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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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Holomorphically Projective Mappings of Kähler Manifolds Preserving The Generalized Einstein Tensor

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Holomorphically projective mappings which preserved the Einstein tensor

$$E_{ij} = R_{ij} - \frac{Rg_{ij}}{n}$$

were studied in [1]. Preserving the stress-energy tensor

$$S_{ij} = R_{ij} - \frac{Rg_{ij}}{2}$$

by conformal mappings was explored in [3], [5]. It's worth for noting that in many classical issues e. g. [2, p. 359], just the latter is referred to as the Einstein tensor.

Let us refer to

$$\mathfrak{E}_{ij} \stackrel{\text{def}}{=} R_{ij} - \kappa Rg_{ij}. \quad (1)$$

as the **generalized Einstein tensor**. Here κ is a constant. Conformal mappings which preserving the introduced tensor were explored in [6].

It is known that a covariant vector ψ_i determining holomorphically projective mapping between two Kähler spaces (V^n, J, g) and (\bar{V}^n, J, \bar{g}) should satisfy the equations

$$\psi_{i,j} = \psi_i \psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{1}{n+2} (\bar{R}_{ij} - R_{ij}). \quad (2)$$

Here we denote by comma "," covariant derivative respect to the metric g of a space (V^n, J, g) . The affnor J_i^n is referred to as a *complex structure*. The structure is the same for both manifolds. The symbols R_{ij} and \bar{R}_{ij} denote Ricci tensors of spaces (V^n, J, g) and (\bar{V}^n, J, \bar{g}) respectively.

It follows from (38) that the deformation of the generalized Einstein tensor can be written as

$$\bar{\mathfrak{E}}_{ij} - \mathfrak{E}_{ij} = \bar{R}_{ij} - \kappa \bar{R}\bar{g}_{ij} - R_{ij} + \kappa Rg_{ij}. \quad (3)$$

Taking account of the preservation requirement, i. e. $\bar{\mathfrak{E}}_{ij} = \mathfrak{E}_{ij}$, from (38) we get

$$\bar{R}_{ij} - R_{ij} = \kappa \bar{R}\bar{g}_{ij} - \kappa Rg_{ij}. \quad (4)$$

Since (38) holds we can rewrite (38) as

$$\psi_{i,j} = \psi_i \psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{\kappa}{n+2} (\overline{Rg}_{ij} - Rg_{ij}). \quad (5)$$

Let us recall that $R = R_{ij} g^{ij}$.

Differentiating (38) covariantly with respect to x^k and the connection Γ which is compatible with the metric g of the manifold (V^n, J, g) , alternating in j and k and using the Ricci identity, we obtain the conditions of integrability:

$$\psi_\alpha W_{ijk}^\alpha = \frac{\kappa}{n+2} (\partial_k \overline{Rg}_{ij} - \partial_j \overline{Rg}_{ik} - \partial_k Rg_{ij} + \partial_j Rg_{ik}), \quad (6)$$

where

$$W_{ijk}^h \stackrel{\text{def}}{=} R_{ijk}^h + \frac{\kappa R}{n+2} (\delta_j^h g_{ik} - \delta_k^h g_{ij} - J_j^h J_{ik} + J_k^h J_{ij} - 2J_i^h J_{jk}). \quad (7)$$

Finally, we can summarize by the theorem

Theorem 1. *If manifolds (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ are in holomorphically projective correspondence and the mapping preserves the tensor $\mathfrak{E}_{ij} = R_{ij} - \kappa Rg_{ij}$, then the function ψ generating the mapping, must satisfy the system of PDE's (38) whose conditions of integrability are (38). Also, the tensor W_{ijk}^h is preserved by the mapping.*

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