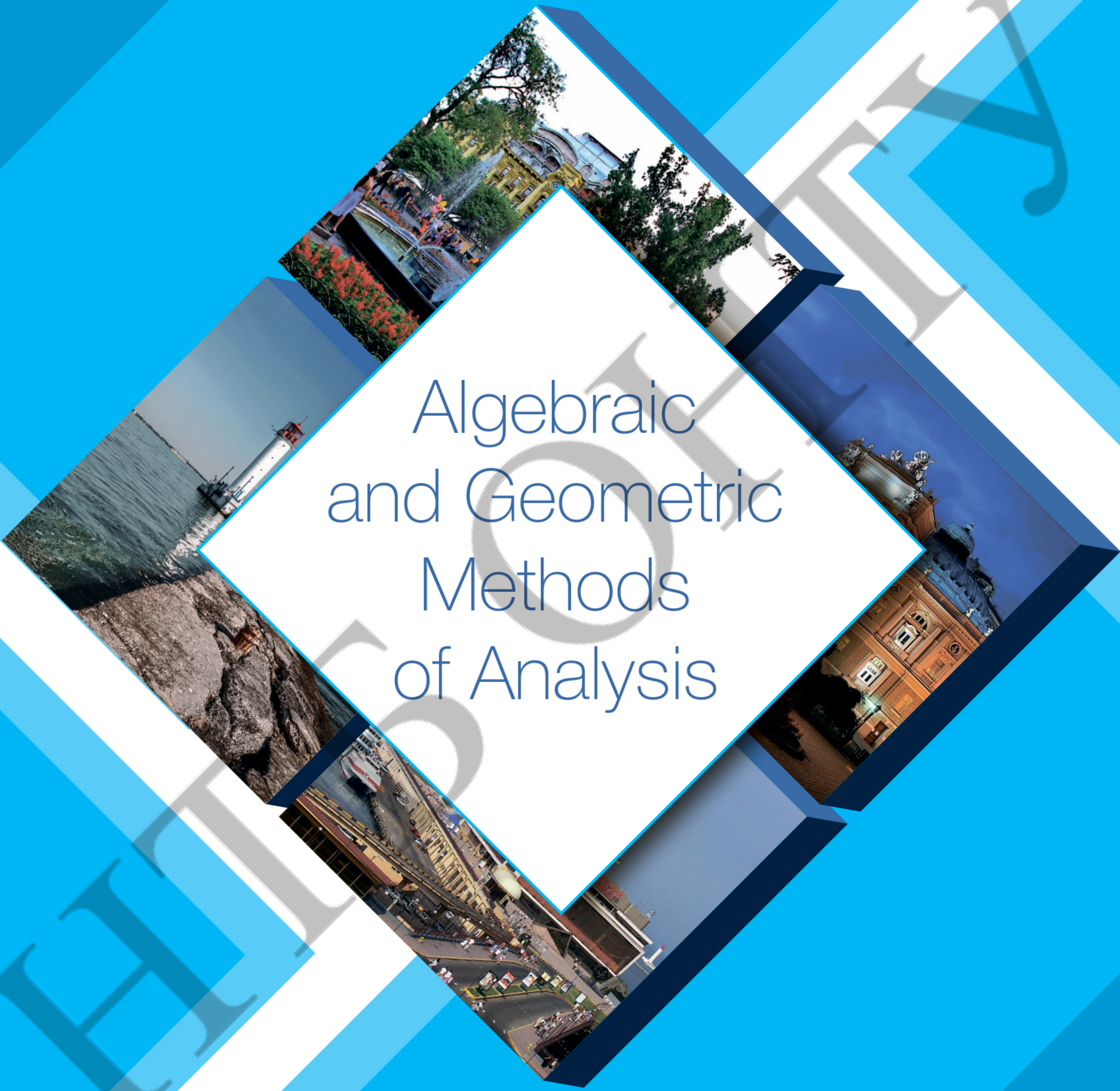
International
Scientific Conference



# Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

## List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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- an *inverse shift-continuous* topology if $(S,\tau)$ is a semitopological semigroup with continuous inversion.

The bicyclic monoid $\mathcal{C}(p,q)$ is the semigroup with the identity 1 generated by two elements $p$ and $q$ subjected only to the condition $pq=1$. The bicyclic monoid admits only the discrete semigroup Hausdorff topology [3]. Bertman and West in [1] extended this result for the case of Hausdorff semitopological semigroups.

We construct two non-discrete inverse semigroup $T_1$-topologies and a compact inverse shift-continuous $T_1$-topology on the bicyclic monoid $\mathcal{C}(p,q)$. Also we give conditions on a $T_1$-topology $\tau$ on $\mathcal{C}(p,q)$ to be discrete.

**Theorem 1.** *Every shift-continuous Baire $T_1$-topology $\tau$ on the bicyclic monoid $\mathcal{C}(p,q)$ is discrete.*

**Theorem 2.** *Let $\tau$ be an inverse semigroup $T_1$-topology on $\mathcal{C}(p,q)$. If there exists a point $q^ip^j \in \mathcal{C}(p,q)$ such that the space $\updownarrow_{\preccurlyeq} q^ip^j$ is quasi-regular at $q^ip^j$, then $\tau$ is discrete.*

**Theorem 3.** *Let $\tau$ be a shift-continuous $T_1$-topology on the bicyclic monoid $\mathcal{C}(p,q)$ such that the maps $\mathcal{C}(p,q) \to E(\mathcal{C}(p,q))$, $x \mapsto xx^{-1}$ and $\mathcal{C}(p,q) \to E(\mathcal{C}(p,q))$, $x \mapsto x^{-1}x$ are continuous. If there exists a point $q^ip^j \in \mathcal{C}(p,q)$ such that the space $\updownarrow_{\preccurlyeq} q^ip^j$ is semiregular at $q^ip^j$, then $\tau$ is discrete.*

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# An Application to Sasaki Extremal metrics via the Berglund-Hübsch rule


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Sasaki-extremal metrics were introduced in [1] as a generalization of metrics with constant scalar curvature, which is obstructed by the Futaki invariant. On this talk we exhibit examples of homotopy spheres and rational homology spheres realized as links of chain-cycle polynomials that do not admit Sasaki extremal metrics in the whole Sasaki cone, which has dimension greater than one. For this, we consider links that are given as 2-fold branched covers of $S^9$ whose branching loci are rational homology 7-spheres which are links of certain invertible polynomials of chain-cycle type studied in [5] and later in [4] through the Berglund-Hübsch rule of classical mirror symmetry. In [2], Boyer and van Coevering defined a relative version of the K-stability of Collins and Székelyhidy [6, 7] and obtain the first examples of Sasaki manifolds with Sasaki cone of dimension greater than one not admitting Sasaki extremal metrics in the whole Sasaki cone. Based on their result we exhibit 37 new families of links with Sasaki cone of dimension 2 such the whole cone does not admit any extremal representative. All the examples produced here are eitheir homotopy 9-spheres, rational homology 9-spheres or manifolds of the form $S^4 \times S^5$. We can easily extrapolate these examples to arbitrary dimensions where the corresponding Sasaki cones have larger dimensions. All these examples are inequivalent to the ones found in [2]. These examples are consequences of the following more general result that we present in this talk:

**Proposition 1** ([5]). *Consider a polynomial of chain-cycle type of the form*

$$f = z_0^{a_0} + z_0 z_1^{a_1} + z_4 z_2^{a_2} + z_2 z_3^{a_3} + z_3 z_4^{a_4}$$

*with* $a_1 = 2$ *whose link* $L_f$ *is a rational homology sphere and that cuts out a projective hypersurface in* $\mathbb{P}(w_0, w_1, \ldots, w_4)$ *such that*

$$(w_0, w_1, w_2, w_3, w_4) = (m_3 v_0, m_3 v_1, m_2 v_2, m_2 v_3, m_2 v_4),$$

*with* $m_3$ *odd,* $\gcd(m_2, m_3) = 1$ *and* $d = m_3 m_2$. *The polynomial*

$$g = f^T + z_5^2 + \ldots + z_n^2$$

*with* $f^T$ *the Berglund-Hübsch transpose of* $f$, *determines a link* $L_g$ *such that*

(1) *If* $n$ *is even, then* $L_g$ *is a rational homology* $(2n-1)$*-sphere.*
(2) *If* $n$ *is odd, then* $L_g$ *is a homotopy* $(2n-1)$*-sphere and* $\Delta_g(-1) = m_3$. *In particular the diffeomorphism type of* $L_g$ *is determined by* $m_3$.
(3) *The Sasaki cone of* $L_g$ *has dimension* $1 + \lfloor \frac{n-3}{2} \rfloor$ *and there are no extremal Sasaki metrics in the Sasaki cone of the link* $L_g$.
(4) *If* $m_3$ *is even, then for* $n = 5$, *the link* $L_g$ *has the form* $S^4 \times S^5$.

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