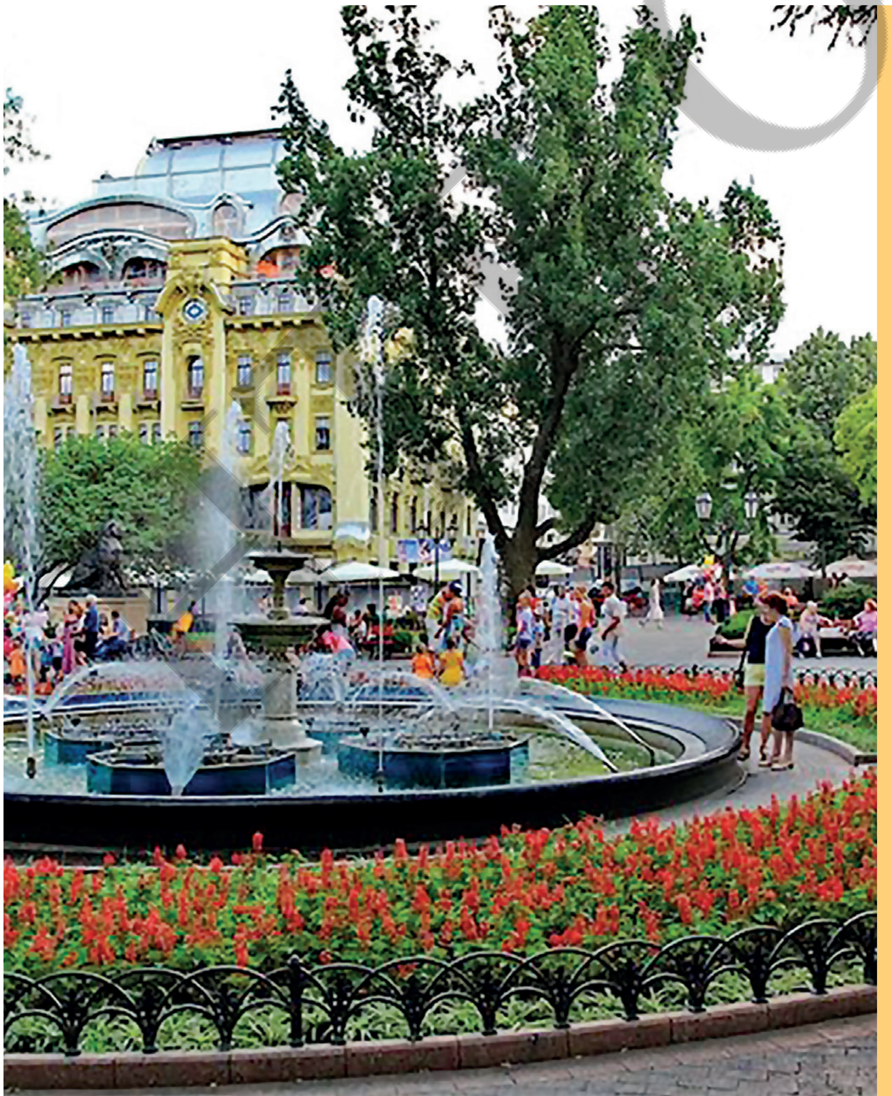
International
Scientific Conference

Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Bolotov D.** (*Kharkiv, Ukraine*)
- **Bondarenko V.** (*Kyiv, Ukraine*)
- **Boychuk O.** (*Kyiv, Ukraine*)
- **Boyko V.** (*Kyiv, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Dorogovtsev A.** (*Kyiv, Ukraine*)
- **Drozd Yu.** (*Kyiv, Ukraine*)
- **Gerasymenko V.** (*Kyiv, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Kiosak V.** (*Odesa, Ukraine*)
- **Kochubei A.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Mikhailets V.** (*Kyiv, Ukraine*)
- **Ostrovskyi V.** (*Kyiv, Ukraine*)
- **Petravchuk A.** (*Kyiv, Ukraine*)
- **Plaksa S.** (*Kyiv, Ukraine*)
- **Portenko M.** (*Kyiv, Ukraine*)
- **Pratsiovytyi M.** (*Kyiv, Ukraine*)
- **Savchenko O.** (*Kherson, Ukraine*)
- **Romanyuk A.** (*Kyiv, Ukraine*)
- **Timokha O.** (*Kyiv, Ukraine*)

Organizing commitee

- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Osadchuk Ye.** (*Odesa, Ukraine*)
- **Sergeeva O.** (*Odesa, Ukraine*)

neighbourhoods. We write $\Omega_{X,p}$ *for the set of geodesics of minimal length connecting a point in* $m(p)$ *with* p *and* $\gamma_{X,p}$ *for such a geodesic originating at* p*. Assume that* $X \subset \Pi \times \mathcal{M}$ *has closed* t*-sections and we have the Kuratowski convergence* $X_t \xrightarrow{K} X_0$*. Then for* $M = \{(t,x) \in \Pi \times \mathcal{M} \mid \exists \gamma_{X_t,p}, \tilde{\gamma}_{X_t,p} \in \Omega_{X_t,p} : \gamma_{X_t,p} \neq \tilde{\gamma}_{X_t,p}\}$*, we have*

$$\liminf_{\pi(M)\ni t\to 0} M_t \supset M_0$$

where the lower limit is understood in the Kuratowski sense:

$$x \in \liminf_{\pi(M)\ni t\to 0} M_t \Leftrightarrow \forall \pi(M) \setminus \{0\} \ni t_\nu \to t_0, \exists M_{t_\nu} \ni x_\nu \to x.$$

We will show how this applies in singularity theory in $\mathbb{R}^n$ giving a criterion for M_X to reach certain singularities of X when X is definable in some o-minimal structure (e.g. semialgebraic), cf. [2].

Finally, we will discuss a counterpart of this theorem in the case of *conflict sets* of finite families of closed, pairwise disjoint sets, instead of the medial axis, cf. [1]. The conflict set of two sets is their set of equidistant points. In case of more than two sets it can be seen as the set of points at which the distance wavefronts emanating from the sets meet.

REFERENCES

[1] Adam Białożyt, Anna Denkowska, Maciej P. Denkowski, *The Kuratowski convergence of medial axes and conflict sets.* arXiv:1602.05422 (2022).

[2] L. Birbrair, M. Denkowski. *Medial axis and singularities.* J. Geom. Anal. 27 no. 3, 2339–2380, 2017.

[3] Harry Blum. *A Transformation for Extracting New Descriptors of Shape.* In: Models for the Perception of Speech and Visual Form. Cambridge: MIT Press, 362–380, 1967.

[4] Frédéric Chazal, Rémi Soufflet. *Stability and finiteness properties of medial axis and skeleton.* J. Dyn. and Control Systems, Vol. 10, No. 2, 149–170, 2004.

The diameter-width-ratio for complete and pseudo-complete sets

Katherina von Dichter
(Brandenburg University of Technology, Cottbus, Germany)
E-mail: `vondicht@b-tu.de`

Any set $A \subset \mathbb{R}^n$ fulfilling $A = t - A$ for some $t \in \mathbb{R}^n$ is called symmetric and 0-symmetric if $t = 0$. We denote the family of all (convex) bodies (full-dimensional compact convex sets) by $\mathcal{K}^n$ and the family of 0-symmetric bodies by $\mathcal{K}^n_0$. For any $K \in \mathcal{K}^n$ the gauge function $\|\cdot\|_K : \mathbb{R}^n \to \mathbb{R}$ is defined as

$$\|x\|_K = \inf\{\rho > 0 : x \in \rho K\}.$$

In case $K \in \mathcal{K}^n_0$ we see that $\|\cdot\|_K$ defines a norm. However, even for a non-symmetric unit ball K, one may approximate the gauge function by the norms induced from symmetrizations of K

$$\|x\|_{\mathrm{conv}(K\cup(-K))} \leq \|x\|_K \leq \|x\|_{K\cap(-K)}.$$

It is natural to request that $K \cap (-K) = K = \mathrm{conv}(K \cup (-K))$ if K is symmetric, which is true if and only if 0 is the center of symmetry of K. This motivates the definition of a meaningful center for general K. We introduce one of the most common asymmetry measures, which is best suited to our purposes, and choose the center matching it.

The *Minkowski asymmetry* of K (denoted by $s(K)$) is defined as

$$s(K) := \inf\{\rho > 0 : K - c \subset \rho(c - K), \quad c \in \mathbb{R}^n\},$$

and a *Minkowski center* of K is any $c \in \mathbb{R}^n$ such that $K - c \subset s(K)(c - K)$. Moreover, if 0 is a Minkowski center, we say K is *Minkowski centered.* It is well-known that $s(K) \in [1, n]$ for all $K \in \mathcal{K}^n$, with $s(K) = 1$ if and only if K is symmetric and $s(K) = n$ if and only if K is a simplex.

Notice that there always exists some $x \in \mathbb{R}^n$ such that $\alpha(K)\|x\|_{K\cap(-K)} = \|x\|_{\mathrm{conv}(K\cup(-K))}$, which means that we have equality in the complete chain in the equality above for that x if $\alpha(K) = 1$. We investigate the region of all possible values for the parameter $\alpha(K)$ for Minkowski centered $K \in \mathcal{K}^2$ in dependence of the asymmetry of K.

We show that $\alpha(K) \geq \frac{2}{s(K)+1}$ for all Minkowski centered K, and that in the planar case $\alpha(K) = 1$ implies $s(K) \leq \varphi$, where $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.61$ denotes the golden ratio.

We give a complete description of the possible α-values of K in the planar case in dependence of its Minkowski asymmetry. Moreover, we derive the (unique) family of convex bodies that fulfill the upper bound of $\alpha(K)$.

K is called complete (w.r.t. C), if any proper superset of it has a greater diameter than K.

We also present an application on the diagram of the α-values of K for the diameter-width ratio for complete and pseudo-complete sets. We extend the results on the bounds for $\alpha(K)$ and describe the region of all possible values for this parameter for Minkowski centered convex compact set K in dependence of the asymmetry of K.

Theorem 1. *Let K be Minkowski centered. Then*

$$\frac{2}{s(K)+1} \leq \alpha(K) \leq \min\left\{1, \frac{s(K)}{s(K)^2 - 1}\right\}.$$

Moreover, for every pair (α, s), such that $\frac{2}{s+1} \leq \alpha \leq \min\left\{1, \frac{s}{s^2-1}\right\}$, there exists a Minkowski centered K, such that $s(K) = s$ and $\alpha(K) = \alpha$.

Consider $K \in \mathcal{K}^n$ and $C \in \mathcal{K}^n_0$. For $s \in \mathbb{R}^n \setminus \{0\}$ the s-breadth of K w.r.t. C is the distance between the two parallel supporting hyperplanes of K with normal vector s, i.e.,

$$b_s(K, C) := \frac{\max_{x,y\in K} s^T(x - y)}{\max_{x\in C} s^T x}.$$

The minimal s-breadth

$$w(K, C) := \min_{s\in mathbbR^n\setminus\{0\}} b_s(K, C)$$

and the maximal s-breadth

$$D(K, C) := \max_{s\in\mathbb{R}^n\setminus\{0\}} b_s(K, C)$$

are called width and diameter of K w.r.t. C, respectively.

We present a quantitative result on the diameter-width ratio for for complete sets.

Theorem 2. *Let K, C be convex compact sets and C be 0-symmetric be such that K is complete w.r.t. C. Then*

$$\frac{D(K, C)}{w(K, C)} \leq \frac{s(K) + 1}{2}.$$

Moreover, for $n > 2$ even and for any $s \in [1, n-1]$ there exists $K, C \in \mathcal{K}^n$ such that K is complete w.r.t. C with $s(K) = s$, such that $\frac{D(K,C)}{w(K,C)} = \frac{s+1}{2}$, while for $n > 2$ odd and any $s \in [1, n]$ there exists $K \in \mathcal{K}^n$ which is complete w.r.t. C with $s(K) = s$, such that $\frac{D(K,C)}{w(K,C)} = \frac{s+1}{2}$.

On the possibility of joining two pairs of points in convex domains using paths

Dovhopiatyi Oleksandr
(Zhytomyr Ivan Franko State University)
E-mail: `Alexdov1111111@gmail.com`

Recall, that a set C is *convex* if any pair of points $x, y \in C$ may be joined by some segment which belongs to C, as well. We define the Euclidean distance between sets and the Euclidean diameter by the formulae

$$d(A,B) = \inf_{x \in A, y \in B} |x - y|\,, \quad d(A) = \sup_{x,y \in A} |x - y|\,.$$

Sometimes we also write $\mathrm{dist}\,(A,B)$ instead $d(A,B)$ and $\mathrm{diam}\,E$ instead $d(E)$, as well. As usually, we set

$$B(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| < r\}\,,$$
$$S(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| = r\}\,.$$

We emphasize that, the results established here have already been obtained in particular case, when a domain is the unit ball [1]. Concerning some applications of modulus inequalities in the mapping theory, see [2], cf. [3]–[4].

Theorem 1. *Let D' be a bounded convex domain in $\mathbb{R}^n$, $n \geqslant 2$, and let $E := B(y_*, \delta_*/2)$ be a ball centered at the point $y_* \in D'$, where $\delta_* := d(y_*, \partial D')$. Let $z_0 \in \partial D'$. Then for any points $A, B \in B(z_0, \delta_*/8) \cap D'$ there are points $C, D \in \overline{B(y_*, \delta_*/2)}$, for which the segments $[A, C]$ and $[B, D]$ are such that*

$$\mathrm{dist}\,([A,C],[B,D]) \geqslant C_0 \cdot |A - B|\,, \tag{1}$$

where $C_0 > 0$ is some constant depending only on δ_* and $d(D')$.

Recall that, a Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *an admissible* for a family Γ of paths γ in $\mathbb{R}^n$, if the relation

$$\int\limits_{\gamma} \rho(x)\,|dx| \geqslant 1 \tag{2}$$

holds for any locally rectifiable path $\gamma \in \Gamma$. *A modulus* of Γ is defined as follows:

$$M(\Gamma) = \inf_{\rho \in \,\mathrm{adm}\,\Gamma} \int\limits_{\mathbb{R}^n} \rho^n(x)\,dm(x)\,. \tag{3}$$

The following statements hold.

Table of contents