b_k \in \mathcal{L}(\mathcal{H}_+^{\odot k}, \mathcal{H}_+)$, $A(\xi) = 1 + \sum_{k=1}^\infty a_k \xi^{\odot k}$, $a_k \in \mathcal{L}(\mathcal{H}_+^{\odot k}, \mathbb{R})$, and the equality (3) is understood as the equality of formal tensor power series in ξ, see [5]. We denote by $\mathbb{S}$ the set of all Sheffer sequences on $\mathcal{H}_-$. We also denote by $\mathbb{A}$ the set of all Appell sequences, i.e., the Sheffer sequences for which $B(\xi) = \xi$ in (3), and we denote by $\mathbb{B}$ the set of all the binomial sequences, i.e., the Sheffer sequences for which $A(\xi) = 1$ in (3).

Since elements of $\mathbb{S}$ were defined through continuous linear operators in $\mathcal{P}(\mathcal{H}_-)$, one can ask a natural question whether a product of two such operators yields a Sheffer sequence. The answer to this question is positive, and furthermore the set $\mathbb{S}$, equipped with this product, becomes a group. Note that the neutral element in this group is the identity operator, equivalently the monomial sequence $p^{(n)}(\omega) = \omega^{\odot n}$. Furthermore, both $\mathbb{A}$ and $\mathbb{B}$ are subgroups $\mathbb{S}$, $\mathbb{A}$ is a normal subgroup of $\mathbb{S}$, and the Sheffer group $\mathbb{S}$ is a semidirect product of the Appell group $\mathbb{A}$ and the binomial group $\mathbb{B}$.

In the talk, we will discuss the following results:

- We will show that $\mathbb{S}$, $\mathbb{A}$ and $\mathbb{B}$ can be described as infinite-dimensional Lie groups, in the sense of Milnor [7], see also [9, Chapter 3].
- We will find the explicit form of the Lie algebra of each of these Lie groups, and we will find a Lie bracket on them.
- We will conclude that the Sheffer group is constructed from two basic operations: gradient of polynomials on $\mathcal{H}_-$ and multiplication by ω.

This is joint result with Dmitri Finkelshtein (Swansea University) and Maria João Oliveira (Universidade Aberta, Lisbon).

References

[1] Bacher, R.: Sur le groupe d'interpolation. arXiv:math/0609736, 2006.
[2] Di Bucchianico, A., Loeb, D.: A selected survey of umbral calculus. *Electron. J. Combin.* 2 (1995), Dynamic Survey 3, 28 pp. (electronic).
[3] Cheon, G.-S., Luzón, A., Morón, M.A. Prieto-Martinez, L.F., Song, M.: Finite and infinite dimensional Lie group structures on Riordan groups. *Adv. Math.* 319 (2017), 522–566.
[4] Costabile, F.A.: Modern umbral calculus. De Gruyter, Berlin/Boston, 2019.
[5] Finkelshtein, D., Kondratiev, Y., Lytvynov, E., Oliveira, M.J.: An infinite dimensional umbral calculus. *J. Funct. Anal.* 276 (2019), 3714–3766.
[6] He, T.-X., Hsu, L.C., Shiue, P.J.-S.: The Sheffer group and the Riordan group. *Discrete Appl. Math.* 155 (2007), 1895–1909.
[7] Milnor, J.: Remarks on infinite-dimensional Lie groups. Relativity, groups and topology, II, pp. 1007–1057, North-Holland, Amsterdam, 1984.
[8] Roman, S.: The umbral calculus. Academic Press, New York, 1984.

[9] Schmeding, A.: An introduction to infinite-dimensional differential geometry. Cambridge University Press, Cambridge, 2023.
[10] Shapiro, L.W., Getu, S., Woan, W.J., Woodson, L.C.: The Riordan group. *Discrete Appl. Math.* 34 (1991), 229–239.

The Gorenstein flat model structure relative to a semidualizing module

Rachid El Maaouy
(CeReMaR Research Center, Faculty of Sciences, B.P. 1014, Mohammed V University in Rabat, Rabat, Morocco)
E-mail: elmaaouy.rachid@gmail.com

Driss Bennis
(CeReMaR Research Center, Faculty of Sciences, B.P. 1014, Mohammed V University in Rabat, Rabat, Morocco)
E-mail: driss.bennis@um5.ac.ma

J. R. Garcia Rozas
(Departamento de Matematicas, Universidad de Almeria, 04071 Almeria, Spain)
E-mail: jrgrozas@ual.es

Luis Oyonarte
(Departamento de Matematicas, Universidad de Almeria, 04071 Almeria, Spain)
E-mail: oyonarte@ual.es

Abstract.
A model structure on a category is a formal way of introducing a homotopy theory on that category, and if the model structure is abelian and hereditary, its homotopy category is known to be triangulated. So a good way to both build and model a triangulated category is to build a hereditary abelian model structure.

Let R be a ring and C be a left R-module. In this talk, we construct a unique hereditary abelian model structure on the category of left R-modules, in which the cofibrations are the monomorphisms with $\mathrm{G_C}$-flat cokernel and the fibrations are the epimorphisms with $\mathcal{C}_C$-cotorsion kernel belonging to the Bass class $\mathcal{B}_C(R)$.

References

[1] D. Bennis, R. El Maaouy, J. R. García Rozas and L. Oyonarte, Relative Gorenstein flat modules and dimension, Comm. Alg., (2022).
[2] D. Bennis, R. El Maaouy, J. R. García Rozas and L. Oyonarte, Relative Gorenstein flat modules and Foxby classes and their model structures, https://arxiv.org/abs/2205.02032.
[3] M. Hovey, *Model categories*, Mathematical Surveys and Monographs 63 (American Mathematical Society, Providence, RI, 1999).
[4] M. Hovey, *Cotorsion pairs, model category structures, and representation theory*, Math. Z. 241 (2002), 553-592.