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Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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On homotopy nilpotency of Moore space

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Given based spaces X_1, X_2 , we use the customary notations $X_1 \times X_2$ for their Cartesian product, $X_1 \vee X_2$ for their wedge and $X_1 \wedge X_2$ for the smash product of X_1, X_2 .

Recall that an *H-space* is a pair (X, μ) , where X is a space and $\mu : X \times X \rightarrow X$ is a map such that the diagram

$$\begin{array}{ccc} X \times X & \xrightarrow{\mu} & X \\ \uparrow & \searrow \nabla & \\ X \vee X & & \end{array}$$

commutes up to homotopy, where $\nabla : X \vee X \rightarrow X$ is the folding map. An *H-space* X is called a *group-like space* if X satisfies all the axioms of groups up to homotopy. From now on, we assume that any *H-space* X is group-like. For an *H-space* X , we write $\varphi_{X,1} = \iota_X$, $\varphi_{X,2} : X \times X \rightarrow X$ for the basic commutator map and $\varphi_{X,n+1} = \varphi_{X,2} \circ (\varphi_{X,n} \times \iota_X)$ for $n \geq 2$.

The *nilpotency class* $\text{nil}(X, \mu)$ of an *H-space* (X, μ) is the least integer $n \geq 0$ for which the map $\varphi_{X,n+1} \simeq *$ is nullhomotopic and we call the homotopy associative *H-space* X homotopy nilpotent. If no such integer exists, we put $\text{nil}(X, \mu) = \infty$. In the sequel, we simply write $\text{nil} X$ for the nilpotency class of an *H-space* X .

In virtue of [2, 2.7. Theorem], we have

Theorem 1. *If X is an H-space then*

$$\text{nil} X = \sup_m \text{nil}[X^m, X] = \sup_m \text{nil}[X^{\wedge m}, X] = \sup_Y \text{nil}[Y, X],$$

where m ranges over all integers and Y over all topological spaces.

Then, by means of [8, Lemma 2.6.1], we may state

Corollary 2. *A connected H-space X is homotopy nilpotent if and only if the functor $[-, X]$ on the category of all spaces is nilpotent group valued.*

With any based space X , we associate the integer $\text{nil}\Omega(X)$ called the *nilpotency class* of X for the loop space $\Omega(X)$ on X . Although many results on the homotopy nilpotency have been obtained, the homotopy nilpotency classes have been determined in very few cases.

Example 3. (1) It is well-known that

$$\text{nil}\Omega(\mathbb{S}^n) = \begin{cases} 3 & \text{for } n \text{ even with } n \neq 2; \\ 2 & \text{for } n \text{ odd with } n \neq 1, 3, 7 \text{ or } n = 2; \\ 1 & \text{for } n = 1, 3, 7 \end{cases}$$

for the n -sphere \mathbb{S}^n .

(2) For the wedge $\mathbb{S}^m \vee \mathbb{S}^n$ of two spheres with $m, n \geq 2$, we have

$$\text{nil}\Omega(\mathbb{S}^m \vee \mathbb{S}^n) = \infty.$$

Write $\mathbb{K}P^m$ for the projective m -space for $\mathbb{K} = \mathbb{R}, \mathbb{C}$, the field of reals or complex numbers and \mathbb{H} , the skew \mathbb{R} -algebra of quaternions. Then, results from [6] have been applied in [3] to study extensively the homotopy nilpotency of the loop spaces of Grassmann and Stiefel manifolds over \mathbb{K} , and their p -localization.

Let $\mathbb{S}_{(p)}^{2m-1}$ be the p -localization of the sphere \mathbb{S}^{2m-1} at a prime p . The main result of the paper [4] is the explicit determination of the homotopy nilpotence class of a wide range of homotopy associative multiplications on localized spheres $\mathbb{S}_{(p)}^{2m-1}$ for $p > 3$.

Next, let A be an Abelian group and n any integer ≥ 2 . A CW -complex X satisfying $\pi_j(X) = 0$ for $j < n$, $\pi_n(X) \approx A$ and $H_i(X) = 0$ for $i > n$ is known as a *Moore space* of type (A, n) , or simply an $M(A, n)$ space. By [7], it is known that a Moore space $M(A, n)$ with $n \geq 2$ exists and, in view of [5, Example 4.34], the homotopy type of a Moore space $M(A, n)$ is uniquely determined by A and $n \geq 2$. This implies that every Moore space $M(A, n)$ with $n \geq 3$, is the suspension $\Sigma M(A, n-1)$. Furthermore, in [1, Section 2], it was shown that also $M(A, 2)$ is the suspension $\Sigma L(A)$ for some CW -complex $L(A)$.

Now, we examine the homotopy nilpotency of $M(A, n)$ with ≥ 2 . Notice that $\mathbb{S}^n = M(\mathbb{Z}, n)$ and the wedge $\mathbb{S}^n \vee \mathbb{S}^n = M(\mathbb{Z} \oplus \mathbb{Z}, n)$ for the integers \mathbb{Z} . Then, by Example 3, we have that $\text{nil } \Omega(\mathbb{S}^n) \leq 3$ but $\text{nil } \Omega(\mathbb{S}^n \vee \mathbb{S}^n) = \infty$ for $n \geq 2$.

First, we show the general fact

Proposition 4. *If the reduced homology $\tilde{H}_*(X, \mathbb{F})$ has at least two primitive generators, where \mathbb{F} is a field then $\Omega\Sigma(X)$ is not homotopy nilpotent.*

Then, we state the main result

Theorem 5. *Let $m \geq 1$, $n_1, \dots, n_m \geq 2$ and $M(A_k, n_k)$ be Moore spaces of type (A_k, n_k) for $k = 1, \dots, m$. Then:*

- (1) $\text{nil } \Omega((M(A_1, n_1) \times \dots \times (M(A_m, n_m))) < \infty$ if and only if A_k are torsion-free groups with rank $r(A_k) = 1$ for $k = 1, \dots, m$;
- (2) $\text{nil } \Omega((M(A_1, n_1) \vee \dots \vee M(A_m, n_m))) < \infty$ if and only if $m = 1$ and A_1 is a torsion-free group with rank $r(A_1) = 1$.

In particular, we derive

Corollary 6. *If $M(A, n)$ is a Moore space with $n \geq 2$ then*

$$\text{nil } \Omega(M(A, n)) < \infty$$

if and only if A is a torsion-free group with rank $r(A) = 1$ or equivalently, A is a subgroup of the rationals \mathbb{Q} .

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