International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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according to the formula

$$(x,y)\widetilde{\varphi} = \big((x)\widehat{\varphi},(y)\widehat{\varphi}\big), \qquad x,y \in G^+.$$

Theorem 2. *Let G be an archimedean totally ordered group. Then the semigroup $\mathbf{End}^o(G)$ of o-endomorphisms of G is isomorphic to the semigroup $\mathbf{End}(\mathscr{B}^+(G))$ of endomorphisms of the monoid $\mathscr{B}^+(G)$.*

We define the category $\mathfrak{TOAG}$ by

(1) $\mathbf{Ob}(\mathfrak{TOAG}) = \{G\colon G$ is an archimedean totally ordered group$\}$;

(2) $\mathbf{Mor}(\mathfrak{TOAG})$ are o-homomorphisms of archimedean totally ordered groups,

and the category $\mathfrak{BETOAG}$ in the following way

(1) $\mathbf{Ob}(\mathfrak{BETOAG})$ are bicyclic extensions $\mathscr{B}^+(G)$ of archimedean totally ordered groups $G \in \mathbf{Ob}(\mathfrak{TOAG})$;

(2) $\mathbf{Mor}(\mathfrak{BETOAG})$ are homomorphisms of monoids $\mathscr{B}^+(G) \in \mathbf{Ob}(\mathfrak{BETOAG})$.

Theorem 3. *The categories $\mathfrak{TOAG}$ and $\mathfrak{BETOAG}$ are isomorphic.*

References

[1] M. R. Darnel, *Theory of Lattice-Ordered Groups*, Marcel Dekker, Inc., New York, 1995.
[2] M. Lawson, *Inverse Semigroups. The Theory of Partial Symmetries*, Singapore, World Scientific, 1998.

The Interaction of an Infinite Number of Eddy Flows

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The Boltzmann kinetic equation plays an important role in the kinetic theory of gases. In paper [2], we consider this equation for a model of hard spheres that describes particles of any gas which move translationally with a certain linear velocity, collide by the laws of classical mechanics and can not rotate. For this model, the equation has the form [1]

$$D(f) = Q(f,f), \tag{1}$$

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{2}$$

$$Q(f,f) \equiv \frac{d^2}{2}\int_{R^3} dV_1 \int_{\Sigma} d\alpha |(V-V_1,\alpha)| \times \Big[f(t,x,V_1')f(t,x,V') - f(t,x,V)f(t,x,V_1)\Big], \tag{3}$$

and V, V_1, V', V_1' are the velocities of particles before and after collision, respectively, determined by the relations

$$V' = V - \alpha(V-V_1,\alpha),$$
$$V_1' = V_1 + \alpha(V-V_1,\alpha).$$

The solution to this equation will be look for in the next form

$$f(t,x,V) = \sum_{i=1}^{\infty} \varphi_i(t,x) M_i(t,x,V). \tag{4}$$

where $M_i(t,x,V)$ are the exact solutions of the equation (1)-(3)

$$D(M_i) = Q(M_i, M_i) = 0$$

and the coefficient functions $\varphi_i(t,x)$ are nonnegative smooth functions on $\mathbb{R}^4$ and $\varphi_i(t,x) \not\equiv 0$.

As a value of the deviation between the parts of equation (1) we will consider a uniform-integral error of the form

$$\Delta = \Delta(\beta_i) = \sup_{(t,x)\in\mathbb{R}^4} \int_{\mathbb{R}^3} \big|D(f) - Q(f,f)\big| dV. \tag{5}$$

In the paper [2], several cases of coefficient functions $\varphi_i(t,x)$ were obtained for which the deviation (5) can be done arbitrarily small. This is possible thanks to a special selection of hydrodynamic flow parameters.

References

[1] C. Cercignani. Theory and Application of the Boltzmann Equation. *Scottish Academic Press*, Edinburgh, 1975.

[2] O.O. Hukalov, V.D. Gordevskyy. The Interaction of an Infinite Number of Eddy Flows for the Hard Spheres Model. *Journal of Mathematical Physics, Analysis, Geometry*, 2: 163–174, 2021.

Semi-Fredholm theory in unital C^*-algebras

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The Fredholm and semi-Fredholm theory on Hilbert and Banach spaces started by studying the integral equations introduced in the pioneering work by Fredholm in 1903 in [5]. After that, the abstract theory of Fredholm and semi-Fredholm operators on Hilbert and Banach spaces was further developed in numerous papers and books such as [1], [2] and [14]. In addition to classical semi-Fredholm theory on Hilbert and Banach spaces, several generalizations of this theory have been considered. Breuer for example started the development of Fredholm theory in von-Neumann algebras as a generalization of the classical Fredholm theory for operators on Hilbert spaces. In [3] and [4] he introduced the notion of a Fredholm operator in a von Neumann algebra and established its main properties. On the other hand, Fredholm theory on Hilbert C^*-modules as another generalization of the classical Fredholm theory on Hilbert spaces was started by Mishchenko and Fomenko. In [13] they introduced the notion of a Fredholm operator on the standard Hilbert C^*-module and proved a generalization in this setting of some of the main results from the classical Fredholm theory. In [6], [7], [8], [9] and [10] we went further in this direction and defined semi-Fredholm and semi-Weyl operators on Hilbert C^*-modules. We investigated and proved several properties of these new semi- Fredholm operators on Hilbert C^*-modules as a generalization of the results from the classical semi-Fredholm theory on Hilbert and Banach spaces. The interest