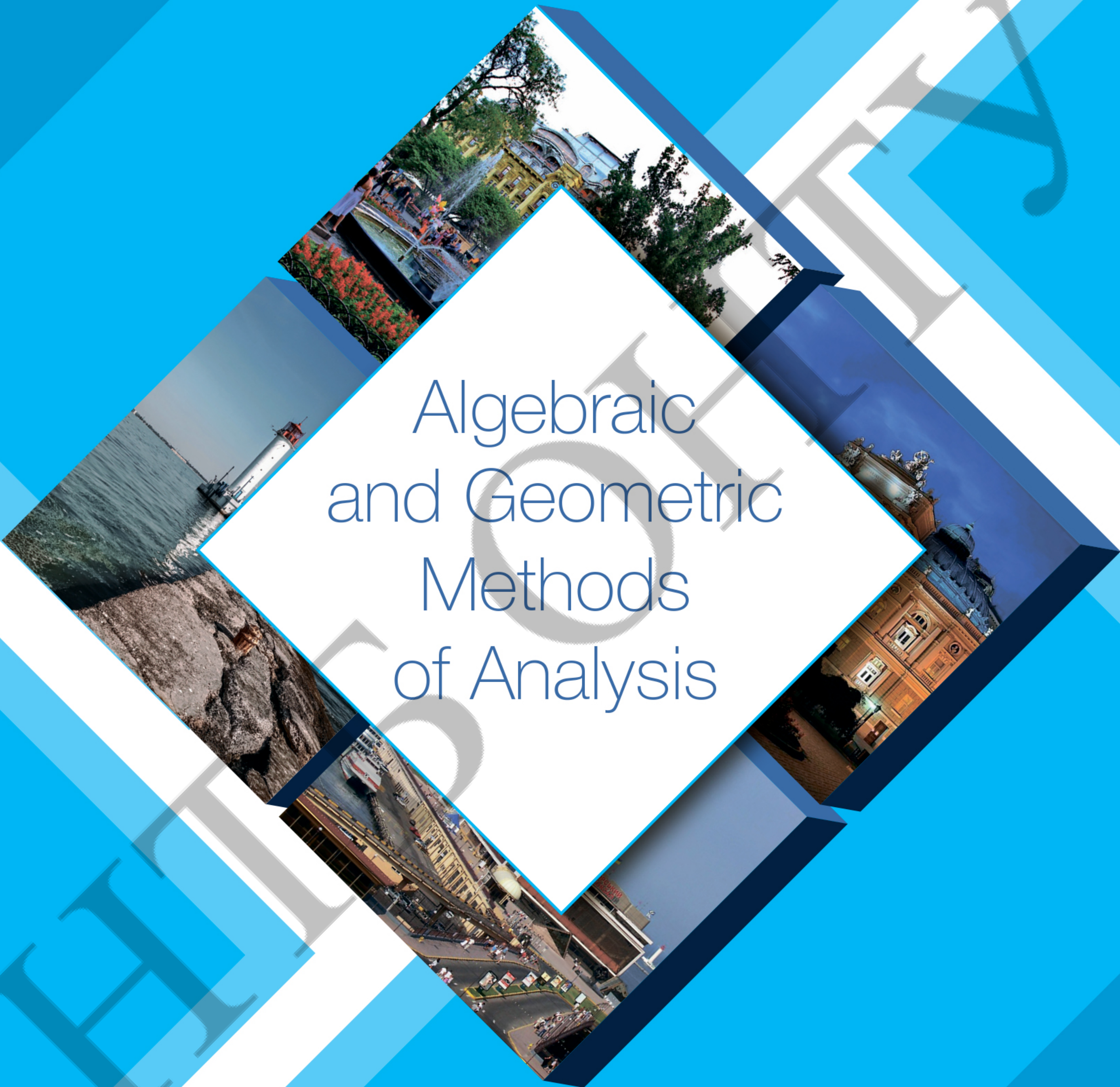
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Invariant transformation of generalized-recurrent-parabolic spaces that are in a quasi-geodesic mapping

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We study diffeomorphisms of pseudo-Riemannian spaces that belong to the intersection of classes of quasi-geodesic mappings (QGM) [2] with the reciprocity condition and almost-geodesic mappings of the second type [1]. We mean that $QGM\ f : (V_n, g_{ij}, F_i^h) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F_i^h)$ satisfies the reciprocity condition if the reverse mapping f^{-1} is also QGM.

The fundamental equations of such a mapping f in the common coordinate system (x^i) with respect to the mapping f has the form:

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \psi_{(i}(x)\delta_{j)}^h + \phi_{(i}(x)F_{j)}^h(x), \tag{1}$$

$$F_i^h(x) = \overline{F}_i^h(x),$$

$$g_{i\alpha}F_j^\alpha = -g_{j\alpha}F_i^\alpha, \qquad \overline{g}_{i\alpha}F_j^\alpha = -\overline{g}_{j\alpha}F_i^\alpha, \tag{2}$$

$$F_{(i,j)}^h = q_{(i}F_{j)}^h, \tag{3}$$

$$F_\alpha^h F_i^\alpha = e\delta_i^h, \qquad e = 0, \pm 1, \tag{4}$$

$$i, h, j, ... = 1, 2, ...n,$$

where $\Gamma_{ij}^h, \overline{\Gamma}_{ij}^h$ are the Christoffel symbols of $V_n, \overline{V}_n$, respectively; $\psi_i(x)$, $\phi_i(x)$, $q_i(x)$, $p_i(x)$ are certain covectors; $F_i^h(x)$ is affinor; brackets (i, j) denote the symmetrization with respect to the corresponding indices; comma «,» is a sign of the covariant derivative in respect to the connection of V_n.

We call an affinor structure F_i^h that satisfies conditions (3) a *generalized-recurrent structure* (of elliptic, hyperbolic, or parabolic type). Let us study the case of a parabolic structure ($e = 0$).

The following holds:

Theorem 1. *If there is a non-trivial QGM of generalized-recurrent-parabolic spaces* $f : (V_n, g_{ij}, F_i^h) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F_i^h)$, *which corresponds to the affinor* F_i^h *and the vector* ϕ_i, *then it generates another non-trivial QGM of other generalized-recurrent-parabolic spaces*

$$f_1 : (\overset{1}{V}_n, \overset{1}{g}_{ij}, \overset{1}{F_i^h}) \longrightarrow (\overset{1}{\overline{V}}_n, \overset{1}{\overline{g}}_{ij}, \overset{1}{F_i^h}),$$

which corresponds to the affinor $\overset{1}{F_i^h}$ *and the vector* $\overset{1}{\phi}_i$ *and preserves the generalized recurrence vector* q_i. *The tensors* $\overset{1}{g}_{ij}$, $\overset{1}{\overline{g}}_{ij}$, $\overset{1}{\phi}_i$, $\overset{1}{F_i^h}$ *are given by the formulas*

$$\overset{1}{g}_{ij} = e^{2\psi}\overline{g}^{\alpha\beta} g_{\alpha i} g_{\beta j},$$

$$\overset{1}{\overline{g}}_{ij} = e^{2\psi} g_{ij},$$

$$\overset{1}{g}_{ij,k} = -\overset{1}{\phi}_{\bar{i}} g_{jk} - \overset{1}{\phi}_{\bar{j}} g_{ik} - \overset{1}{\phi}_i F_{jk} - \overset{1}{\phi}_j F_{ik},$$

$$\overset{1}{F_i^h} \overset{def}{=} F_\alpha^h \tilde{B}_i^\alpha = F_i^\alpha \tilde{B}_\alpha^h,$$

$$B_i^h = e^{2\psi}\overline{g}^{h\alpha} g_{\alpha i}, \qquad \tilde{B}_i^h = e^{-2\psi} g^{h\alpha}\overline{g}_{\alpha i}.$$

Theorem 2. *If there is a non-trivial QGM of generalized-recurrent-parabolic spaces* $f : (V_n, g_{ij}, F_i^h) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F_i^h)$, *which corresponds to the affinor* F_i^h *and the vector* ϕ_i, *then it generates an infinite sequence of non-trivial QGM of other generalized-recurrent-parabolic spaces*

$$(\overset{1}{V}_n, \overset{1}{g}_{ij}, \overset{1}{F_i^h}) \xrightarrow{f_1} (\overset{1}{\overline{V}}_n, \overset{1}{\overline{g}}_{ij}, \overset{1}{F_i^h}),$$

$$\downarrow$$

$$(\overset{2}{V}_n, \overset{2}{g}_{ij}, \overset{2}{F_i^h}) \xrightarrow{f_2} (\overset{2}{\overline{V}}_n, \overset{2}{\overline{g}}_{ij}, \overset{2}{F_i^h}),$$

$$\downarrow$$

....

$$\downarrow$$

$$(\overset{s}{V}_n, \overset{s}{g}_{ij}, \overset{s}{F_i^h}) \xrightarrow{f_s} (\overset{s}{\overline{V}}_n, \overset{s}{\overline{g}}_{ij}, \overset{s}{F_i^h}),$$

$$\downarrow$$

....

which correspond to the affinor $\overset{s}{F_i^h}$ *and the vector* $\overset{s}{\phi}_i$ *and preserve the generalized recurrence vector* q_i. *The tensors* $\overset{s}{g}_{ij}$, $\overset{s}{\overline{g}}_{ij}$, $\overset{s}{\phi}_i$, $\overset{s}{F_i^h}$, *are given by the formulas*

$$\overset{s}{g}_{ij} = \overset{(s)}{B_i^\alpha} g_{\alpha j}, \qquad \overset{s}{\overline{g}}_{ij} = e^{2\psi} \overset{(s-1)}{B_i^\alpha} g_{\alpha j}.$$

$$\overset{s}{\phi}_i = \phi_\alpha \overset{(s)}{B_i^\alpha}, \qquad \overset{s}{F_i^h} = F_\alpha^h \overset{(s)}{\tilde{B}_i^\alpha},$$

where $\overset{(s)}{B_i^h}$ *is the s-th degree of the affinor* B_i^h *and we mean* $\overset{(0)}{B_i^h} = \delta_i^h$.

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Notes on the Quality of Non-compactness for Non-compact Sobolev Embeddings

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It is well known that when a Sobolev space on a bounded domain is embedded into the smallest possible Lebesgue or Lorentz space, the resulting embedding is non-compact. In this talk, we will closely examine non-compact Sobolev embeddings and describe the quality of their non-compactness from different points of view.

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Ordinary linear differential operators and connections. Application to curvilinear webs

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The framework is real analytic or holomorphic, the field $\mathbb{K}$ denoting $\mathbb{R}$ or $\mathbb{C}$ according to this framework.

We are given two vector bundles E and F, respectively of rank p and q, over a n-dimensional manifold V. We assume $q < p(n+1)$ (the rank of F is smaller than the rank of J^1E). A linear homogeneous differential operator of order one[1] is a linear morphism of vector bundles $D\colon J^1E \to F$. Associated to it is the partial order equation

$$(*) \quad \mathcal{D}s = 0, \text{ where we set, for any section } s \text{ of } E, \mathcal{D}s := D(j^1 s).$$

[1] The theory works also for differential operators of higher order.