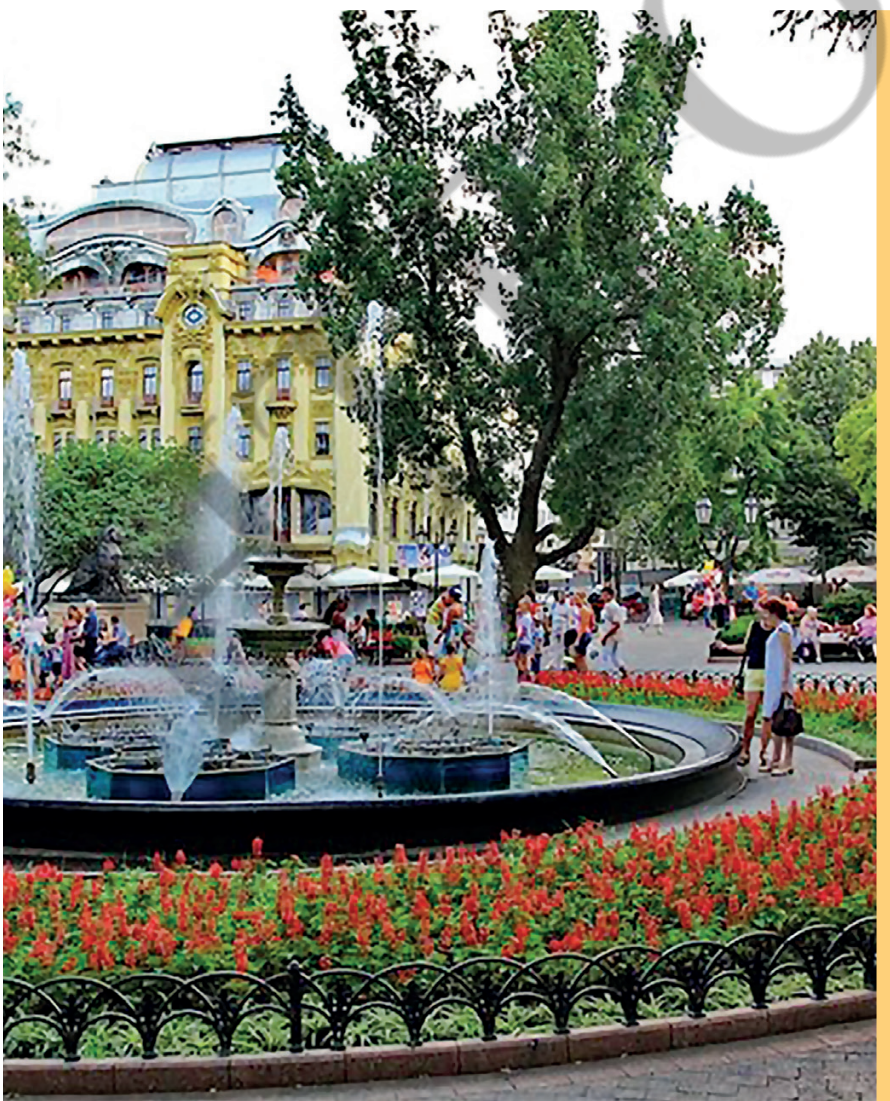
International
Scientific Conference

Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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- Institute of Mathematics of the National Academy of Sciences of Ukraine
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Reeb graph invariants of Morse functions, manifolds and groups

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The Reeb graph of a Morse function on a closed manifold is obtained by contracting each connected component of its level sets. There are two necessary and sufficient conditions for a finite graph to be realized as the Reeb graph of a Morse function on a given closed manifold: it needs to have the so-called good orientation and its first Betti number cannot exceed the corank of the fundamental group of the manifold. Moreover, any free quotient of this group can be represented as the Reeb epimorphism of a Morse function which is induced on fundamental groups by the quotient map from the manifold to the Reeb graph. It leads to the study of relations between the notions of equivalence of epimorphisms onto free groups, cobordism of systems of hypersurfaces and topological conjugation of Morse functions.manifold to the Reeb graph.

However, the realization of a graph as the Reeb graph of a Morse function is possible only up to a homeomorphism of graphs in general. The minimum number of degree 2 vertices in Reeb graphs of Morse functions is a strong invariant of the topology of manifold. It has three essentially different lower bounds in terms of the fundamental group, homology groups and Lusternik-Schnirelmann category. In the case of orientable 3-manifolds all of them can be improved by the inequality involving the Heegaard genus, and there is also another lower bound by a new invariant defined in terms of finite presentations of the fundamental group. We use Freiheitssatz, a fundamental fact from one-relator groups, to calculate it in some cases. The equalities in these bounds are closely related with the problem of finding a function such that the first Betti number of its Reeb graph is equal to corank. It is a one of potential geometric methods of calculating the corank, which is quite a complicated task in practise.

References

[1] W. Marzantowicz and Ł. P. Michalak, *Relations between Reeb graphs, systems of hypersufaces and epimorphisms onto free groups*, preprint (2020), arXiv:2002.02388.

[2] Ł. P. Michalak, *Combinatorial modifications of Reeb graphs and the realization problem*, Discrete Comput. Geom. 65 (2021), 1038–1060.

[3] Ł. P. Michalak, *Reeb graph invariants of Morse functions and 3-manifold groups*, preprint (2023), https://www.researchgate.net/publication/367568801_Reeb_graph_invariants_of_Morse_functions_and_3-manifold_groups