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**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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A connection between L -index of vector-valued entire function and L -index of each its component

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The present talk is devoted to the properties of entire vector-valued functions of bounded L -index in joint variables. We need some notations and definitions. Let $\mathbf{L} : \mathbb{C}^n \rightarrow \mathbb{R}_+^n$ be any fixed continuous function. We consider a class of vector-valued entire functions $F = (f_1, \dots, f_p) : \mathbb{C}^n \rightarrow \mathbb{C}^p$. For this class of functions there was introduced a concept of boundedness of L -index in joint variables.

Let $\|\cdot\|_0$ be a norm in \mathbb{C}^p . Let $\mathbf{L}(z) = (l_1(z), \dots, l_n(z))$, where $l_j(z) : \mathbb{C}^n \rightarrow \mathbb{R}_+$ is a positive continuous function. An entire vector-valued function $F : \mathbb{C}^n \rightarrow \mathbb{C}^p$ is said to be of bounded L -index in joint variables, if there exists $n_0 \in \mathbb{Z}_+$ such that $(\forall z = (z_1, \dots, z_n) \in \mathbb{C}^n)(\forall J \in \mathbb{Z}_+^n)$:

$$\frac{\|F^{(J)}(z)\|_0}{J! \mathbf{L}^J(z)} \leq \max \left\{ \frac{\|F^{(K)}(z)\|_0}{K! \mathbf{L}^K(z)} : K \in \mathbb{Z}_+^n, \|K\| \leq n_0 \right\},$$

where $F^{(J)}(z) = (f_1^{(J)}(z), \dots, f_p^{(J)}(z))$, $f_k^{(J)}(z) = \frac{\partial^{\|J\|}}{\partial z_1^{j_1} \dots \partial z_n^{j_n}} f_k(z)$, $\|J\| = j_1 + \dots + j_n$, $J! = j_1! \cdot \dots \cdot j_n!$ for $J = (j_1, \dots, j_n)$, $k \in \{1, \dots, p\}$. The least such integer n_0 is called the L -index in joint variables and is denoted by $N(F, \mathbf{L})$.

Denote by $\mathbb{D}^n[z_0, R/\mathbf{L}(z_0)] = \{z = (z_1, \dots, z_n) \in \mathbb{C}^n : |z_j - z_{j0}| \leq r_j/l_j(z_0) \text{ for every } j \in \{1, \dots, n\}\}$ the closed polydisc in \mathbb{C}^n . Let Q^n be a class of continuous functions $\mathbf{L} : \mathbb{C}^n \rightarrow \mathbb{R}_+^n$ such that $0 < \lambda_{1,j}(R) \leq \lambda_{2,j}(R) < \infty$ for any $j \in \{1, 2, \dots, n\}$ and $\forall R = (r_1, \dots, r_n) \in \mathbb{R}_+^n$, where $\lambda_{1,j}(R) = \inf_{z_0 \in \mathbb{C}^n} \inf \{l_j(z)/l_j(z_0) : z \in \mathbb{D}^n[z_0, R/\mathbf{L}(z_0)]\}$, $\lambda_{2,j}(R)$ is defined analogously with replacement \inf by \sup .

For $F : \mathbb{C}^n \rightarrow \mathbb{C}^p$ let us introduce the sup-norm $|F(z)|_p = \max_{1 \leq j \leq p} \{|F_j(z)|\}$. The notation $A \leq B$ for $A = (a_1, \dots, a_n)$, $B = (b_1, \dots, b_n) \in \mathbb{R}^n$ means that $a_j \leq b_j$ for every $j \in \{1, \dots, n\}$. The following proposition was firstly deduced for analytic curves in [1]. Similar proposition was also obtained for analytic vector-valued functions $F : \mathbb{B}^2 \rightarrow \mathbb{C}^2$ in the unit ball $\mathbb{B}^2 = \{z \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 < 1\}$ [2]. Here we present it for vector-valued entire functions $F : \mathbb{C}^n \rightarrow \mathbb{C}^p$.

Proposition 1. *Let $\mathbf{L} = (l_1(z), \dots, l_n(z))$ be a positive continuous function in \mathbb{C}^n . If each component f_s of an entire vector-valued function $F = (f_1, \dots, f_p) : \mathbb{C}^n \rightarrow \mathbb{C}^p$ is of bounded L -index $N(\mathbf{L}, f_s)$ in joint variables then F is of bounded L -index in joint variables in every norm, in particular, in the sup-norm and $N(\mathbf{L}; F) \leq \max\{N(\mathbf{L}, f_s) : 1 \leq s \leq p\}$, and also F is of bounded L_* -index in the Euclidean norm with $\mathbf{L}_*(z, w) \geq \sqrt{p} \mathbf{L}(z, w)$ and $N_E(\mathbf{L}_*, F) \leq \max\{N(\mathbf{L}, f_s) : 1 \leq s \leq p\}$. (Here $N(\mathbf{L}, F)$ and $N_E(\mathbf{L}_*, F)$ are the L -index and the L_* -index in joint variables with the sup-norm and the Euclidean norm, respectively.)*

Theorem 2 ([3]). *Let $\mathbf{L} \in Q^n$. An entire vector-valued function $F : \mathbb{C}^n \rightarrow \mathbb{C}^p$ has bounded L -index in joint variables if and only if for every $R \in \mathbb{R}_+^n$ there exist $n_0 \in \mathbb{Z}_+$, $p_0 > 0$ such that for all $z_0 \in \mathbb{C}^n$ there exists $K_0 \in \mathbb{Z}_+^n$, $\|K_0\| \leq n_0$, satisfying inequality*

$$\max\left\{\frac{|F^{(K)}(z)|_p}{K!\mathbf{L}^K(z)} : \|K\| \leq n_0, z \in \mathbb{D}^n[z_0, R/\mathbf{L}(z_0)]\right\} \leq p_0 \frac{|F^{(K_0)}(z_0)|_p}{K_0!\mathbf{L}^{K_0}(z_0)}.$$

This theorem is basic in the theory of functions of bounded index. Theorem 2 implies also the following corollary.

Corollary 3. *Let $\mathbf{L} \in Q^n$. An entire vector-function $F: \mathbb{C}^n \rightarrow \mathbb{C}^p$ has bounded \mathbf{L} -index in joint variables in the sup-norm if and only if it has bounded \mathbf{L} -index in joint variables in the norm $\|\cdot\|_0$.*

Theorem 4. *Let $\mathbf{L} \in Q^n$. In order that an entire vector-valued function $F: \mathbb{C}^n \rightarrow \mathbb{C}^p$ be of bounded \mathbf{L} -index in joint variables it is necessary that for all $R \in \mathbb{R}_+^n$ there exist $n_0 \in \mathbb{Z}_+$, $p_1 \geq 1$ such that for all $z_0 \in \mathbb{C}^n$ there exists $K_0 \in \mathbb{Z}_+^n$, $\|K_0\| \leq n_0$, satisfying inequality*

$$\max\{|F^{(K_0)}(z)|_p : z \in \mathbb{D}^n[z_0, R/\mathbf{L}(z_0)]\} \leq p_1 |F^{(K_0)}(z_0)|_p \quad (1)$$

and it is sufficient that for all $R \in \mathbb{R}_+^n$ there exist $n_0 \in \mathbb{Z}_+$, $p_1 \geq 1 \forall z_0 \in \mathbb{C}^n \exists K_1^0 = (k_1^0, 0, \dots, 0)$, $\exists K_2^0 = (0, k_2^0, 0, \dots, 0), \dots, \exists K_n^0 = (0, \dots, 0, k_n^0) : k_j^0 \leq n_0$, and

$$(\forall j \in \{1, \dots, n\}) : \max\{|F^{(K_j^0)}(z)|_p : z \in \mathbb{D}^n[z_0, R/\mathbf{L}(z_0)]\} \leq p_1 |F^{(K_j^0)}(z_0)|_p \quad (2)$$

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Зміст

E. Afanas'eva <i>Finitely bi-Lipschitz homeomorphisms between Finsler manifolds</i>	3
Aliyev Yagub <i>About longest and shortest chords passing through a fixed point</i>	5
S. Antonyan <i>Some equivariant properties of Milnor's construction</i>	6
K. Antoshyna, S. Kozerenko <i>Commuting sets for topological set operators</i>	7
B. Apanasov <i>Asymptotic analysis of quasi-regular mappings in space</i>	8
M. J. Atteya <i>Generalized (σ, τ)-derivations on associative rings satisfying certain identities</i>	10
V. Balan <i>The Tucker HO-SVD and the anisotropy of Finslerian geometric models</i>	11
V. Balashchenko, D. Vylegzhanin <i>Invariant structures on homogeneous Φ-spaces and Lie groups</i>	13
T. Banakh <i>Every 2-dimensional Banach space has the Mazur-Ulam property</i>	15
A. Bandura, V. Baksa, O. Skaskiv <i>A connection between L-index of vector-valued entire function and L-index of each its component</i>	16
B. Baratov, Yu. Eshkabilov <i>Separable cubic stochastic operators</i>	18
V. Bilet, O. Dovgoshey <i>Asymptotically equivalent subspaces of metric spaces</i>	20
E. Bonacci <i>Isomorphic issues about the CTCs in Quantum Physics</i>	22
P. Petrenko, A. Andreev <i>Geometrical Langlands Ramifications and Differential Operators Classification by Verma Module Extensions</i>	23
Y. Cherevko, V. Berezovski, J. Mikeš, Y. Fedchenko <i>Conharmonic Transformations of Locally Conformal Kähler Manifolds</i>	24
V. Chernov <i>Applications of Linking to the Study of Causality</i>	26
A. Bakhtin, I. Denega <i>Problem on extremal decomposition of the complex plane</i>	27
A. Dikarev, A. S. Galaev <i>Parallel spinors on Lorentzian Weyl spaces</i>	29
Yu. A. Drozd <i>Matrix problems, triangulated categories and stable homotopy types</i>	30
V. S. Dryuma <i>On the properties smooth manifolds defined by intersections</i>	31
K. Eftekharinasab <i>Some applications of transversality for infinite dimensional manifolds</i>	33
S. Favorov <i>Uniqueness theorems for almost periodic objects</i>	34
V. Fedorchuk, V. Fedorchuk <i>On symmetry reduction and some classes of invariant solutions of the $(1 + 3)$-dimensional homogeneous Monge-Ampère equation</i>	35
B. Feshchenko <i>Deformations of circle-valued Morse functions on 2-torus</i>	37